Neutrinos in Cosmology

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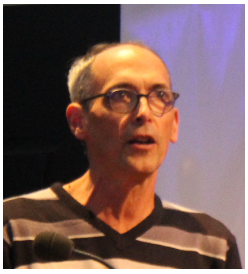

We summarize the role of neutrinos in the standard ΛCDM cosmology. Their most important job was to regulate neutron-proton thermal equilibrium in the early Universe, thus setting the initial conditions for primordial nucleosynthesis. More recently, some of them have become non-relativistic, slightly affecting the formation of structures, allowing one to use cosmological measurements to set limits on neutrino masses.

1 Introduction

In the standard ΛCDM cosmological model [1], today's Universe consists, in decreasing order of energy density, of vacuum energy, cold dark matter (CDM), baryonic matter, light neutrinos, and Cosmic Microwave Background (CMB) photons. In this model, the expanding Universe goes through a series of phases where, after an initial inflationary epoch, the energy density is successively dominated by relativistic particles (photons, neutrinos....), non-relativistic particles (baryons and cold dark matter, and vacuum energy (Λ). This history is summarized in Fig. 1 showing density vs time in a log-log plot. Important events are the formation of nuclei (nucleosynthesis) in the radiation epoch and the formation of atoms (recombination) early in the matter epoch. We live near the beginning (on a log scale) of the vacuum epoch.

The origin or the CDM, the baryons, and the vacuum energy is unknown but the CMB photons and neutrinos are well-understood products of electro-weak interactions in the hot early Universe. These interactions, when sufficiently rapid compared to the expansion rate, led to thermal (blackbody) spectra of photons and three species of neutrinos. At that time, the number density of neutrinos differed from that of photons only by a factor 3/4 coming from the fact that neutrinos are fermions rather than bosons.

With the dilution of the expanding Universe, the interaction rates eventually fell below the expansion rate, at which point collisions ceased and could no longer guarantee a thermal distribution. However, even in the absence of collisions, the massless photons retain their blackbody form, now at $T \sim 2.7\,\mathrm{K}$. The three neutrino species, $(\nu_e, \nu_\mu, \nu_\tau)$ decoherred into their mass eigenstates (ν_1, ν_2, ν_3), of which at least one is non-relativistic today. The non-relativistic neutrinos

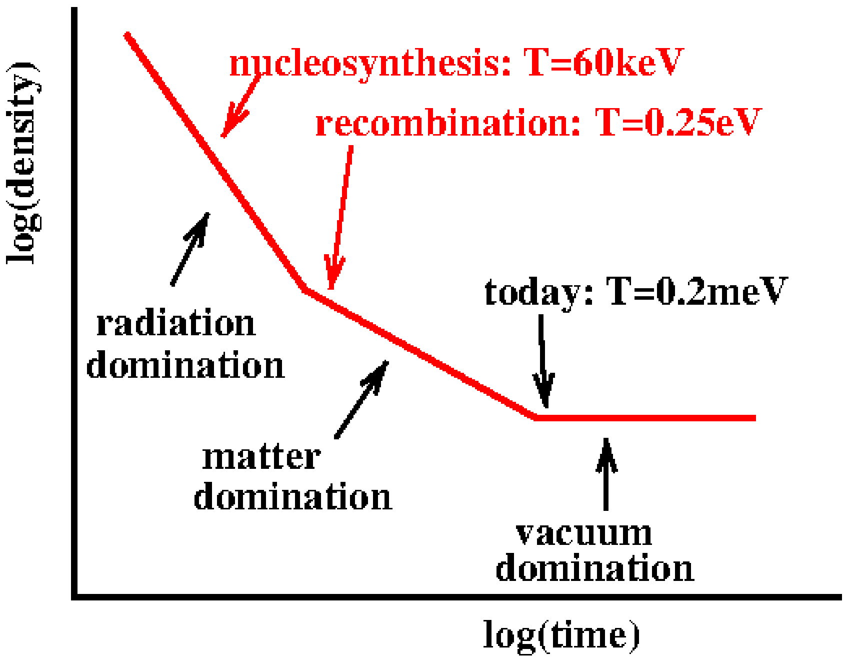

Figure 1 – This history of the ΛCDM Universe described by the density vs. time. After (presumably) an initial inflationary epoch, the Universe goes through successively epochs when the density is dominated by radiation, non-relativistic matter, and vacuum energy. Nucleosynthesis occurs during the radiation epoch (≈ 3 min after inflation) and recombination (formation of atoms) occurs during the matter epoch ($\approx 4\times10^5$ yr after inflation).

would no longer have a nearly uniform density because gravity would enhance there density in bound structures like the Milky Way.

In spite of their very weak interactions with other particles, neutrinos played an important role in cosmological history. In the early Universe, the presence of thermal neutrinos had two important effects

- At $T\sim$MeV reactions involving ν_e can change neutrons into protons and vice versa giving them an important role in primordial nucleosynthesis [3].
- The non-negligible energy density of the neutrinos increased the expansion rate, $H\propto\sqrt{G\rho}$ (where ρ is the total energy density), over that without neutrinos, lowering the time available for nucleosynthesis and atomic recombination.

At late times, the importance of neutrinos would become increasingly negligible except for the fact that they are massive and eventually become non-relativistic dark matter. This means that the present mass density is greater than what would be deduced from CMB observations that yield the mass density at recombination when only baryons and CDM were non-relativistic. Compared to the case when neutrinos are massless, this has two effects:

- The expansion rate is modified when neutrinos become non-relativistic, modifying the distance-redshift relation.
- Structure formation on scales below the neutrino free-streaming scale is diminished.

In the following two sections, we go into some details.

2 Relativistic neutrinos in the early Universe

The well-known cosmic microwave background (CMB) has a blackbody spectrum with a present temperature of ~ 2.7 K and an energy density $\propto T^4$ about 4 orders of magnitude less than the matter or vacuum densities. Going back in time, the increasing temperature means that thermal photons dominated the energy density for $T > 1\,eV$ ($t < 2 \times 10^4$ yr).

Though cosmological neutrinos are yet to be detected, the simple argument given here indicates that they would have been produced and reached a density corresponding to thermal equilibrium. General kinetic theory tells us that a species reaches this density if and only if the reactions that produce the species proceed at a rate that produces the required number of particles in a time corresponding to a significant change in the temperature. This is equivalent to the much quoted requirement of "reaction rate greater than expansion rate". At temperatures $T > m_e$, the rate for $\gamma\gamma \to e^+e^-$ was sufficiently high that blackbodies of electrons and positrons were created. The gas of e^+e^- can produce neutrinos of all three species through $e^+e^- \to \nu\bar{\nu}$. This has a production rate proportional to the number density of electrons and positrons ($\sim T^3$ for $T > m_e$) and the cross section ($\sim E^2 \sim T^2$ for $T < m_W$). The neutrino production thus goes like T^5 and so, going back in time, eventually becomes greater than the expansion rate $\propto \sqrt{G\rho} \propto T^2$. Putting in the numbers one finds that the interaction rate for neutrinos is greater than the expansion rate for $T >\sim 1\,\mathrm{MeV}$ ($t < 1$ sec). Thus, the standard requirement for thermal equilibrium is met at high temperature and the formation of a thermal distributions of neutrinos is assured, one for each neutrino species.

If neutrinos are Dirac particles, one could ask whether there is also a thermal distribution of "wrong helicity" neutrinos. In fact, the cross section for the production of such particles is so small that they would never have had a production rate greater than the expansion rate. So even if they exist as a state in in the model of particle physics, there is no reason to suppose that they are present in number comparable to that of the correct-helicity neutrinos.

For $T > 1\,\mathrm{MeV}$ ($t < 1$ sec), the temperature of the neutrinos is equal to that of the photons. After the neutrinos decouple, their temperature redshifts along with that of the photons, but the photons are "reheated" through "entropy" injection from the annihilation of the e^+e^- that occurs when the temperature drops below the electron mass. The fact that the neutrino temperature fell below that of the photons coupled with the fact that neutrinos are fermions with a slightly different thermal distribution than the bosonic photons means that the number density of neutrinos plus that of antineutrinos of each species is $3/11$ that of the photons. Today there are about 100 neutrinos and antineutrinos of each species per cm^3 compared to 400 photons. Of course the neutrinos have long since forgotten what reactions produced them so it is best to think of that as a number of neutrinos of a given mass, ν_1, ν_2, ν_3, rather than of a given flavor.

What of interest did these neutrinos do during the time they were still coupled to the other particles? Their most important role was to maintain a chemical equilibrium between neutrons and protons via reactions like:

$$\bar{\nu}_e p \leftrightarrow e^+ n \qquad \nu_e n \leftrightarrow e^- p \tag{1}$$

The rates for these reactions are greater than the expansion rate for $T >\mathrm{MeV}$ and maintain a neutron-proton ratio equal to the Boltzmann factor, $\exp(-(m_n - m_p)c^2/kT)$. This is illustrated in Fig. 2 showing the ratio as a function of temperature. For $T \gg (m_n - m_p) = 1.2\,\mathrm{MeV}$, the ratio is equal to unity and then declines steadily until the reaction rate falls below the expansion rate. After this "decoupling", the only thing left for neutrons to do is decay, which they do until they are incorporated into nuclei, first deuterium (^2H) and then ^3H, ^3He and, mostly, ^4He.

The original simulations in 1948 of nucleosynthesis in the early Universe by Alpher and Gamow[5] assumed a primordial Universe consisting of nucleons and photons. The high density of photons compared to nucleons in their model kept the nucleons unbound until the temperature dropped to $\sim 60\,\mathrm{keV}$, considerably below the binding energy of the deuteron, 2.2 MeV. This had the effect of giving neutrons time to decay (about three minutes), ensuring an excess of protons over neutrons and a Universe dominated by hydrogen. The inclusion of neutrinos in the cosmological mix by Hayashi[6] in 1950 and by Alpher, Folin and Herman[7] in 1953 further reduced the number of neutrons available by maintaining thermal equilibrium with protons until

$T \sim 1\,\mathrm{MeV}$ with the resulting Boltzmann factor. Whereas in the original Alpher-Gamow model, the efficiency for the production of heavy elements was mostly determined by the initial nucleon-photon ratio, inclusion of neutrinos made helium production mostly sensitive to the strength of the weak-interactions. As Hoyle and Taylor [8] put it in 1964, "the weak interaction cross-sections turn out to be just of the right order of magnitude for interesting effects to occur in the time-scale available", i.e. to create a universe that is neither entirely hydrogen, nor entirely heavier elements.

The fraction of primordial elements heavier than hydrogen (mostly ^4He) is observed to be about 22 % be weight. This measurement can be used to deduce the number of species of neutrinos that were in thermal equilibrium in the early Universe. The number of neutrons available for nucleosynthesis is determined by the number of neutrons that decay between decoupling ($T \sim \mathrm{MeV}$) and deuterium formation ($T \sim 60\,\mathrm{keV}$). Since the neutron lifetime is known (presumably it was the same in the early Universe), if we know how many nuclei were formed we can deduce the time between $T \sim \mathrm{MeV}$ and $T \sim 60\,\mathrm{keV}$ and thus the expansion rate. The Friedman equation gives the expansion rate in terms of the energy density so we can deduce the latter which, at the time of nucleosynthesis, is due to photons and neutrinos. We can thus deduce the number of neutrino species in thermal equilibrium. This idea was applied as early as 1964 by Hoyle and Taylor [8]: "Similarly, if there were more than two kinds of neutrino the expansion would have to be faster.....(and) the larger the ratio of He/H turns out to be." At the time, the amount of primordial helium was controversial and estimates varied by a factor two. However, one could still deduce $N_\nu < 16$, though this appears to be a conclusion of the Particle Data Group [4] rather than explicitly of Hoyle and Taylor. By 1977 the primordial abundances were settling down to their modern values and Steigman, Schramm and Gunn [9] deduced $N_\nu < 5$. A recent analysis [10] gives

$$2.67 < N_\nu < 3.85 \quad (68\% CL) \tag{2}$$

Before the high energy studies of $e^+e^- \to Z^0 \to \nu\bar{\nu}$ or $\nu\bar{\nu}\gamma$ at high-energy colliders, the limit on the number of neutrino species coming from particle physics experiment was very weak. The first search for $e^+e^- \to \nu\bar{\nu}\gamma$ at PEP (Stanford collider) [11] gave $N_\nu < 17$. Eventually, LEP (Large Electron Positron collider at CERN) experiments confirmed the expected number of three, and the Particle Data Group [4] now gives $N_\nu = 2.9840 \pm 0.0082$. It should however be emphasized that the cosmological limit is more general, referring to any particle with a relativistic black body spectrum. The LEP limit refers only to light particles coupling with full strength to the Z^0 boson.

The nucleosynthesis limit on the number of neutrino species comes from using the neutron-decay "clock" to measure the expansion rate at the time of nucleosynthesis. A similar measurement can by made using CMB anisotropies by using the recombination process as a clock.

Before recombination, photons are tightly coupled to the electrons through Compton scattering so the photon mean-free path is very small (compared to, say, the Hubble length at the time). As the electrons start to combine with protons to form hydrogen, the photon mean free path increases and they random walk from hot regions to cold regions, This smears out the temperature map one sees (Fig. 3), so no temperature inhomogeneities are seen below a certain scale, about 0.1deg. This scale depends on the expansion rate at recombination: the larger the expansion rate, the faster recombination and the less time for photon random walking. The observed blurring scale allowed the Planck collaboration [2] to estimate the expansion rate and deduce the number of neutrinos:

$$N_\nu = 2.94 \pm 0.38 \tag{3}$$

This confirms and improves on the nucleosynthesis limit using a different clock at a different time. It means that any other relativistic degree of freedom could not have been in thermal equilibrium. It conflicts with the implications of the "reactor anomaly" which imply a fourth neutrino mixing with the ν_e with a mixing angle $\sim 10^{-2}$. This angle is sufficiently large to ensure that the fourth neutrino would have thermalized in the early Universe, yielding $N_\nu = 4$.

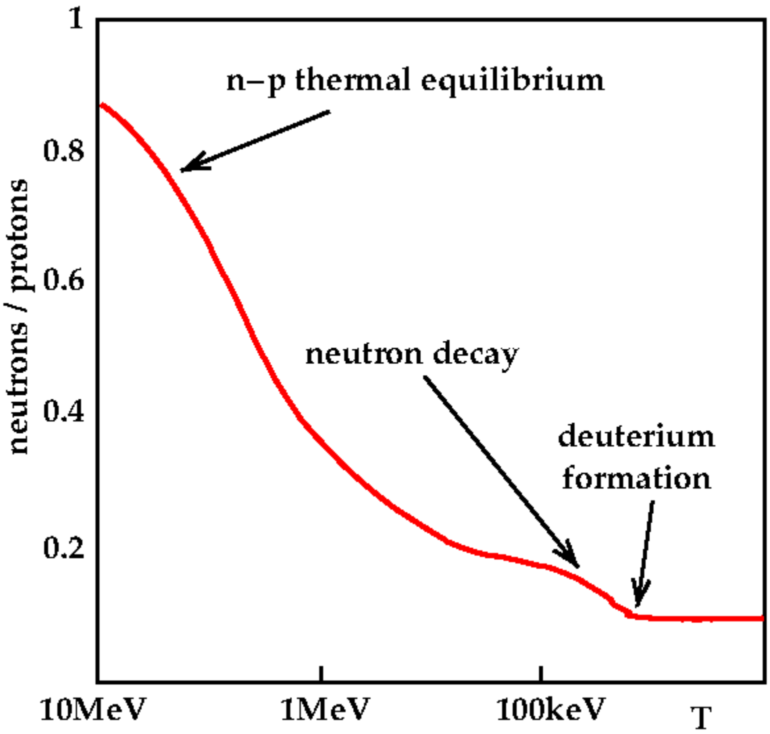

Figure 2 – The neutron-proton ratio vs temperature during the epoch of nucleosynthesis. Neutrons and protons are in thermal equilibrium for $T <$ MeV. When the rate of weak-interaction reactions falls below the expansion rate, only neutron decay remains active which continues until $T \sim 60\,\mathrm{keV}$ ($t \approx 3$ min) when the remaining neutrons combine with protons to form deuterium. Most of this deuterium ends up combining to form ^4He.

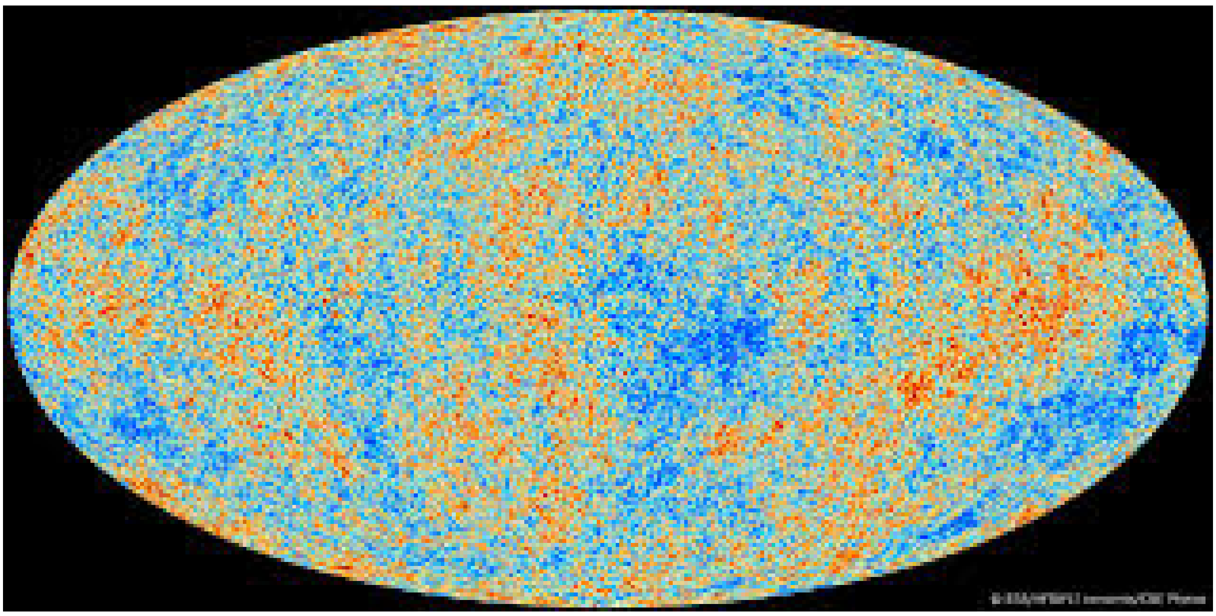

Figure 3 – Sky map of the CMB temperature measured by the Planck satellite showing hot and cold regions. Structures appear on scales $> 0.1\,\mathrm{deg}$, below which photon random walks on the last-scattering surface (and the Planck telescope angular resolution) blur the image.

3 Massive neutrinos and structure formation

The relativistic neutrinos in the early Universe eventually become non-relativistic if they are massive. The idea that a massive neutrino species could constitute the dark matter of the Universe goes back at least to Gershtein and Zel'dovich [12]. Since the number density of each species is fixed at $n \sim 100\ cm^{-3}$, the present mass density of a species of mass m_ν is

$$\Omega_\nu = \frac{\rho_\nu}{3H_0^2/8\pi G} = h^{-2}\frac{m_\nu}{91.5\,eV} \tag{4}$$

where the denominator is the critical density separating open and closed Universes and h is the normalized present expansion rate (Hubble constant), $H_0 = h \times 100\,\mathrm{km\,sec^{-1}Mpc^{-1}}$. For $\Omega_M \sim 0.3$ and $h \sim 0.7$, a neutrino of mass $m_\nu \sim 15\,\mathrm{eV}$ would do the trick.

Neutrino dark matter is called "hot dark matter" in distinction with the more popular candidate "cold dark matter". Hot dark matter is relativistic when they stop interacting with other particles, $T \sim \mathrm{MeV}$ for neutrinos. After decoupling, inhomogeneities of the neutrino density present at decoupling would be smeared out the as neutrinos randomly move about at the speed of light. This movement ceases when they become non-relativistic and the distance they can travel between decoupling and this moment defines the "free streaming" scale. Below this scale, initial inhomogenities are erased. For a neutrino of mass $\sim 10\,\mathrm{eV}$, this scale is about 10 Mpc today. The amount of matter contained in the corresponding volume is about that of a large cluster of galaxies. Initial homogenities containing an amount of matter corresponding to a galaxy would have been erased. Today's galaxies would only have been formed from the fragmentation of larger structures. This is in contrast with CDM structure formation where small-scale inhomogenities are not erased and the small structures for first, later merging to form larger clusters. CDM structure formation is therefore called "bottom-up" compared to the "top-down" hot dark matter scenario.

As late as the 1990's, the lack of quality data on the spectrum of present-day inhomogeneities and the absence of data on CMB anisotropies allowed one to take seriously the hot-dark matter scenario, even without a well developed scenario of how clusters fragment to galaxies. This possibility was a primary motivation [13] for searches for $\nu_\mu - \nu_\tau$ oscillations with $(\nu_3 - \nu_2)^2 \sim 100\,eV^2$ and $(\nu_{\tau,\nu} \sim \nu_{3,2})$. Oscillations at this mass scale were not found, in particular by the NOMAD [14] experiment at CERN. Instead, atmospheric oscillations pointed to a lower mass scale for $m_3 - m_2$ and solar neutrino neutrino experiments gave an even lower value for $m_2 - m_1$. Combined with decreasing limits on the absolute value of m_1 from tritium end-point experiments strongly suggested that hot dark matter could not be the standard neutrinos. At any rate, the large 21$^{\mathrm{st}}$ century redshift surveys and CMB anisotropies measurements clearly pointed toward a CDM model.

Even if most of the dark matter does not consist of neutrinos, the non-zero neutrino masses implied by neutrino oscillation experiments imply that at least one neutrino species is non-relativistic today. This would change slightly the redshift-distance relation and the spectrum of today's inhomogenities from what would be expected in the absence of neutrino masses. Basically, the free-streaming of the neutrinos before becoming non-relativistic would decrease today's inhomogenities over what would be they would be in the absence of a mass.

To derive a limit on the mass, one procedure is to use the observed CMB temperature anisotropies to predict the low-redshift inhomogenity spectrum as a function of neutrino mass. The spectrum of small-scale inhomogenities can be derived from the fluctuations of absorption in the Lyα forest of quasars using techniques pioneered by McDonald [15]. A recent analysis using quasar spectra from the SDSS [16] gave a limit

$$\sum m_\nu < 0.012\,eV \qquad 95\% CL \tag{5}$$

where the sum is over all species. Future observations should yield a limit which can clearly distinguish between the normal and inverse ordering of neutrino masses implied by oscillation data. In the former case there is one heavy species with $m_\nu \sim 0.006\,\mathrm{eV}$ and in the latter case there are two species.

Intermediate between hot and cold dark matter is “warm dark matter” that decoupled at a time such their free-streaming length is intermediate between the $\sim 10\,\mathrm{Mpc}$ scale of galaxy clusters and the $\sim 10\,\mathrm{kpc}$ scale of small galaxies. Such would be the case for neutrinos if they have $m_\nu \sim$keV. For $n_\nu \sim 100\,cm^{-3}$, this would give a present-day density far in excess of observations so such keV neutrino dark matter must not have been in thermal equilibrium. It could have been produced by weakly coupled oscillations as originally suggested by Dodelson and Widrow [17]. Such neutrinos are subject to radiative decays, $\nu_4 \to \nu_3\gamma$, and have been hinted at in astronomical spectra around $E_\gamma \sim 3.55\,\mathrm{keV}$ implying $m_\nu \sim 7.1\,\mathrm{keV}$. Constraints on such scenarios have been derived from the spectrum of inhomogeneities seen in the spectra of Lyα forests [18].

4 Cosmological neutrino detection

We conclude this review with a short discussion of the possibility of detecting cosmological relic neutrinos [19]. Because of their very low energies, they can only be detected by neutral-current reactions or by charged-current reactions using radioactive targets. A possible scheme was proposed by Weinberg [20] through the reaction

$$\nu + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^- \tag{6}$$

The final state electron will have an energy $E_e \sim (m_H + m_\nu) - (m_{He} + m_e) \sim 17\,\mathrm{keV}$. A 100 gram tritium target should have about 10 such events per year. Of course tritium is radioactive and there will be $\sim 10^{23}$ tritium decays for each neutrino capture. The electron from neutrino capture will have an energy $\sim m_\nu$ greater than the endpoint of tritium decay. Good energy resolution is thus necessary to distinguish capture electrons from decay electrons. The Ptolemy project [21] is investigating the experimental possibilities.

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