Horava-Lifshitz cosmology revisited

Shinji Mukohyama
(YITP, Kyoto U)

Based on arXiv:1709.07084 (PRD97, 043512 (2018)) w/
S.Bramberger, A.Coates, J.Magueijo, R.Namba, Y.Watanabe
Also on CQG27 (2010) 223101 & JCAP0906 (2009) 001

Implication of GW170817 on gravity theories @ late time

- $|(c_{gw} - c_\gamma)/ c_\gamma| < 10^{-15}$
- Horndeski theoy (scalar-tensor theory with 2nd-order eom):
  Among 4 free functions, $G_4(\phi,X)$ & $G_5(\phi,X)$ are strongly constrained. Still $G_2(\phi,X)$ & $G_3(\phi,X)$ are free. $X = -\partial^\mu\phi\partial_\mu\phi$
- Generalized Proca theory (vector-tensor theory):
  Among 6 (or more) free functions, $G_4(X)$ & $G_5(X)$ are strongly constrained. Still $G_2(X,F,Y,U)$, $G_3(X)$, $G_6(X)$, $g_5(X)$ are free. $X = -A^\mu A_\mu$
- Horava-Lifshitz theory (renormalizable quantum gravity):
  The coefficient of $R^{(3)}$ is strongly constrained
  → IR fixed point with $c_{gw} = c_\gamma$? How to speed up the RG flow?
- Ghost condensation (simplest Higgs phase of gravity):
  No additional constraint
- Massive gravity (simplest modification of GR):
  Upper bound on graviton mass ≈ 10^{-22}eV
  Much weaker than the requirement from acceleration

c.f. "All" gravity theories (including general relativity):
The cosmological constant is strongly constrained ≈ 10^{-120}.

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Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

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- **Scaling dim of ϕ**

  $t \rightarrow b\, t$ ($E \rightarrow b^{-1} E$)

  $x \rightarrow b\, x$

  $\phi \rightarrow b^s\, \phi$

  $1+3-2+2s = 0$

  $s = -1$

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$$I \supset \int dt dx^3 \dot{\phi}^2 \qquad \int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

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  $n \leq 4$

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- Renormalizability

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- Gravity is highly non-linear and thus non-renormalizable

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- **Gravity becomes renormalizable!?**

Horava-Lifshitz gravity

- HL gravity realizes **z=3 scaling @ UV** and thus is power-counting renormalizable
- **Renormalizability was recently proved with any number of spacetime dimensions** [Barvinsky, et al. 2016]
- Ostrogradsky ghost is absent and thus HL gravity is **likely to be unitary**
- **In 2+1 dimensions HL gravity is asymptotically free.**
- Lorentz-invariance is broken @ UV
- **Lorentz-invariant IR fixed-point** is generic [Chadha & Nielsen 1983] (and may apply to GW as well; cf. $|c_{gw}^2 - c_\gamma^2| < 10^{-15}$ from GW170817) but running is slow (logarithmic)
- SUSY or/and strong dynamics can speed-up the RG running towards Lorentz-invariant IR fixed-point

Cosmological implications

Horava-Lifshitz Cosmology: A Review, arXiv: 1007.5199

- The z=3 scaling solves the horizon problem and leads to (almost) scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The initial condition with z=3 scaling may actually solve the flatness problem. (Bramberger, Coates, Magueijo, Mukohyama, Namba and Watanabe 2017)
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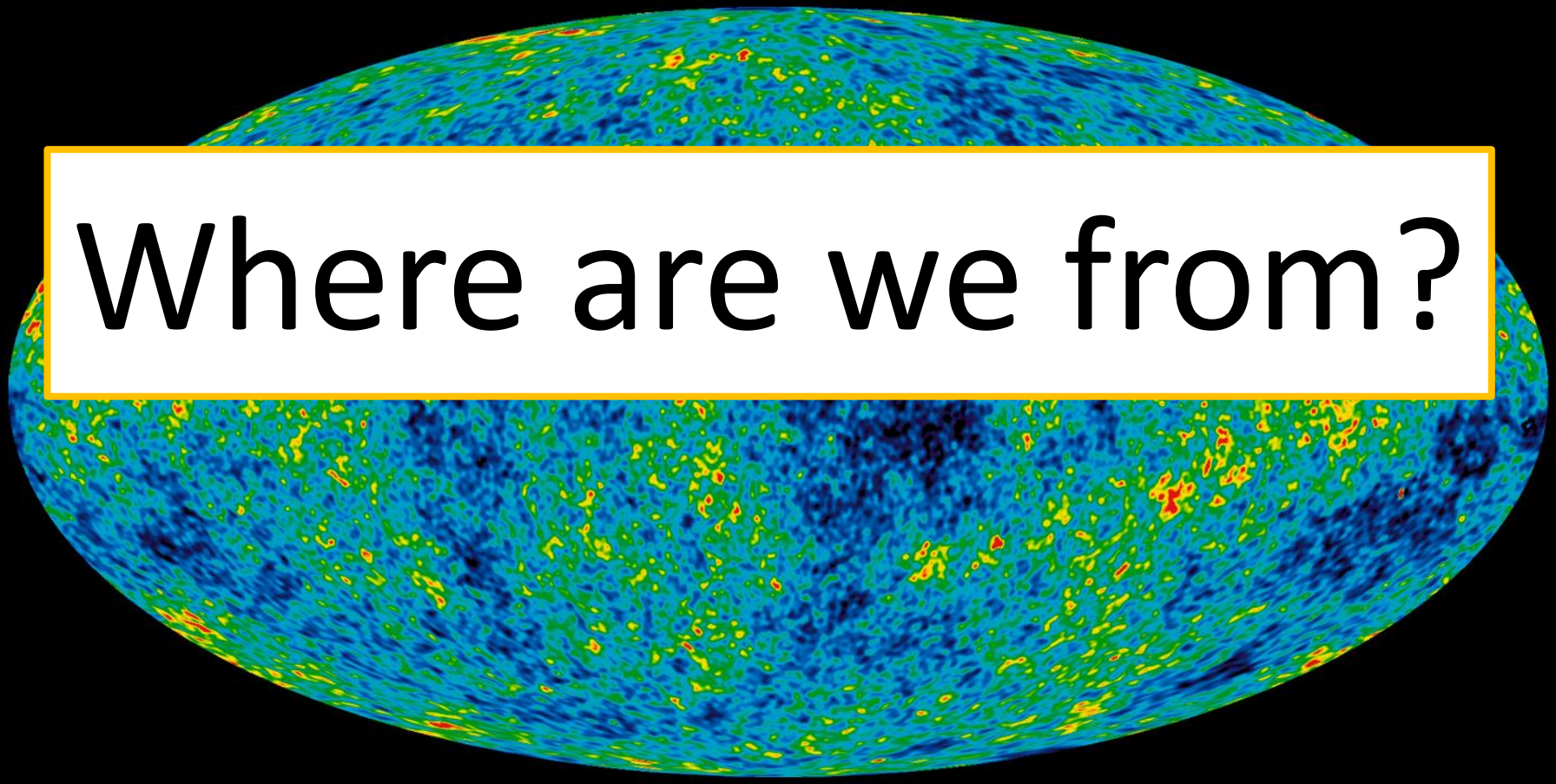
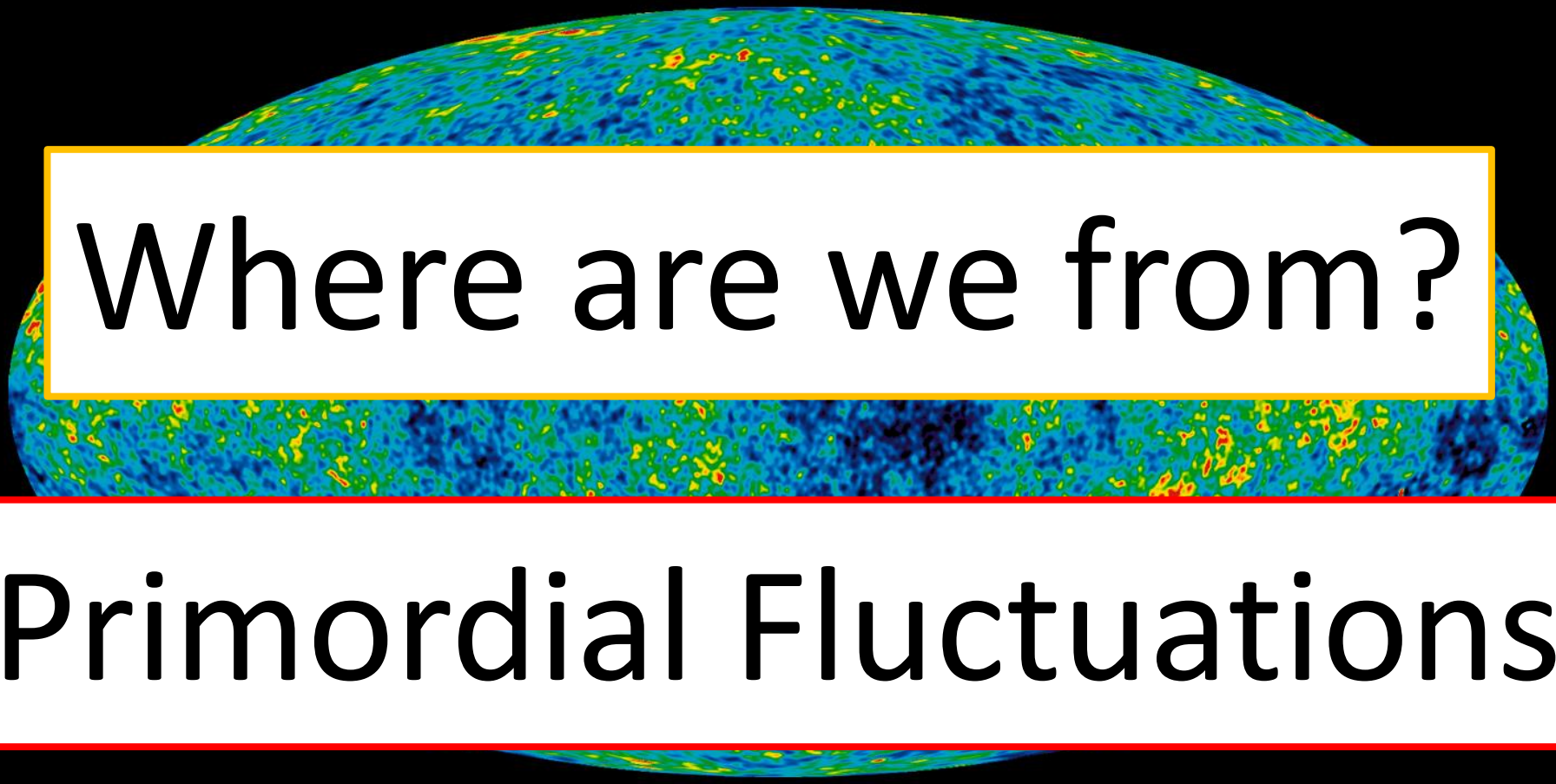


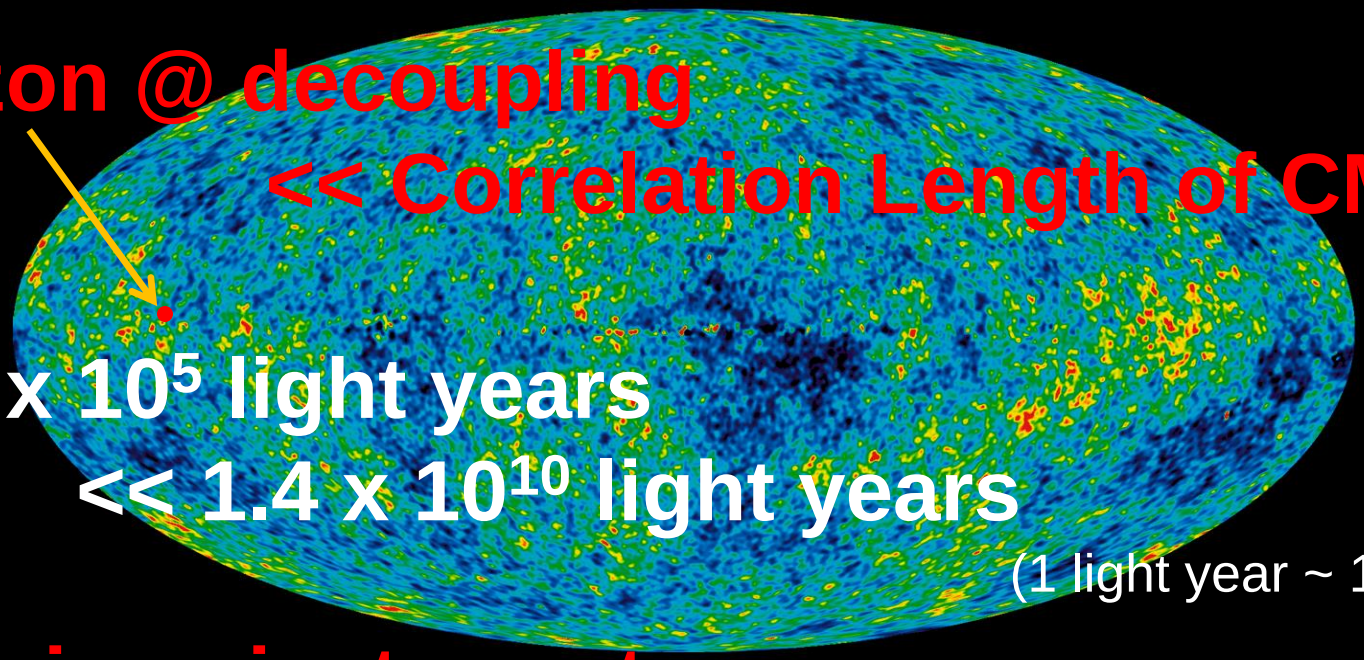
Where are we from?



Where are we from?

Primordial Fluctuations

Horizon Problem & Scale-Invariance



Horizon @ decoupling

<< Correlation Length of CMB

3.8×10^5 light years

<< 1.4×10^{10} light years

(1 light year ~ 10^{18} cm)

Scale-invariant spectrum

$\Delta \sim$ constant

$$\left\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \right\rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') \frac{\Delta}{|\vec{k}|^3}$$

Usual story

- $\omega^2 >> H^2$: oscillate $\qquad H = (da/dt) / a$

  $\omega^2 << H^2$: freeze $\qquad a$: scale factor

  oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

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- Scaling law

  $t \rightarrow b\,t$ ($E \rightarrow b^{-1}E$)

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  $\phi \rightarrow b^{-1}\,\phi$

  $\Rightarrow \quad \delta\phi \propto E \sim H$

  Scale-invariance requires almost const. H, i.e. inflation.

New story with z=3

Mukohyama 2009

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  OK for $a \sim t^p$ with $p > 1/3$

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  Scale-invariant fluctuations!

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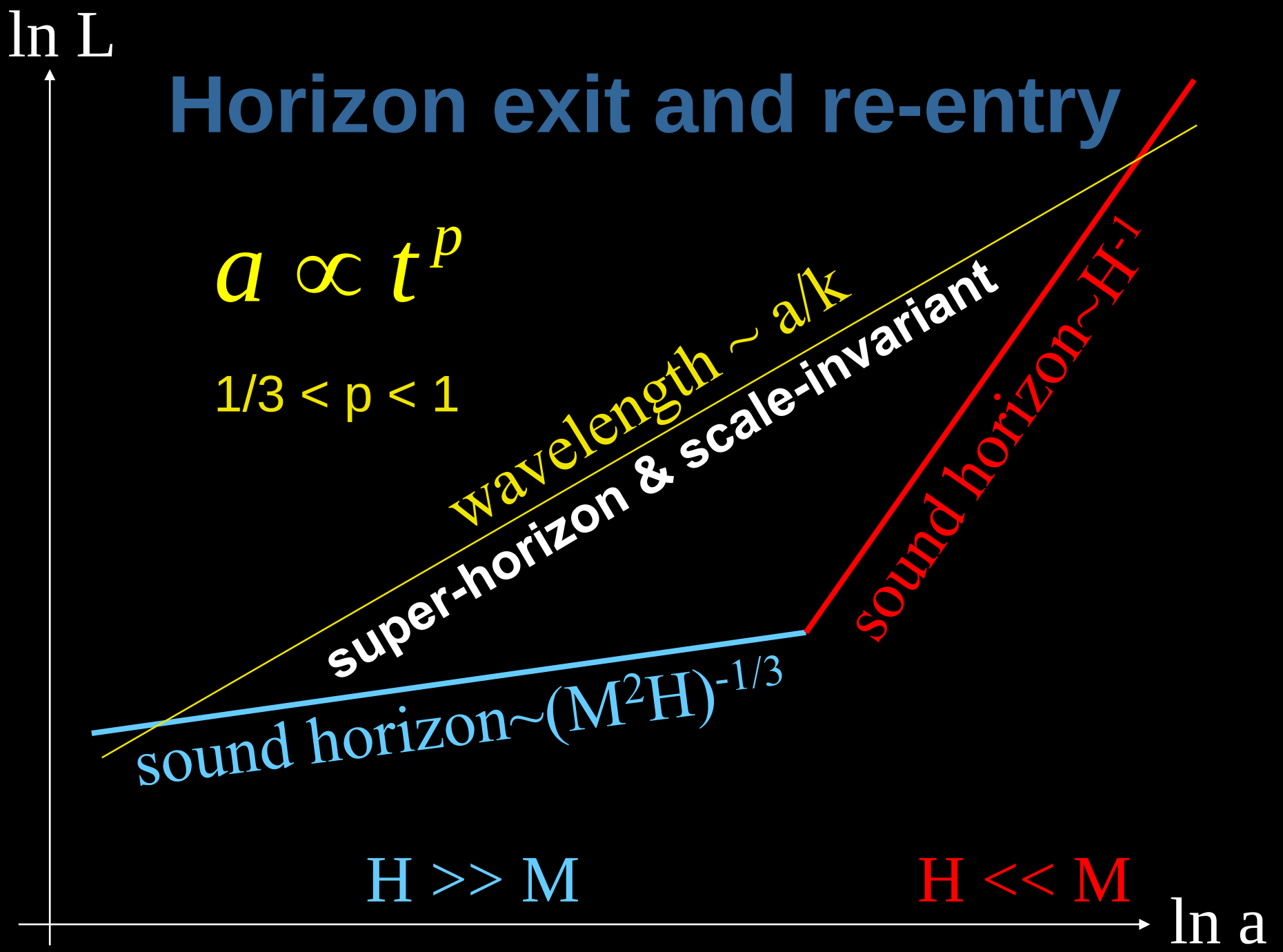
  Scale-invariant fluctuations!
- Tensor perturbation $P_h \sim M^2/M_{Pl}^2$

Horizon exit and re-entry



New Quantum Gravity

New Mechanism of Primordial Fluctuations

- ✔ **Horizon Problem Solved**
- ✔ **Scale-Invariance Guaranteed**
- ✔ **Slight scale-dependence calculable**
- ✔ **Predicts relatively large non-Gaussianity**

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“Vainshtein screening” in projectable (N=N(t)) HL gravity

- Perturbative expansion breaks down in the $\lambda \rightarrow 1+0$ limit.

$$L_{kin} = K^{ij}K_{ij} - \lambda K^2$$

- Non-perturbative analysis shows continuity and GR is recovered in the $\lambda \rightarrow 1+0$ limit.

Screening scalar graviton

$$L = \left[f\left(\frac{\zeta}{\lambda - 1}\right) + g(\zeta, \lambda) \right] \frac{M_{Pl}^2 \dot{\zeta}^2}{\lambda - 1} - V(\zeta, D_i) \quad + \text{matter}$$

subleading

Independent of λ
No time derivative

Local in time, no time derivative
Non-local in space, each term has the same # of spatial derivatives in denominator and numerator

λ → 1

$$\boxed{L \sim \dot{\zeta}_c^2} \quad + \text{matter}$$

"Canonically normalized" scalar graviton decouples from the rest of the world.

Analogue of Vainshtein screening

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  - ✓ Spherically-sym, static, vacuum (Mukohyama 2010)
  - ✓ Spherically-sym, dynamical, vacuum (Mukohyama 201?)
  - ✓ Spherically-sym, static, with matter (Mukohyama 201?)
  - ✓ General super-horizon perturbations with matter (Izumi-Mukohyama 2011; Gumrukcuoglu-Mukohyama-Wang 2011)

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  - ✓ General super-horizon perturbations with matter (Izumi-Mukohyama 2011; Gumrukcuoglu-Mukohyama-Wang 2011)
- "Vainshtein radius" can be pushed to infinity in the $\lambda \to 1+0$ limit.

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cc & flatness problems

$$3H^2 = 8\pi G\rho - \frac{3K}{a^2} + \Lambda$$

- Λ does not decay → cc problem "Why is Λ as small as $8\pi G\rho$ now?"
- K/a^2 decays but only slowly → flatness problem "Why is K/a^2 smaller than $8\pi G\rho$ now?"

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We shall consider the flatness problem.

Two ways to tackle flatness problem

$$3H^2 = 8\pi G\rho - \frac{3K}{a^2}$$

- If ρ does not decay for an extended period then flatness problem solved → **Inflation**
- If $K/a^2 << 8\pi G\rho$ initially then flatness problem solved → **Quantum cosmology**

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We shall consider the second possibility.

Usual story

- Initial condition set by e.g. quantum tunneling
- O(4) symmetric instanton
  → **$T \sim L$, where $T \sim 1/H$, $L \sim a/|K|^{1/2}$**
- Three terms in $3H^2 = 8\pi G\rho - 3K/a^2$
  are of the same order initially.
- **Flatness problem exists unless inflation occurs.**

New story with z=3

- Initial condition set by e.g. quantum tunneling
- Instanton with z=3 anisotropic scaling, which we call an anisotropic instanton
  → **$T \propto L^3$, where $T \sim 1/H$, $L \sim a/|K|^{1/2}$**
  → $T \sim M^2 L^3$
- $T \ll L$ if $L \ll 1/M$
- **Flatness problem may be solved if the anisotropic instanton is small.**

Summary

- Horava-Lifshitz gravity is renormalizable and likely to be unitary, and thus is a candidate for UV complete theory of quantum gravity.
- Lorentz-invariance can be restored at IR fixed-point. SUSY or/and strong dynamics can speed-up the RG running to match with phenomenology.
- It is likely that GR (+DM) is recovered in the $\lambda \to 1$ limit due to nonlinear effects. [c.f. Vainshtein effect]
- Horizon problem can be solved and (almost) scale-invariant cosmological perturbations can be generated without inflation.
- Flatness problem can be solved by equipartition in highly trans-Planckian regime.

Gravitational collapse in Einstein-aether theory

Shinji Mukohyama
(YITP, Kyoto U)

Based on arXiv:1806.00142 w/ M.Bhattacharjee, M.-B. Wan and A.Wang

Einstein-aether theory

- LV gravity: GR + timelike unit vector u^μ
  → Useful for test of GR
- Low-E limit of non-projectable HL gravity
  → predictions of a quantum gravity theory
- 4 parameters (c_1, c_2, c_3, c_4)

$$\boxed{M^{\alpha\beta}{}_{\mu\nu} = c_1 g^{\alpha\beta} g_{\mu\nu} + c_2 \delta^\alpha_\mu \delta^\beta_\nu + c_3 \delta^\alpha_\nu \delta^\beta_\mu - c_4 u^\alpha u^\beta g_{\mu\nu}}$$

$$\mathcal{L}_{æ} \equiv -M^{\alpha\beta}{}_{\mu\nu} \left(D_\alpha u^\mu\right)\left(D_\beta u^\nu\right) + \lambda\left(g_{\alpha\beta} u^\alpha u^\beta + 1\right)$$

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int \sqrt{-g}\, d^4x \left[R(g_{\mu\nu}) + \mathcal{L}_{æ}\left(g_{\mu\nu}, u^\lambda\right)\right]$$

$$S = S_{æ} + \int \sqrt{-g}\, d^4x \left[\mathcal{L}_m\left(g_{\mu\nu}, \psi\right)\right]$$

Some formulae

$$c_{ij} \equiv c_i + c_j \qquad c_{ijk} = c_i + c_j + c_k$$

- Stability & Gravi-Cerenkov & GW170817

$$q_S = \frac{(1-c_{13})(2+c_{13}+3c_2)}{c_{123}} \qquad q_V = c_{14} \qquad q_T = 1-c_{13}$$

$$c_S^2 = \frac{c_{123}(2-c_{14})}{c_{14}(1-c_{13})(2+c_{13}+3c_2)} \qquad c_V^2 = \frac{2c_1 - c_{13}(2c_1-c_{13})}{2c_{14}(1-c_{13})} \qquad c_T^2 = \frac{1}{1-c_{13}}$$

- BBN

$$G_N = \frac{G_{æ}}{1-\frac{1}{2}c_{14}} \qquad G_{\rm cos} = \frac{G_{æ}}{1+\frac{1}{2}(c_{13}+3c_2)}$$

- PPN

$$\alpha_1 = -\frac{8(c_3^2+c_1c_4)}{2c_1-c_1^2+c_3^2}$$

$$\alpha_2 = \frac{1}{2}\alpha_1 - \frac{(c_1+2c_3-c_4)(2c_1+3c_2+c_3+c_4)}{c_{123}(2-c_{14})}$$

- Pulsars

$$\hat{\alpha}_1 = \alpha_1 + \frac{c_-(8+\alpha_1)\sigma_{æ}}{2c_1}$$

$$\hat{\alpha}_2 = \alpha_2 + \frac{\hat{\alpha}_1-\alpha_1}{2} - \frac{(c_{14}-2)(\alpha_1-2\alpha_2)\sigma_{æ}}{2(c_{14}-2c_{13})}$$

Constraints on (c_1, c_2, c_3, c_4)

arXiv: 1802.04303 w/ Jacob Oost & Anzhong Wang

- Strongest constraint ← GW170817

$$|c_{13}| < 10^{-15}$$

$$c_{ij} \equiv c_i + c_j$$

$$c_{ijk} = c_i + c_j + c_k$$

- (c_1, c_{14})-plane

$$0 < c_{14} \leq 2.5 \times 10^{-5}$$

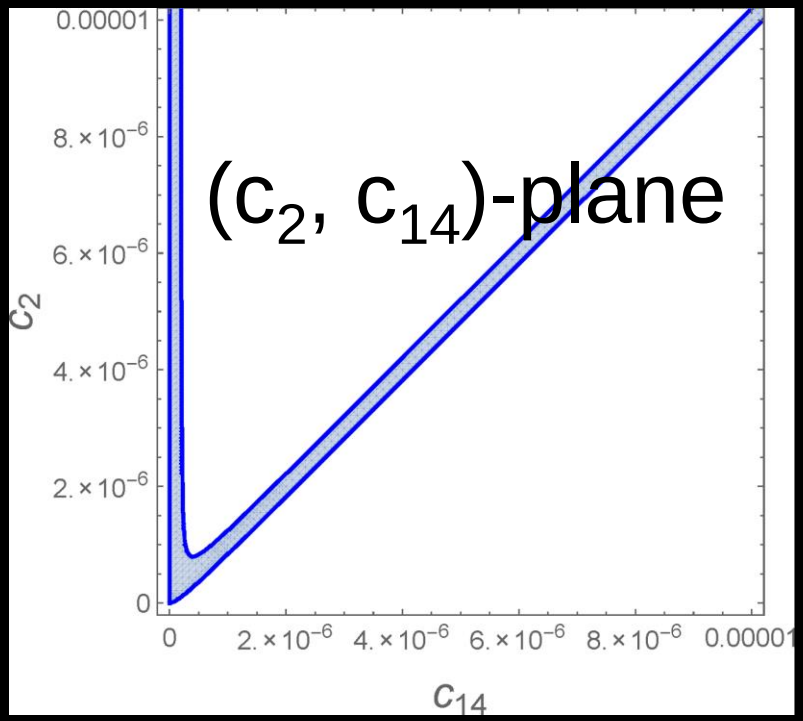
$$c_{14} \lesssim c_1$$

- (c_2, c_{14})-plane

$$c_{14} \leq 2.5 \times 10^{-5}$$

$$c_2 \lesssim 0.095$$

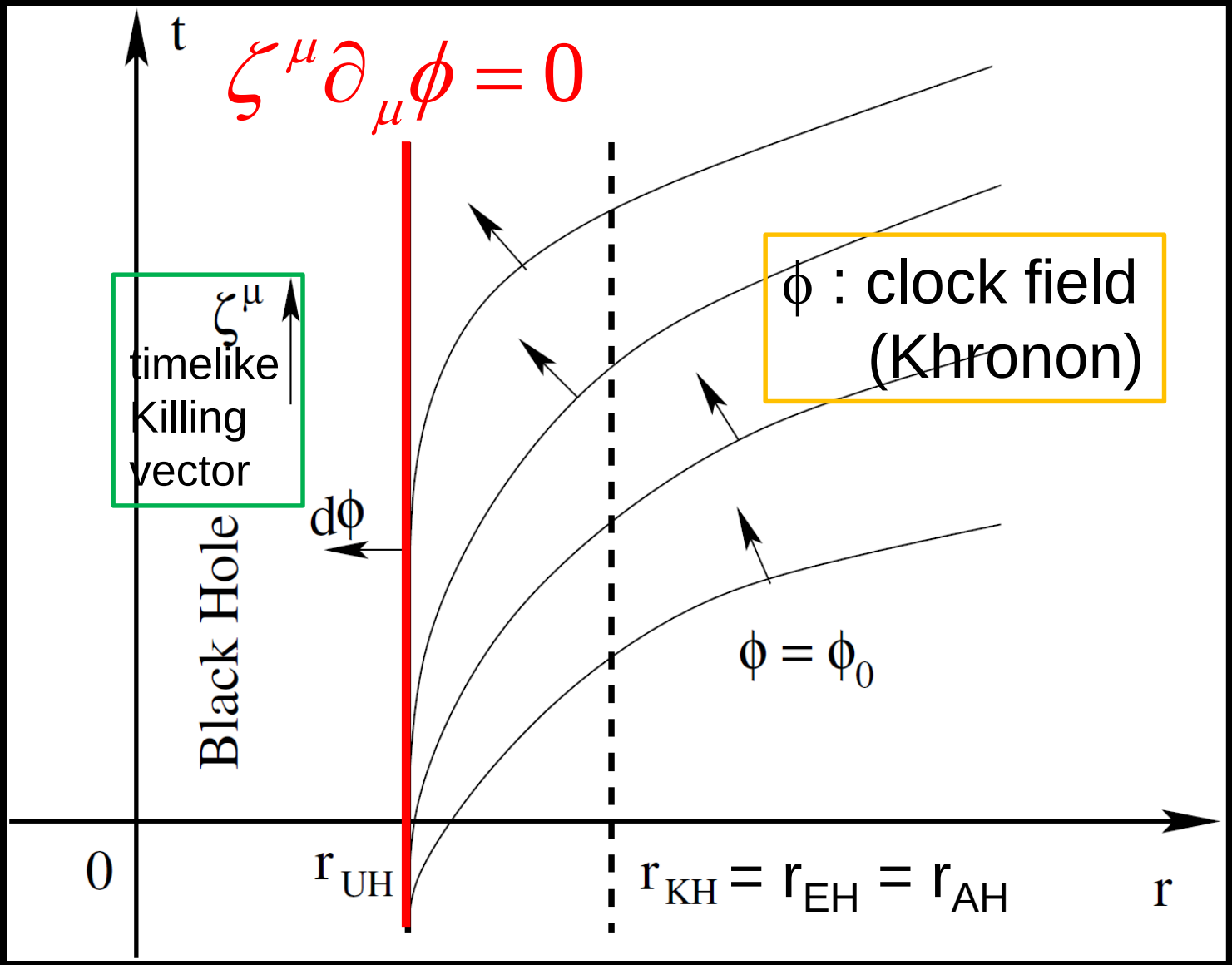
- Sensitivity $\sigma_{æ}$ not known in those range



BH in LV gravity theories

- Light cone structure is only emergent @ IR
- Usual event/apparent horizons are also emergent @ IR
- Causal structure in the sense of past & future still exists → clock field (Khronon).
- There may still be the absolute causal boundary, called "universal horizon".

Universal horizon for static BH



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- There may still be the absolute causal boundary, called "universal horizon".
- Does "universal horizon" form from gravitational collapse? → Numerical study of massless scalar with spherical symmetry

Dynamical universal horizon?

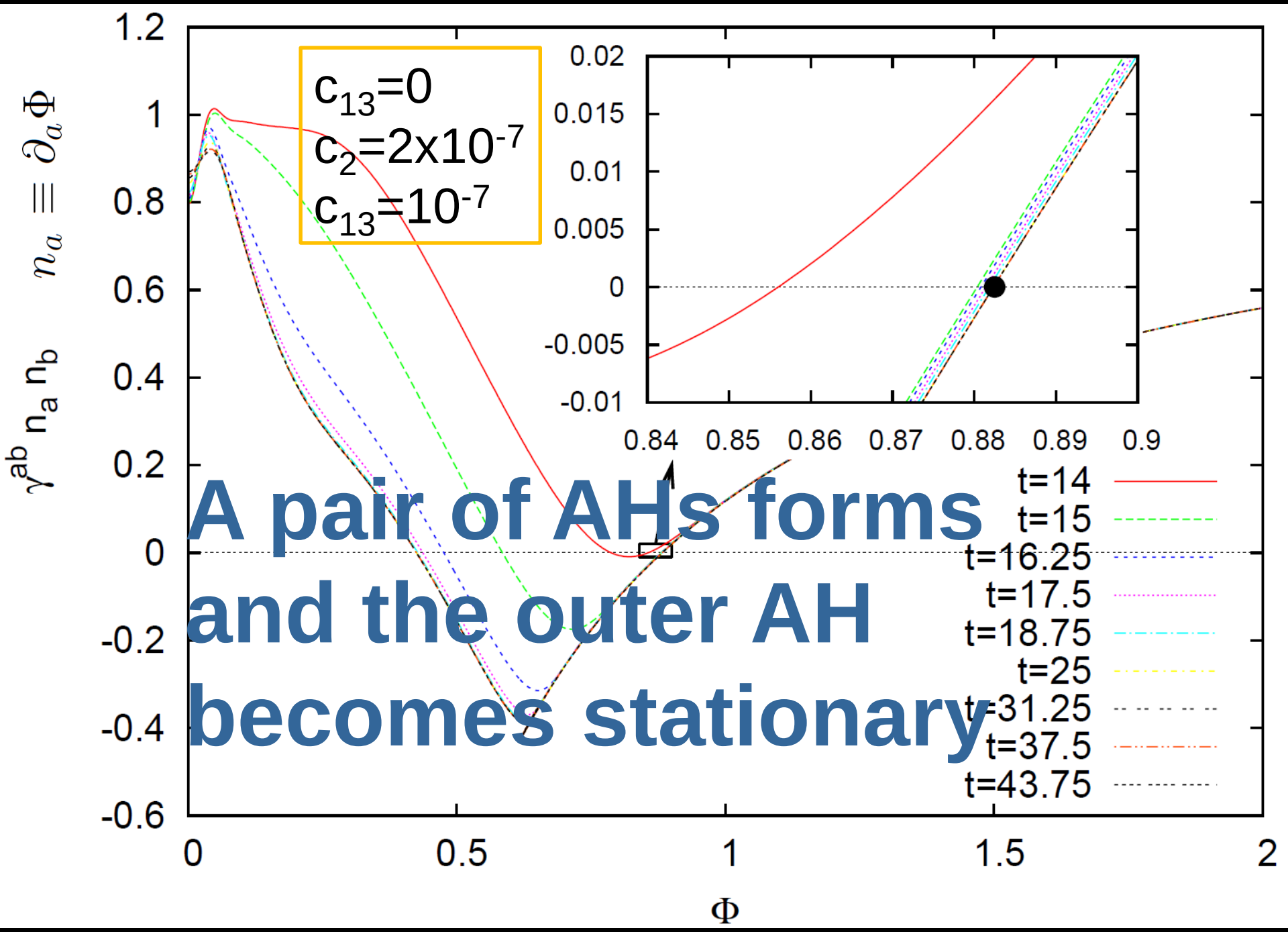
- For static BHs, a "universal horizon" is characterized by $\zeta^{\mu}\partial_{\mu}\phi = 0$, where ζ^{μ} is the timelike Killing vector and ϕ is the clock field (Khronon).
- A natural extension of ζ^{μ} to spherically symmetric dynamical spacetime

$$ds^2 = \gamma_{ab}dx^a dx^b + \Phi^2\left(d\theta^2 + \sin\theta^2 d\varphi^2\right)$$

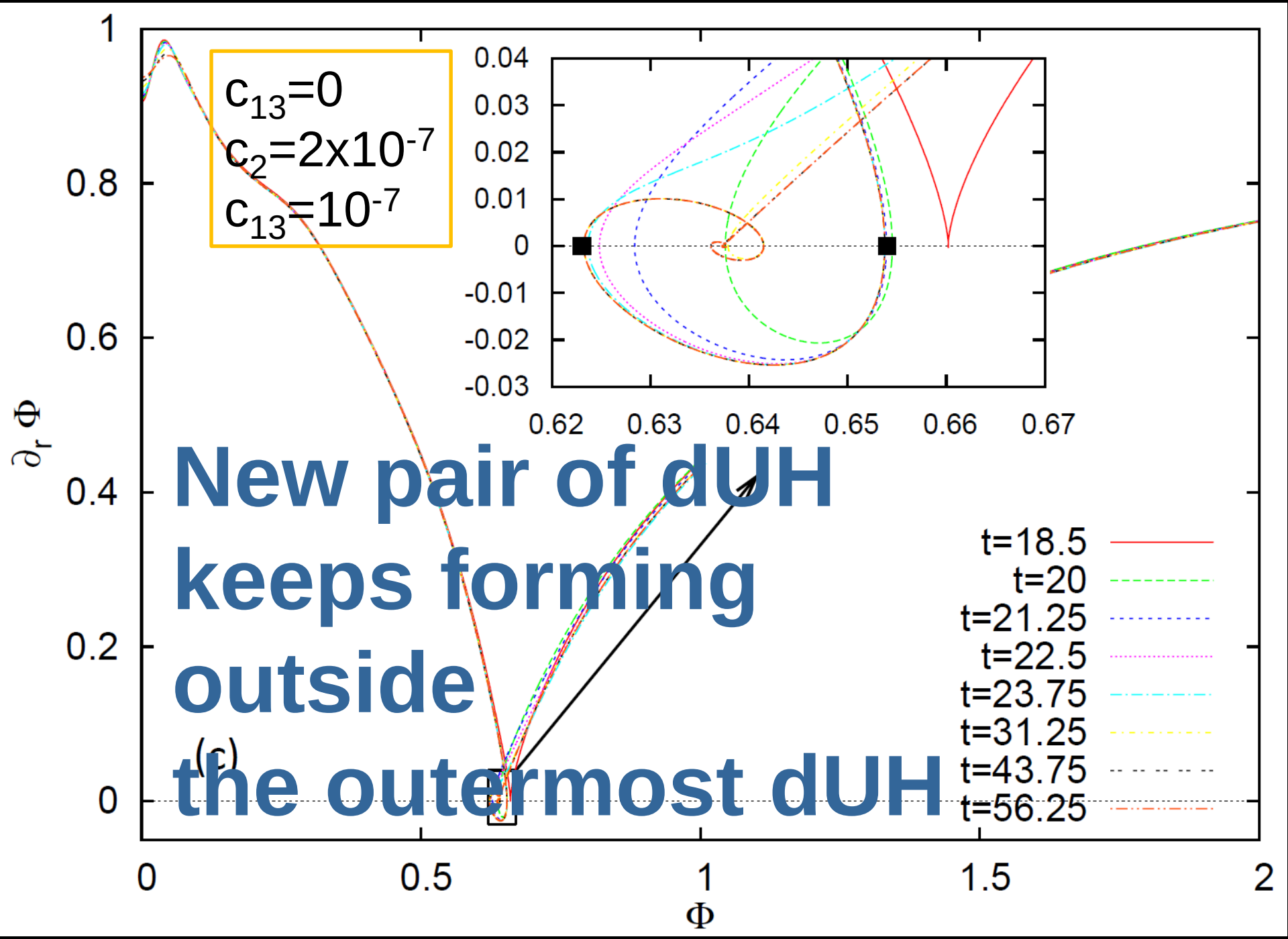
is Kodama vector $k^a = \varepsilon_{\perp}^{ab}\partial_b\Phi \qquad \varepsilon_{\perp}^{01} = \frac{1}{\sqrt{\gamma}}$

- Dynamical universal horizon:

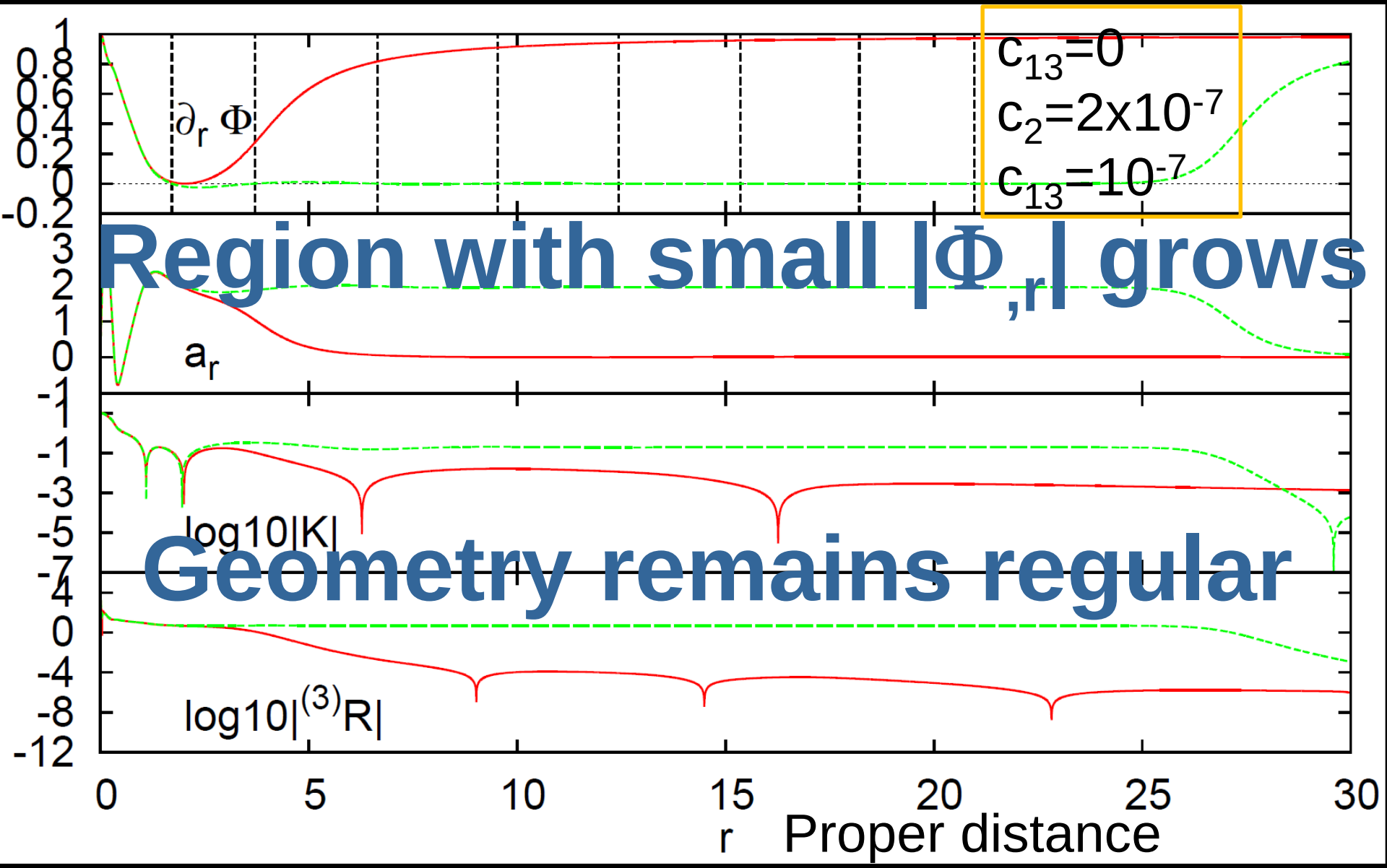
$$k^a\partial_a\phi = 0 \quad \Longleftrightarrow \quad \partial\Phi / \partial r = 0$$



A pair of AHs forms and the outer AH becomes stationary



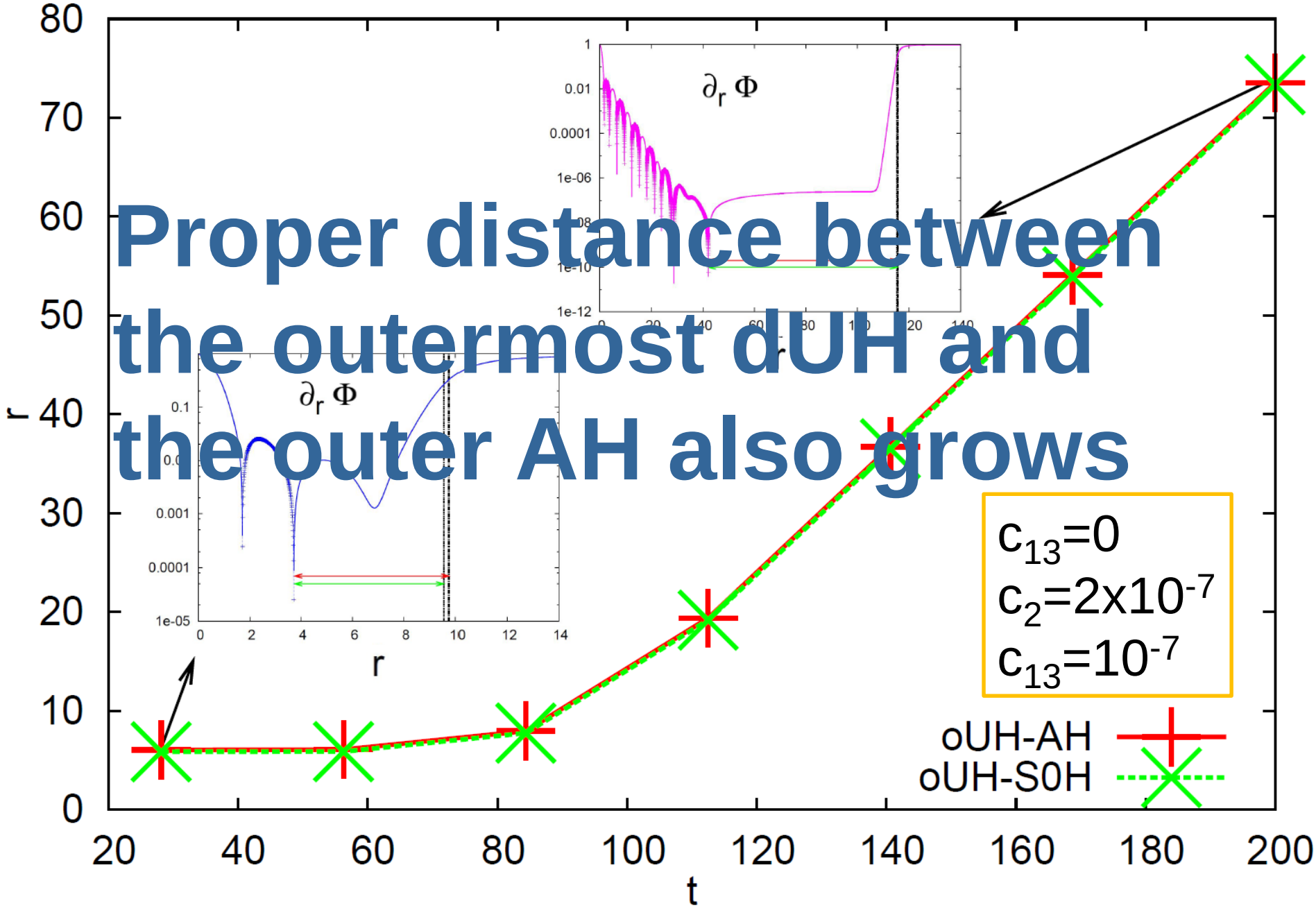
New pair of dUH keeps forming outside the outermost dUH



Region with small $|\Phi_{,r}|$ grows

Geometry remains regular

Proper distance between the outermost dUH and the outer AH also grows



Summary

- Numerical study of gravitational collapse of massless scalar with spherical symmetry
- Apparent horizon (AH) and dynamical universal horizon (dUH) form
- Geometry outside the outer AH becomes stationary
- Proper distance between the outer AH and the outermost dUH along the aether-orthogonal slicings keep increasing → the outermost dUH is evolving into a causal boundary? → dUH = precursor of the universal horizon?

Evidences of UH formation

- Formation of multiple pairs of dUHs
- New pair of dUHs keeps forming outside the outermost dUH (but inside the outer AH)
- No sign of disappearance of any of them
- Proper distance between the outer AH and the outermost dUH keeps increasing and there is no sign of slowdown
- Proper distance between the outermost and innermost dUHs also keep increasing
- Geometry on and outside dUHs stays regular

BACKUP SLIDES

Horava-Lifshitz gravity

- HL gravity realizes **z=3 scaling @ UV** and thus is power-counting renormalizable
- **Renormalizability was recently proved** [Barvinsky, et al. 2016]
- Ostrogradsky ghost is absent and thus HL gravity is **likely to be unitary**
- Lorentz-invariance is broken @ UV
- **Lorentz-invariant IR fixed-point** is generic [Chadha & Nielsen 1983] (and may apply to GW as well; cf. $|c_{gw}^2 - c_\gamma^2| < 10^{-15}$ from GW170817) but running is slow (logarithmic)
- SUSY or/and strong dynamics can speed-up the RG running towards Lorentz-invariant IR fixed-point

3 ways to recover LI @ IR

i. RG flow → LI @ IR [Chadha & Nielsen 1983]. RG running may be speeded up e.g. by strong dynamics.

ii. Supersymmetry protects low-E LI [Groot Nibbelink & Pospelov 2004].

iii. Low Lifshitz scale $M << M_{pl}$ may suppresses LV [Pospelov & Shang 2012] .

2 ways to recover LI @ IR

i. RG flow → LI @ IR [Chadha & Nielsen 1983]. RG running may be speeded up e.g. by strong dynamics.

ii. Supersymmetry protects low-E LI [Groot Nibbelink & Pospelov 2004].

iii. ~~Low Lifshitz scale $M \ll M_{pl}$ may suppresses LV [Pospelov & Shang 2012]. Unfortunately, shift-loops spoil this mechanism [Coates, Melby-Thompson & Mukohyama 2018].~~

Minimal Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z\, t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$N dt \qquad \sqrt{g}\, d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R \quad R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad \right]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other terms

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

- Total action

$$I_g = \frac{M_{\rm Pl}^2}{2}\int N dt \sqrt{g} d^3\vec{x}\left(K^{ij}K_{ij} - \lambda K^2 - 2\Lambda + R + L_{z>1}\right)$$

$$\frac{M_{\rm Pl}^2}{2}L_{z>1} = \left(c_1 D_i R_{jk} D^i R^{jk} + c_2 D_i R D^i R + c_3 R_i^j R_j^k R_k^i + c_4 R R_i^j R_j^i + c_5 R^3\right) + \left(c_6 R_i^j R_j^i + c_7 R^2\right)$$

- 3d space may consist of several connected pieces Σ_α

$$\boxed{\int d^3\vec{x} = \sum_\alpha \int_{\Sigma_\alpha} d^3\vec{x}}$$

- Common lapse N(t) and set of lapses & set of 3d metrics

$$N^i = N^i_\alpha(t,\vec{x})\,, \quad g_{ij} = g^\alpha_{ij}(t,\vec{x})\,, \quad (\vec{x}\in\Sigma_\alpha)$$

- We are interested in one of Σ_α

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FLRW universe in vacuum

- No local Hamiltonian constraint
- Dynamical eq

$$\frac{3\lambda-1}{2}\left(2\frac{\partial_t H_\alpha}{N}+3H_\alpha^2\right)=\frac{\alpha_3 K_\alpha^3}{a_\alpha^6}+\frac{\alpha_2 K_\alpha^2}{a_\alpha^4}-\frac{K_\alpha}{a_\alpha^2}+\Lambda$$

$$\alpha_2 = 4(c_6+3c_7)/M_{\rm Pl}^2 \qquad \alpha_3 = 24(c_3+3c_4+9c_5)/M_{\rm Pl}^2$$

- 1st integral → effective Friedmann eq

$$\frac{3(3\lambda-1)}{2}H_\alpha^2=\frac{C_\alpha}{a_\alpha^3}-\frac{\alpha_3 K_\alpha^3}{a_\alpha^6}-\frac{3\alpha_2 K_\alpha^2}{a_\alpha^4}-\frac{3K_\alpha}{a_\alpha^2}+\Lambda$$

C_α/a_α^3 : "dark matter as an integration constant"

(Mukohyama 2009)

- Global Hamiltonian constraint

$$\sum_\alpha C_\alpha = 0$$

Anisotropic instanton

- Effective Friedmann eq with $\alpha_2 = 0 = \Lambda$

$$\frac{3(3\lambda-1)}{2}H^2 = \frac{C}{a^3} - \frac{\alpha_3 K^3}{a^6} - \frac{3K}{a^2}$$

- Imaginary-time $\tau = i\int^t N(t')dt'$, K = 1

$$\frac{3(3\lambda-1)}{2}\frac{(\partial_\tau a)^2}{a^2} = -\frac{C}{a^3} + \frac{\alpha_3}{a^6} + \frac{3}{a^2}$$

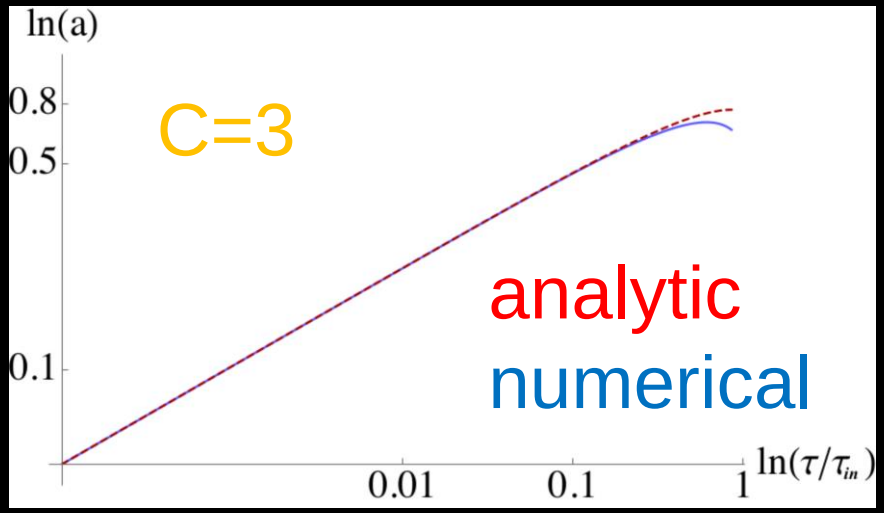
- For small a, the last term can be dropped

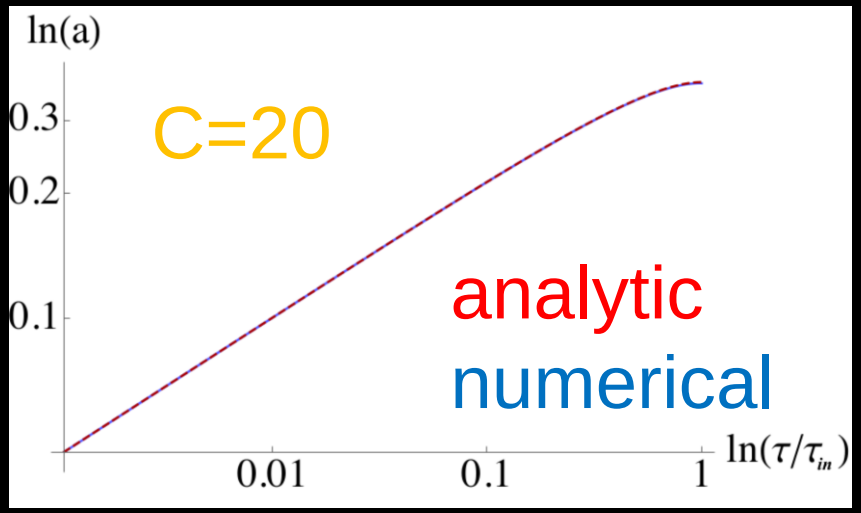
$$\frac{3(3\lambda-1)}{2}\frac{(\partial_\tau a)^2}{a^2} \simeq -\frac{C}{a^3} + \frac{\alpha_3}{a^6}$$

$$\Rightarrow \quad \boxed{a \simeq \left[3\sqrt{\alpha_3}\mathcal{T} - \frac{9}{4}C\mathcal{T}^2\right]^{1/3}, \quad \mathcal{T} = \sqrt{\frac{2}{3(3\lambda-1)}}\tau}$$

Numerical result

$\lambda = 2,\ \alpha_3 = 1,\ \alpha_2 = 0$



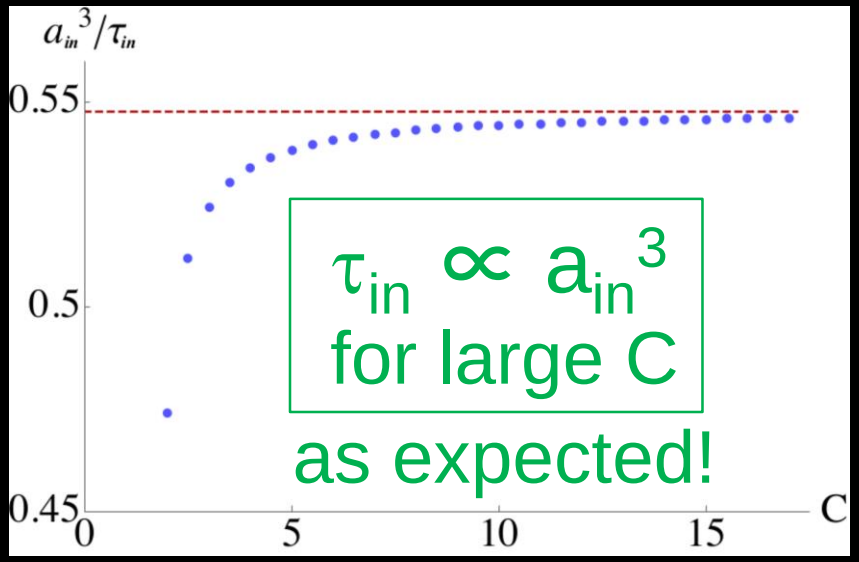


Wick rotation @ $\tau = \tau_{in}$

$$\partial_\tau a|_{\tau=\tau_{\rm in}} = 0$$

$$a_{\rm in} \equiv a(\tau_{\rm in})$$

large C ⇔ small τ_{in} & a_{in}



Some remarks

- Solution is singular @ a=0 → unable to rely on semi-classical formula for tunneling rate
- Quantum effects such as RG running should be taken into account near a=0
- The classical solution away from a=0 is unique up to a constant shift of τ
- The scaling $T \propto L^3$ is robust and thus the flatness problem may be solved

General argument

- Generalized Friedmann eq

$$H^2 = \frac{1}{3}\frac{\rho}{M_{\rm Pl}^2} \pm M^2 f(|K|/a^2M^2)$$

$$\dot{\rho} + 3H(1+w)\rho = 0 \qquad f(x) = \begin{cases} x\,, & (0 \le x \ll 1) \\ x^z\,, & (x \gg 1) \end{cases}$$

- Curvature as fluid with

  $\rho_K = \pm 3M_{Pl}^2M^2f(|K|/a^2M^2)$

- **Equipartition**

  $\rho \approx \rho_K \sim \rho_{in}$ @ quantum-classical transition

- z=1 Friedmann eq recovered @ $\rho = \rho_{z\to 1}$

$$\rho_{z\to 1} \sim \rho_{\rm in}\left(\frac{M_{\rm Pl}^2M^2}{\rho_{\rm in}}\right)^{\frac{3(1+w)}{2z}}$$

- For $\rho_{z\to 1} \ll \rho \ll \rho_{\rm in}$

$$\Omega_K = \frac{\rho_K}{\rho + \rho_K} \propto a^{3(1+w)-2z}$$

- To solve the flatness problem, we impose

$$\Omega_K \ll \Omega_{\rm sup} \equiv z_{\rm eq} \left(\frac{T_{\rm CMB}}{M_{\rm Pl}}\right)^2 \left(\frac{M_{\rm Pl}^4}{\rho_{z\to 1}}\right)^{\frac{1}{2}}$$

i.e.

$$\boxed{\frac{\rho_{\rm in}}{M_{\rm Pl}^2 M^2} \gg \left[\frac{1}{z_{\rm eq}} \frac{M_{\rm Pl} M}{T_{\rm CMB}^2}\right]^{\frac{4z}{2z-3(1+w)}}}$$

- If we set z=3, w=0, M=M_{Pl} then the requirement is

$$\frac{\rho_{\rm in}^{1/4}}{M_{\rm Pl}} \gg \frac{1}{z_{\rm eq}} \left(\frac{M_{\rm Pl}}{T_{\rm CMB}}\right)^2 \sim 10^{58}$$

- In UV complete theory we do not afraid of going into highly trans-Planckian regime.

Dynamical case (Mukohyama 201?)

- IR action

$$I_g = \frac{M_{Pl}^2}{2}\int N dt\,\sqrt{g} d^3\vec{x}\left(K_{ij}K^{ij} - \lambda K^2 - 2\Lambda + R\right)$$

- Dynamical ansatz with N=1 & $N^i = 0$

$$g_{ij}dx^i dx^j = e^{2B(t,x)}\left[e^{2A(t,x)}dx^2 + d\Omega_2^2\right]$$

- Change of variable $\tilde{B} \equiv \frac{3\lambda-1}{2}B + \frac{\lambda-1}{2}A$
- EOM

$$\begin{aligned}
\partial_t A &= \frac{(3\lambda-1)\partial_r\partial_t\tilde{B}}{2\partial_x\tilde{B} - (\lambda-1)\partial_x A} \\
\partial_t^2\tilde{B} &= \frac{1}{2(3\lambda-1)^2}\Big[-6(3\lambda-1)(\partial_t\tilde{B})^2 + 4e^C(\partial_x\tilde{B})^2 \\
&\quad -4(\lambda-1)e^C\partial_x A\partial_x\tilde{B} - (3\lambda-1)(\lambda-1)(\partial_t A)^2 \\
&\quad +(\lambda-1)^2e^C(\partial_x A)^2 + (3\lambda-1)^2(\Lambda - e^D)\Big]
\end{aligned}$$

$$C \equiv -\frac{4}{3\lambda-1}(\lambda A + \tilde{B}) \qquad D \equiv \frac{2}{3\lambda-1}[(\lambda-1)A - 2\tilde{B}]$$

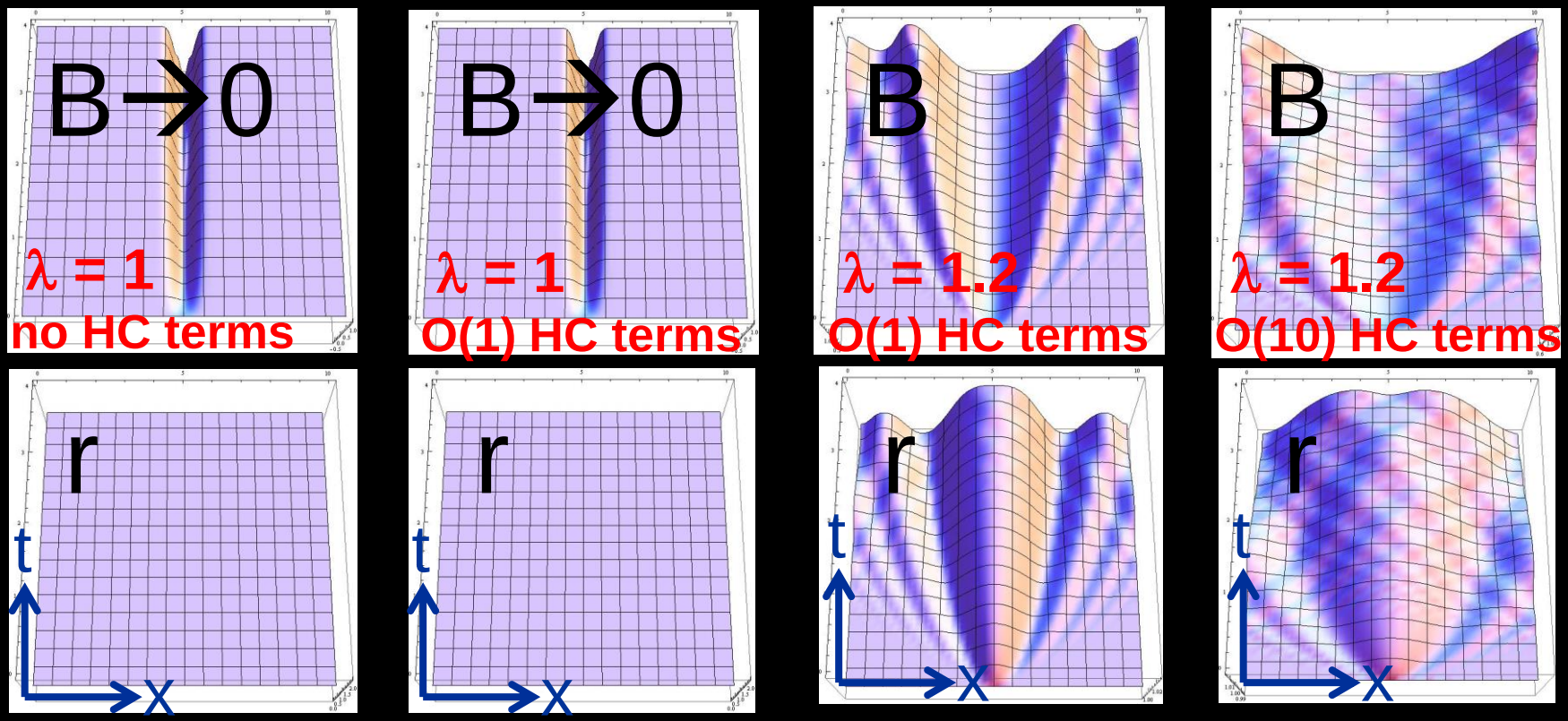
- The $\lambda \to$ 1+0 limit is continuous and recovers GR+DM.

Caustic avoidance

(preliminary)

$$N = 1 \qquad N_i = 0$$

$$ds^2 = B(t,x)dx^2 + r^2(t,x)(dy^2 + dz^2)$$



Scalar graviton and $\lambda \to 1$

- GR may recover @ IR if $\lambda \to 1$
- The minimal HL gravity has a scalar graviton in addition to tensor graviton.
- What happens to the scalar graviton in the limit $\lambda \to 1$?

IR action

- **UV: z=3** , power-counting renormalizability

  ⇓ RG flow

- **IR: z=1** , seems to recover GR iff $\lambda \to 1$

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\,d^3x\left(\overbrace{K_{ij}K^{ij}-\lambda K^2}^{\text{kinetic term}}+\underbrace{c_g^2 R-2\Lambda}_{\text{IR potential}}\right)$$

note:

Renormalizability was recently proved.

RG flow has not yet been investigated.

Physical d.o.f.

- $(6 + 3) – 3 – 3 = 3$
  g_{ij} : 6 components
  N^i : 3 components
  $x^i \rightarrow x'^i(t,x)$: 3 gauge d.o.f.
  $\delta I/\delta N^i=0$: 3 constraints
- $3 = 2 + 1$
  tensor graviton: 2 d.o.f.
  scalar graviton: 1 d.o.f.

Different versions of HL gravity

- There are versions w/wo the projectability condition.
- Horava's original proposal was **with the projectability condition, N=N(t).**
- **Naïve non-projectable extension is inconsistent** [c.f. Henneaux, et.al. 2009].
- **Inclusion of $a_i = (\ln N)_{,i}$** (and thus more terms) in the action can cure the non-projectable extension [Blas, Pujolas and Sibiryakov 2009].
- U(1) extension [Horava & Melby-Thompson 2010]

This talk is based on the projectable version without U(1) extension.

Linear instability of scalar graviton

- Sign of (time) kinetic term $(\lambda-1)/(3\lambda-1) > 0$.
- The dispersion relation in flat background
  $\omega^2 = c_s^2 k^2 \times [1+ O(k^2/M^2)]$ with $c_s^2 = -(\lambda-1)/(3\lambda-1)<0$
  → IR instability in linear level
  (Wang&Maartens; Blas,et.al.; Koyama&Arroja 2009)
- Slower than Jeans instability if
  $t_J \sim (G_N\rho)^{-1/2} < t_L \sim L/|c_s|$.
- Tamed by Hubble friction or/and $O(k^2/M^2)$ terms if
  $H^{-1} < t_L$ or/and $L < 1/M$.
- Thus, the linear instability does not show up if
  $|c_s| = |(\lambda-1)/(3\lambda-1)|^{1/2} < \mathrm{Max}\,[|\Phi|^{1/2}, HL]$. ($\Phi \sim -G_N\rho L^2$)
  for $L > \mathrm{Max}[0.01\mathrm{mm}, 1/M]$
  (Shorter scales → similar to spacetime foam)
- Phenomenological constraint on properties of RG flow.

Perturbative vs non-perturbative regimes

$$N = 1, \quad N_i = \partial_i B + n_i, \quad g_{ij} = a^2 e^{2\zeta_T} \left(e^h\right)_{ij}$$

$$\zeta_T = O(q), \quad h_{ij} = O(q), \quad \boxed{B = O(q^0), \quad n_i = O(q^0)}$$

Momentum constraint

$$\Rightarrow \quad B = \frac{O(1)}{\boxed{O(\lambda - 1) + O(q)}} \partial_t \zeta_T$$

- Perturbative regime: q << (λ-1)
  breakdown in the λ → 1 limit
- Non-perturbative regime: (λ-1) << q << 1
  responsible for recovery of GR

Vainshtein effect in massive gravity

- Linearized analysis results in vDVZ discontinuity of the massless limit.
- However, perturbative expansion breaks down in this limit and cannot be trusted.
- Non-perturbative analysis shows continuity and GR is recovered in the massless limit.
- Continuity is not uniform w.r.t. distance. (e.g. 1/r expansion does not work.) However, Vainshtein radius can be pushed to infinity in the massless limit.

Analogue of Vainshtein effect (mukohyama 2010)

- Spherically symmetric, static ansatz

$$N = 1, \quad N_i dx^i = \beta(x) dx, \quad g_{ij} dx^i dx^j = dx^2 + r(x)^2 d\Omega_2^2$$

$$R \equiv \beta^{(\lambda-1)/(2\lambda)} r$$

without z>1 terms

$$R'' + \frac{\lambda - 1}{\lambda}\left[\frac{(3\lambda-1)(\beta')^2 R}{4\lambda^2\beta^2} + \frac{(\lambda-1)\beta' R'}{\lambda\beta} - \frac{(R')^2}{R}\right] = 0$$

$$\frac{\beta'}{\beta} - \frac{(\lambda-1)R}{4\lambda R'}\left(\frac{\beta'}{\beta}\right)^2 + \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}} = 0$$

- Two branches

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$

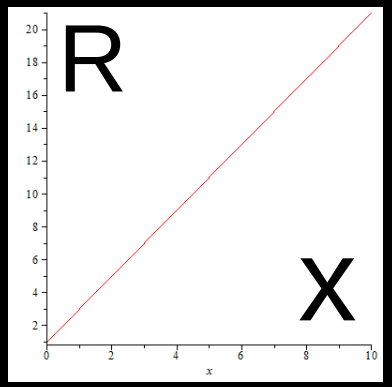
$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}}$$

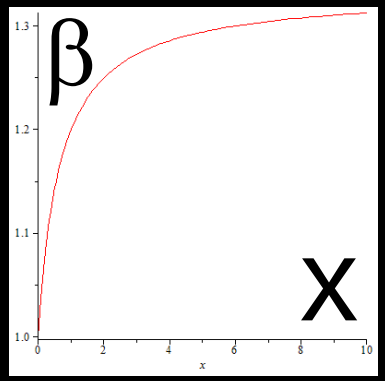
- "-" branch recovers GR in the $\lambda \to 1$ limit

Analogue of Vainshtein effect

- Numerical integration in the "-" branch with $\beta(x=0)=1$, $r(x=0)=1$, $r'(x=0)$ given

for
$\lambda-1=10^{-6}$
$r'(x=0)=2$



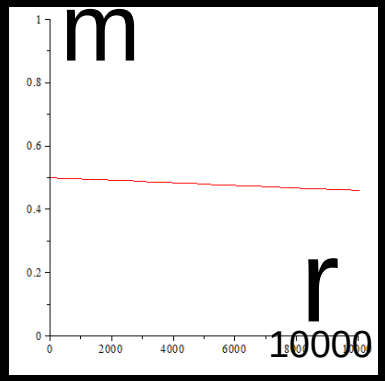


- Misner-Sharp energy

$$m \equiv \frac{r}{2}\left[1-\left(1-\beta^2\right)\left(r'\right)^2\right]$$

almost constant

GR is recovered!



Analogue of Vainshtein effect (mukohyama 2010)

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$ ➡ choose the "-" branch

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'} \frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}}$$

- $(3\lambda-1)\beta^2 << (\lambda-1)$
  perturbative regime, 1/r expansion
- $(3\lambda-1)\beta^2 >> (\lambda-1)$
  non-perturvative regime, recovery of GR
- $(3\lambda-1)\beta^2 \sim (\lambda-1)$ with $\beta^2 \sim r_g/r$ → $r \sim r_g/(\lambda-1)$
  analogue of Vainshtein radius

Fate of scalar graviton

$$L = \left[f\left(\frac{\zeta}{\lambda - 1}\right) + g(\zeta, \lambda) \right] \frac{M_{Pl}^2 \dot{\zeta}^2}{\lambda - 1} - V(\zeta, D_i)$$

subleading

Independent of λ
No time derivative

Local in time, no time derivative
Non-local in space, each term has the same # of spatial derivatives in denominator and numerator

λ → 1

$$\boxed{L \sim \dot{\zeta}_c^2}$$

- Looks like a minimally coupled FREE field with sound speed = 0
- **Scalar Graviton → "Dark Matter"**

Nonlinear cosmological perturbation and $\lambda \to 1$

arXiv: 1105.0246 [hep-th] with K.Izumi
arXiv: 1109.2609 [hep-th] with E.Gumruhcuglu & A.Wang

- HL gravity + a scalar matter field
- Flat FRW background
- **Nonlinear cosmological perturbation**
- Gradient expansion up to any order
- **Regular and continuous in the $\lambda \to 1$ limit**
- **Recovers GR+DM+scalar field in the $\lambda \to 1$ limit**