Baryogenesis from Helical Magnetic Fields

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based on work with Kohei Kamada 1606.08891 (PRD) & 1610.03074 (PRD)
The “Ordinary Matter” Problem

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But this is not true … in fact, we don’t understand 100%!

The “ordinary matter” problem = why is there more matter than anti-matter? → baryogenesis is the endeavor to solve this problem
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Matter / Antimatter Asymmetry
\[ \Omega_b \sim 5\% \quad \Rightarrow \quad n_B/s \sim 10^{-10} \]

hiding in the “cracks” (error bars):
Primordial Magnetic Field
In this talk, I’m going to …

… assume that a helical magnetic field was created in the early universe prior to the EW epoch. (e.g., arises naturally in axion inflation)

… show that the decaying helicity of this field gives rise to a baryon asymmetry through the Standard Model B+L anomaly (builds on earlier work by Joyce, Shaposhnikov, Giovannini, Bamba, Geng, Ho, …)

… calculate the evolution of the magnetic field and baryon asymmetry from magnetogenesis until today, while paying particular attention to the EW crossover (this is my work with Kohei Kamada; see also Fujita & Kamada, 2016)

… conclude that the predicted relic baryon asymmetry suffers from a large theoretical uncertainty, because we don’t understand well how magnetic fields behave at the EW crossover (even though this is just SM physics!)
Standard Model anomalies & primordial magnetic fields
Thermal fluctuations of the weak isospin field (EW sphaleron), provide support for the SU(2)\(_L\) term.

The sphaleron is responsible for B-violation in many models of baryogenesis, including EW baryogenesis & leptogenesis.

Can we use the U(1)\(_Y\) term to accomplish baryogenesis?

Kuzmin, Rubakov, Shaposhnikov (1985)
A hypermagnetic field provides support for the $U(1)_Y$ term.

$$\langle Y_{\mu\nu} \tilde{Y}^{\mu\nu} \rangle = -4 E_Y \cdot B_Y = 2 \left[ \frac{\partial}{\partial t} (A \cdot B) + \nabla \cdot (\phi B + E \times A) \right]$$

To induce a change in B-number, the helicity must change

$$\Delta Q_B = -N_{\text{gen}} \frac{g'^2}{16\pi^2} \Delta \mathcal{H}_Y \quad \text{where} \quad \mathcal{H}_Y = \int d^3x \ A_Y \cdot B_Y$$

In a plasma, the helicity decays because of ohmic losses

$$E_Y = j_Y / \sigma_Y \approx \nabla \times B_Y / \sigma_Y$$

$$\langle Y_{\mu\nu} \tilde{Y}^{\mu\nu} \rangle = -4 \langle B_Y \cdot \nabla \times B_Y \rangle / \sigma_Y$$
E.g., field generation via axion inflation

For example, a helical magnetic field may be generated during inflation from a pseudo-scalar inflaton (or spectator field).

\[
-\mathcal{L}_{\text{int}} = \frac{\varphi}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu} = \frac{d\varphi/ dt}{2f} \mathbf{A} \cdot \mathbf{B} + \cdots \left( \frac{\partial^2}{\partial \eta^2} + k^2 \pm \frac{k}{\eta} \xi \right) A_{\pm}(\eta, k) = 0
\]

\[ B_{\text{today}} \sim 10^{-13} \text{ Gauss} \]

\[ \lambda_{\text{today}} \sim 10 \text{ pc} \]

Lattice simulation of B-field growth during preheating after axion inflation

Adshead, Giblin, Scully, Sfakianakis (2016)
How do we formulate the problem?
Background B-Field Evolution

Simplified model for the background B-field:

\[ P_E(t, k) = \pm P_H(t, k) \]

\[ \langle B \cdot \nabla \times B \rangle \approx \pm B_p(t)^2 / \lambda_B(t) \]

MHD evolution of B-field leads to inverse cascade scaling behavior.

\[ B_p(t) = \left( \frac{a}{a_0} \right)^{-2} \left( \frac{\tau}{\tau_{\text{rec}}} \right)^{-1/3} B_0 \]

\[ \lambda_B(t) = \left( \frac{a}{a_0} \right) \left( \frac{\tau}{\tau_{\text{rec}}} \right)^{2/3} \lambda_0 \]

Frisch, Pouquet, Leorat, Mazure, 75,76
Banerjee & Jedamzik, 2004
Campenelli, 2007
Kahniashvilli et. al. 2013
Baryon Asymmetry Evolution

Roughly speaking, you integrate the anomaly equation to obtain the kinetic equation for B-number:

\[
\partial_\mu j_\mu^B = N_{\text{gen}} \left( \frac{g^2}{16\pi^2} \text{Tr}[W_{\mu\nu} \tilde{W}^{\mu\nu}] - \frac{1}{2} \frac{g'^2}{16\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right)
\]

\[
\frac{d}{dt} n_B = -\Gamma_{\text{sphaleron}} n_B + S_{\text{hypermagnetic}}
\]

This glosses over Yukawa interactions which communicate B-number violation between the left- and right-chiral fermions.
SM Boltzmann eqns. w/ anomaly terms

\[
\begin{align*}
\frac{d\eta_{u_L}}{dx} &= -S_{\text{UDW}}^{i} - \sum_{j=1}^{N_g} (S_{\text{u}u}^{ij} + S_{\text{u}u}^{ji} + S_{\text{u}d}^{ij}) - S_{\text{s,ph}} - \frac{N_c}{2} S_{\text{w,ph}}, \\
\frac{d\eta_{d_L}}{dx} &= S_{\text{UDW}}^{i} - \sum_{j=1}^{N_g} (S_{\text{d}d}^{ij} + S_{\text{d}d}^{ji} + S_{\text{d}u}^{ij}) - S_{\text{s,ph}} - \frac{N_c}{2} S_{\text{w,ph}}, \\
\frac{d\eta_{e_R}}{dx} &= S_{\text{EW}}^{i} - \sum_{j=1}^{N_g} (S_{\text{e}e}^{ij} + S_{\text{e}e}^{ji} + S_{\text{e}h}^{ij}) - \frac{1}{2} S_{\text{w,ph}}, \\
\frac{d\eta_{\nu_R}}{dx} &= \sum_{j=1}^{N_g} (S_{\text{u}u}^{ij} + S_{\text{u}u}^{ji} + S_{\text{u}d}^{ij} + S_{\text{s,ph}} - N_c y_{u,R} S_{\text{y,bkg}}^{i}), \\
\frac{d\eta_{\nu_L}}{dx} &= \sum_{j=1}^{N_g} (S_{\text{d}d}^{ij} + S_{\text{d}d}^{ji} + S_{\text{d}u}^{ij} + S_{\text{s,ph}} - N_c y_{d,R} S_{\text{y,bkg}}^{i}), \\
\frac{d\eta_{\phi^+}}{dx} &= \left( S_{\text{hww}} + S_{\text{h}} \right) + \sum_{i,j=1}^{N_g} \left( -S_{\text{phu}}^{ij} + S_{\text{phd}}^{ij} + S_{\text{phe}}^{ij} \right), \\
\frac{d\eta_{\phi^0}}{dx} &= S_{\text{hww}} - S_{\text{h}} + \sum_{i,j=1}^{N_g} \left( -S_{\text{phu}}^{ij} + S_{\text{phd}}^{ij} + S_{\text{phe}}^{ij} \right), \\
\frac{d\eta_{W^+}}{dx} &= \left( S_{\text{hww}} + S_{\text{h}} \right) + \sum_{i=1}^{N_g} (S_{\text{UDW}}^{i} + S_{\text{EW}}^{i}).
\end{align*}
\]

\[
\begin{align*}
S_{\text{UDW}}^{ij} &= \frac{\alpha_y}{s t} \frac{4 X}{2} \frac{1}{4 \pi} \frac{1}{4 \pi} \left( Y_{\mu\nu} \right) \left( Y_{\rho\sigma} \right),
S_{\text{EWW}}^{ij} &= \frac{1}{s t} \frac{2}{4 \pi} \frac{1}{4 \pi} \left( W_{\mu\nu} \right) \left( W_{\rho\sigma} \right),
S_{\text{y,bkg}}^{i} &= \frac{1}{s t} \frac{2}{4 \pi} \frac{1}{4 \pi} \left( g_{\mu\nu} \right) \left( W_{\rho\sigma} \right).
\end{align*}
\]

\[
S_{\text{hww}} = \gamma_{\text{hww}} \left( \eta_{\phi^+} + \eta_{\phi^0} \right),
S_{\text{h}} = \gamma_{\text{h}} \left( \eta_{\phi^+} + \eta_{\phi^0} \right).
\]

Related work: Giovanni & Shaposhnikov; Fujita & Kamada; AL, Sabancilar, & Vachaspati; Semikoz, Dvornikov, Smirnov, Sokoloff, Valle.
Numerical Results
Evolution before EW crossover

Temperature: \( T \) (GeV)

\[
B_0 = 10^{-16} \text{ G} \\
\lambda_0 = 10^{-2} \text{ pc} \\
f_{h\rightarrow ee} = f_{\text{flip}} = 1
\]

\[
x = M_0/T
\]

field strength & coherence length today
source term balances against washout by sphaleron
we’ll discuss evolution through the EW crossover next

turn on B-field at different times
Evolution before EW crossover

\[ \eta_B \approx \frac{\# \frac{\alpha_y}{s_T} (B_Y \cdot \nabla \times B_Y) / \sigma_Y \# |y_e|^2 m^2_h(T) / T^2 + \# \frac{\alpha_y^2}{T^3} |B_Y|^2 / \sigma_Y}{\# \frac{\alpha_y^2}{T^3} |B_Y|^2 / \sigma_Y} \sim (4 \times 10^{-12}) \frac{B_{14}^2}{\lambda_1} \frac{(T/T_w)^{4/3}}{0.08 m^2_h(T) / T^2 + B_{14}^2 (T/T_w)^{2/3}} \]

( \( B_{14} \equiv B_0 / (10^{-14} \text{ G}) \), \( \lambda_1 \equiv \lambda_0 / (1 \text{ pc}) \), \( T_w \equiv 162 \text{ GeV} \) )

Turn on B-field at different times

Equilibrium baryon asymmetry: \( n_B / s \)

Source from decaying magnetic helicity

Washout due to sphaleron + Yukawa interactions + chiral magnetic effect

Field strength & coherence length today

Source term balances against washout by sphaleron

We’ll discuss evolution through the EW crossover next

Order 1 numbers

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Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

... while keeping $\left( \frac{\lambda_0}{1 \text{ pc}} \right) \sim \left( \frac{B_0}{10^{-14} \text{ Gauss}} \right)$
Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU... while keeping \[ \left( \frac{\lambda_0}{1 \text{ pc}} \right) \sim \left( \frac{B_0}{10^{-14} \text{ Gauss}} \right) \]

we’ll discuss EW crossover next
Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

... while keeping $\left( \frac{\lambda_0}{1 \text{ pc}} \right) \sim \left( \frac{B_0}{10^{-14} \text{ Gauss}} \right)$

$B_0 = 5 \times 10^{-17}$ G ..., $\lambda_0 = 5 \times 10^{-3}$ pc

we’ll discuss EW crossover next
Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

... while keeping $\left( \frac{\lambda_0}{\text{1 pc}} \right) \sim \left( \frac{B_0}{10^{-14} \text{ Gauss}} \right)$

![Graph showing temperature and B-asymmetry with x-axis labeled $x = M_0/T$ and y-axis labeled B-asym: $\eta_B = n_B/s$. The graph includes a note: we’ll discuss EW crossover next.]
Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

... while keeping

$$\left( \frac{\lambda_0}{1 \, \text{pc}} \right) \sim \left( \frac{B_0}{10^{-14} \, \text{Gauss}} \right)$$

we’ll discuss EW crossover next
Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

... while keeping $\left( \frac{\lambda_0}{1 \text{ pc}} \right) \sim \left( \frac{B_0}{10^{-14} \text{ Gauss}} \right)$

Temperature: $T$ (GeV)

$B_0 = 5 \times 10^{-14}$ G ... $\lambda_0 = 5 \times 10^0$ pc

we’ll discuss EW crossover next
Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

\[ ... \text{while keeping} \quad \left( \frac{\lambda_0}{1 \text{ pc}} \right) \sim \left( \frac{B_0}{10^{-14} \text{ Gauss}} \right) \]

\[ B_0 = 5 \times 10^{-13} \text{ G} \quad \lambda_0 = 5 \times 10^{1} \text{ pc} \]

we’ll discuss EW crossover next

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Let’s play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

... while keeping \[ \left( \frac{\lambda_0}{1 \text{ pc}} \right) \sim \left( \frac{B_0}{10^{-14} \text{ Gauss}} \right) \]

\[ B_0 = 5 \times 10^{-13} \text{ G} \ldots \lambda_0 = 5 \times 10^1 \text{ pc} \]

\[ \eta_B = \eta_B / s \]

\[ \eta_B^{(eq)} \approx \frac{\# \frac{\alpha_y}{sT} (B_Y \cdot \nabla \times B_Y)/\sigma_Y}{\# |y_e|^2 m_h^2(T)/T^2 + \# \frac{\alpha^2}{T^3} |B_Y|^2/\sigma_Y} \approx \left( 4 \times 10^{-12} \right) \frac{B^2_{14}}{\lambda_1} 0.08 m_h^2(T)/T^2 + B^2_{14} (T/T_w)^{2/3} \]

Washout induced by chiral magnetic effect … prevents $\eta_B$ from reaching $10^{-10}$ for large $B_0$.

This behavior was overlooked in some previous studies. The CME cannot be neglected!
What happens at the EW crossover?
**Evolution through EW Crossover**

\[
\frac{d}{dt} n_B = -\Gamma_{\text{sphaleron}} n_B + S_{\text{hypermagnetic}}
\]

At this time…

… the source shuts off, because the U(1)_\gamma field is converted into a U(1)_\text{em} field, which does not source B-number.

\[
\partial j_B \sim W\tilde{W} - Y\tilde{Y} \neq F\tilde{F}
\]

… the washout shuts off, because the W-boson mass grows, suppressing EW sphaleron transitions.

\[
\Gamma_{\text{sph}} \propto \exp \left[ -\# \frac{M_W(T)}{\alpha_W} \right]
\]
Crossover Evolution Scenarios

Increasing time, decreasing temperature

Baryon asymmetry

Turn on helical B-field

Washout processes come into equilibrium, & suppress the baryon asymmetry

At the EW crossover, both the source & washout processes go out of equilibrium

Source & washout shut off simultaneously (Fujita & Kamada, 2016)
**Crossover Evolution Scenarios**

At the EW crossover, both the source & washout processes go out of equilibrium.

*Source & washout shut off simultaneously (Fujita & Kamada, 2016)*

*Washout remains active after source has shut off (Kamada & Long, 2016a)*

- Increasing time, decreasing temperature
- Turn on helical B-field
- Washout processes come into equilibrium, & suppress the baryon asymmetry

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**Crossover Evolution Scenarios**

As time increases and temperature decreases, washout processes come into equilibrium, & suppress the baryon asymmetry.

- **Increase time, decrease temperature**
- **Turn on helical B-field**
- **At the EW crossover, both the source & washout processes go out of equilibrium**

Source remains active after washout shuts off
(Kamada & Long, 2016b)

Source & washout shut off simultaneously
(Fujita & Kamada, 2016)

Washout remains active after source has shut off
(Kamada & Long, 2016a)
Model the $U(1)_Y$ to $U(1)_{em}$ conversion

\[
\langle W^1_\mu(x) \rangle = \langle W^2_\mu(x) \rangle = 0
\]
\[
\langle W^3_\mu(x) \rangle = \sin \theta_W(t) A_\mu(x)
\]
\[
\langle Y_\mu(x) \rangle = \cos \theta_W(t) A_\mu(x)
\]

$B_Y$ $B_{em}$ $B_{A}$ $\theta_W(t)$ $B_{W^3}$ $B_Z$

**Z-$\gamma$ mixing is proxy for $B_Y$ to $B_{em}$**

- **Entirely $B_Y$**
- **Entirely $B_{em}$**

**Dots & error bars** = lattice simulation from D'Onofrio & Rummukainen (2015).

**Black dashed** = analytic approx. from Kajantie, Laine, Rummukainen, & Shaposhnikov (1996).

**Colored curves** = we use tanh functions to model the crossover.

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BAU Evolution through EW Crossover

\[ \frac{\eta_B}{\eta_B} \approx \frac{11}{37} \left( g'^2 \cos^2 \theta_w S_{\text{BdB}} + \frac{d\theta_w}{d\ln x} \sin 2\theta_w S_{\text{AB}} \right) + \frac{1}{2} \left( \gamma_{\text{Ehe}}^{11} + \gamma_{\nu_{\text{he}}}^{11} \right) + \gamma_{\text{Ee}}^{11} + g'^4 \cos^4 \theta_w \gamma^{\text{CME}} \]

\[ B \cdot \nabla \times B \]
Result: Predicted Baryon Asymmetry

The conversion of $U(1)_Y$ B-field into $U(1)_{em}$ B-field at the EW crossover is not well-understood.

However, the relic baryon asymmetry depends sensitively on these details.

Consequently, the predicted baryon asymmetry is very uncertain.

Need to understand the crossover better!
Where to go from here?

Refinements:
Study conversion of magnetic fields at EW crossover
- main message in this talk
Calculate helicity decay directly with MHD simulations
- “not difficult” to implement

Implications & Applications:
Study baryogenesis from axion inflation (etc.) self-consistently
- Various studies: Anber & Sabancilar (2015); Cado & Sabancilar (2016); Jimenez, Kamada, Schmitz, & Xu (2017)
Observation side – develop new probes of relic (helical) magnetic fields
- E.g., using cascade halos around TeV blazars
Study “dark” magnetic field (hidden sector U1) and/or dark matter production
- E.g., Cado & Sabancilar (2016)
In this talk, I’m going to …

… assume that a helical magnetic field was created in the early universe prior to the EW epoch. (e.g., arises naturally in axion inflation)

… show that the decaying helicity of this field gives rise to a baryon asymmetry through the Standard Model B+L anomaly (builds on earlier work by Joyce, Shaposhnikov, Giovannini, Bamba, Geng, Ho, …)

… calculate the evolution of the magnetic field and baryon asymmetry from magnetogenesis until today, while paying particular attention to the EW crossover (this is my work with Kohei Kamada; see also Fujita & Kamada, 2016)

… conclude that the predicted relic baryon asymmetry suffers from a large theoretical uncertainty, because we don’t understand well how magnetic fields behave at the EW crossover (even though this is just SM physics!)
backup
Implications & Applications

models considered in this work live along black arrow

Field Strength, today : $B_0$ (Gauss)

Coherence length, today : $\lambda_0$ (pc)

inconsistent with MHD evolution

conflict with CMB

will be probed by CMB measurements
(see John Kovac’s talk)

cannot explain blazar observations

will be probed by blazar halo measurements

axion inflation magnetogenesis

Application:
--oscillation of photons into keV axion dark matter
--see Kamada & Nakai (2017)

Application:

(figure adapted from Durrer & Neronov, 2013)
Magnetic Field Scaling Law

Comoving quantities:
\[ \tilde{B}(\tau) = a(t)^2 B_p(t) \quad \tilde{\lambda}(\tau) = a(t)^{-1} \lambda_B(t) \]

Adiabatic evolution after recombination:
\[ \tilde{B}_{\text{rec}} = B_0 \quad \tilde{\lambda}_{\text{rec}} = \lambda_0 \]

Coherence length tracks eddy scale:
\[ \tilde{\lambda}(\tau) = C \nu_A(\tau) \tau \]
\[ \nu_A(\tau) = c/\sqrt{1 + (\rho + P)/(2P_m)} \propto \tilde{B}(\tau) \]
\[ P_m(\tau) = \tilde{B}(\tau)^2/2 \]

Helicity is quasi-conserved:
\[ \tilde{\lambda} \tilde{B}^2 = \tilde{\lambda}_{\text{rec}} \tilde{B}_{\text{rec}}^2 \quad \text{H} \sim \lambda B^2 \text{ for maximally helical field} \]

Solution:
\[ B_p = (a/a_0)^{-2} (\tau/\tau_{\text{rec}})^{-1/3} B_0 \quad \text{“inverse cascade”} \]
\[ \lambda_B = (a/a_0) (\tau/\tau_{\text{rec}})^{2/3} \lambda_0 \]
Baryogenesis without (B-L)?

Recall that (B-L) = 0 at all times! But, Kuzmin, Rubakov, & Shaposhnikov (‘85) taught us that B → 0 and L → 0 in equilibrium. **How is washout avoided?**

In the **symmetric phase** (T > 160 GeV), the EW sphaleron tries to drive (B+L) to zero, but the U(1)$_Y$ field sources (B+L) and prevents B,L → 0.

$$\partial j_B \sim W \tilde{W} - Y \tilde{Y}$$

In the **broken phase** (T < 160 GeV), the EW sphaleron remains in equilibrium until T~140 GeV. Since the U(1)$_{em}$ field doesn’t source B-number (because, vector-like interactions), why doesn’t B-number washout? ... The U(1)$_{em}$ field sources chiral charge (like in QED) and prevents B-washout in the R-chiral fermions.

\[
\begin{align*}
\frac{d\eta_L}{dx} &= -\gamma_{sph}\eta_L + \gamma_{flip}(\eta_R - \eta_L) - S_{em} \\
\frac{d\eta_R}{dx} &= -\gamma_{flip}(\eta_R - \eta_L) + S_{em}
\end{align*}
\]

\[
\begin{align*}
\eta_{L,eq} &= 0 \\
\eta_{R,eq} &= \frac{S_{em}}{\gamma_{flip}}
\end{align*}
\]