Two recent determinations of photon mass upper limits Perspectives at low radio frequencies Report on research activities

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Plan of the talk

- Motivations and considerations.
- The experimental state of affairs.
- The de Broglie-Proca (dBP) theory.
- Cluster data analysis.
- FRB data analysis.
- Low radio frequencies: LOFAR NENUFAR OLFAR.
- Current investigations (see Luca poster): Heisenberg-Euler and magnetars; Massive photons resulting from SuSY and LS breaking; Radiation from Born-Infeld.

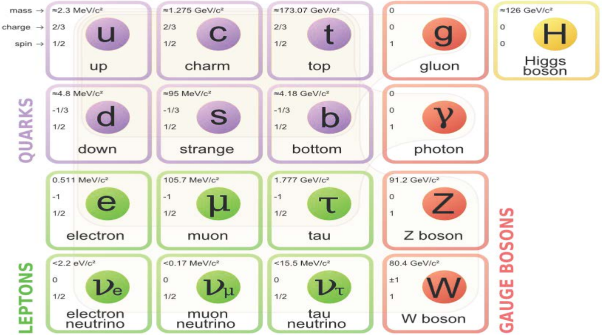
Bonetti L., Ellis J., Mavromatos N.E., Sakharov A.S., Sarkisyan-Grinbaum E.K.G., SPALLICCI A., 2016. Photon mass limits from Fast Radio Bursts, Phys. Lett. B, 757, 548. arXiv:1602.09135 [astro-ph.HE]
Retinò A., SPALLICCI A., Vaivads A., 2016. Solar wind test of the de Broglie-Proca's massive photon with Cluster multi-spacecraft data, to appear in Astropart. Phys., arXiv:1302.6168 [hep-ph]

Perez-Bergliaffa S., Bonetti L., SPALLICCI A., 2016. Electromagnetic shift in the Euler-Heisenberg dipole.
Bentum M., Bonetti L, SPALLICCI A., 2016. Dispersion by pulsars, magnetars and non-Maxwellian electromagnetism at very low radio frequencies.
Bonetti L., dos Santos Rodolfo L., Helayl-Neto A. J., SPALLICCI A., Massive photon and dispersion relations originated by supersymmetry and Lorentz symmetry breaking.

Bonetti L., Piazza F., Perez-Bergliaffa S., SPALLICCI A., Radiation in Born-Infeld electromagnetism.

Investigating non-Maxwellian (nM) theories: motivations

- Understanding of the universe based on electromagnetic observations.
- As photons are the main messengers, fundamental physics has a concern in testing the foundations of electromagnetism.
- 96% of the universe is unknown, and yet precision cosmology.
- Striking contrast: complex and multi-parameterised cosmology - electromagnetism from the 19^{th} century (1826-1867).
- Conversely to the graviton, a mass for the photon isn't frequently assumed.



nM theories: considerations

- non-Maxwellian theories are non-linear (initiated by Born and Infeld; Heisenberg and Euler) or massive photon based (dBP).
- Massive photon and yet gauge invariant theories include: Bopp, Laudé, Podolsky, Stueckelberg, Chern and Simons.
- Always pursued topic: four large reviews from 2005 but none on the thirty ? nM theories.
- Impact on relativity? Difficult answer: variety of the theories above; removal of ordinary landmarks and rising of interwoven implications.
- Experimentalists have mostly conveyed their efforts towards the dBP photon. The upper mass limits of dBP photon mass cannot be generalised to other massive photon theories.
- Massive photons evoked for dark matter, inflation, charge conservation, magnetic monopoles, Higgs boson, redshifts; in applied physics, superconductors and "light shining through walls" experiments. The mass can be considered effective, if depending on given parameters.

Experimental limits 1: Goldhaber and Nieto, Rev. Mod. Phys., 2000

TABLE I. A list of the most significant mass limits of various types for the photon and graviton.

Description of method	$\lambda_C \gtrsim$ (m)	$\mu \lesssim$ (eV)	$\mu \lesssim$ (kg)	Comments
1. Secure photon mass limits:				
Dispersion in the ionosphere (Kroll, 1971a)	8×10^5	3×10^{-13}	10^{-49}	
Coulomb's law (Williams, Faller, and Hill, 1971)	2×10^7	10^{-14}	2×10^{-50}	
Jupiter's magnetic field (Davis, Goldhaber, and Nieto, 1975)	5×10^8	4×10^{-16}	7×10^{-52}	
Solar wind magnetic field (Ryutov, 2007)	2×10^{11} (1.3 AU)	10^{-18}	2×10^{-54}	
2. Speculative photon-mass limits:				
Extended Lakes method (Lakes, 1998; Luo, Tu, Hu, and Luan, 2003a, 2003b; Goldhaber and Nieto, 2003)	$3\times10^9 \Leftrightarrow 3\times10^{12}$	$7\times10^{-7} \Leftrightarrow 7\times10^{-20}$	$10^{-52} \Leftrightarrow 10^{-55}$	$\lambda_C \sim 4R_\odot$ to 20 AU, depending on **B** speculations
Higgs mass for photon (Adelberger, Dvali, and Gruzinov, 2007)	No limit feasible			Strong constraints on 3D Higgs parameter space
Cosmic magnetic fields (Yamaguchi, 1959; Chibisov, 1976; Adelberger, Dvali, and Gruzinov, 2007)	3×10^{19} (10^3 pc)	6×10^{-27}	10^{-62}	Needs const **B** in galaxy regions

Experimental limits 2: What about the graviton?

- LIGO upper limit 2×10^{-58} kg
- Often determination of graviton mass upper limit supposes massless photons

3. Graviton mass limits:

Gravitation wave dispersion (Finn and Sutton, 2002)	3×10^{12}	8×10^{-20}	10^{-55}	Question mark for scalar graviton
Pulsar timing (Baskaran *et al.*, 2008)	2×10^{16}	9×10^{-24}	2×10^{-59}	Fluctuations due to graviton phase velocity
Gravity over cluster sizes (Goldhaber and Nieto, 1974)	2×10^{22}	10^{-29}	2×10^{-65}	
Near field constraints (Gruzinov, 2005)	3×10^{24} (10^{8} pc)	6×10^{-32}	10^{-67}	For DGP model
Far field constraints (Dvali, Gruzinov, and Zaldarriaga, 2003)	3×10^{26} (10^{10} pc)	6×10^{-34}	10^{-69}	For DGP model

Experimental limits 3: dBP photon

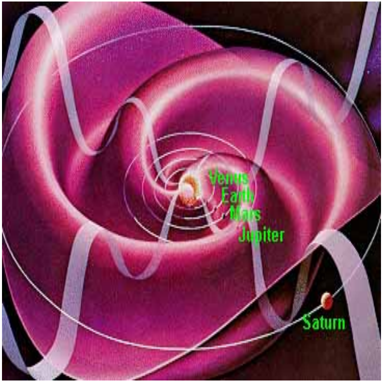
- Laboratory experiment (Coulomb's law) 2×10^{-50} kg.
- Dispersion-based limit 3×10^{-49} kg (lower energy photons travel at lower speed). Note: quantum gravity affects high frequencies (GRB, Amelino-Camelia).
- Ryutov finds $m_\gamma < 10^{-52}$ kg in the solar wind at 1 AU, and $m_\gamma < 1.5 \times 10^{-54}$ kg at 40 AU (PDG value). These values come partly from *ad hoc* models. Limits: (i) the magnetic field is assumed exactly always and everywhere a Parker's spiral; (ii) the accuracy of particle data measurements (from e.g. Pioneer or Voyager) has not been discussed; (iii) there is no error analysis, nor data presentation.
- Speculative lower limits from modelling the galactic magnetic field: 3×10^{-63} kg include differences of ten orders of magnitude on same data.
- New theoretical limits from black holes stability, gravitational light bending, CPT violation.

Experimental limits 4: Warnings

- Quote "Quoted photon-mass limits have at times been overly optimistic in the strengths of their characterisations. This is perhaps due to the temptation to assert too strongly something one knows to be true. A look at the summary of the Particle Data Group (Amsler et al.. 2008) hints at this. In such a spirit, we give here our understanding of both secure and speculative mass limits." Goldhaber and Nieto, Rev. Mod. Phys., 2000
- The lowest theoretical limit on the measurement of any mass is dictated by the Heisenberg's principle $m \geq \hbar \Delta t c^2$, and gives 3.8×10^{-69} kg, where Δt is the supposed age of the Universe.

Experimental limits 5: Parker's spiral

As the Sun rotates, its magnetic field twists into an Archimedean spiral, as it extends through the solar system. This phenomenon is named after Eugene Parker's work: he predicted the solar wind and many of its associated phenomena in the 1950s. The spiral nature of the heliospheric magnetic field had been noted earlier by Hannes Alfvén.



- The concept of a massive photon has been vigorously pursued by Louis de Broglie from 1922 throughout his life. He defines the value of the mass to be lower than 10^{-53} kg. A comprehensive work of 1940 contains the modified Maxwells equations and the related Lagrangian.
- Instead, the original aim of Alexandru Proca, de Broglie's student, was the description of electrons and positrons. Despite Proca's several assertions on the photons being massless, his work has been used.

de Broglie-Proca (dBP) theory 2: SI equations

$$\mathcal{L} = -\frac{q\hbar}{cm_e}\left[\frac{c^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{c^2\mathcal{M}^2}{2}A_\mu A^\mu + \frac{j^\mu}{\epsilon_0}A_\mu\right]^{1/2} \quad (1)$$

$F_{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu$. Minimal action (Euler-Lagrange) $\rightarrow$ inhomogeneous eqs.
Ricci Curbastro-Bianchi identity $\partial^\lambda F^{\mu\nu} + \partial^\nu F^{\lambda\mu}\partial^\mu F^{\nu\lambda} = 0 \rightarrow$ homogeneous eqs.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} - \mathcal{M}^2\phi \ , \quad (2)$$

$$\nabla \times \vec{B} = \mu_0\vec{j} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} - \mathcal{M}^2\vec{A} \ , \quad (3)$$

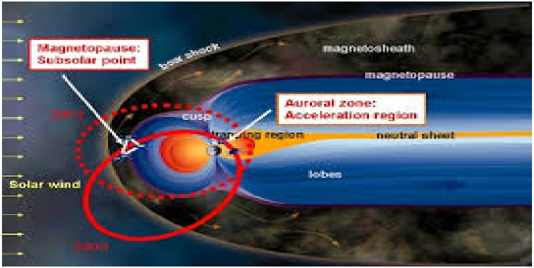
$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t} \ , \quad (4)$$

$$\nabla \cdot \vec{B} = 0 \ , \quad (5)$$

ϵ_0 permittivity, μ_0 permeability, ρ charge density, $\vec{j}$ current, ϕ and $\vec{A}$ potential.
$\mathcal{M} = m_\gamma c/\hbar = 2\pi/\lambda$, $\hbar$ reduced Planck (or Dirac) constant, c speed of light, λ Compton wavelength, m_γ photon mass.

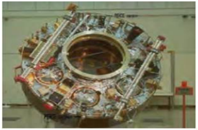
Eqs. (2, 3) are Lorentz-Poincaré transformation but not Lorenz gauge invariant, though in static regime they are not coupled through the potential.

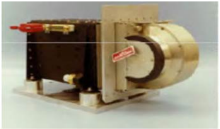
Cluster data analysis 1: the mission



Highly elliptical evolving orbits in tetrahedron: perigee 4 $R_\oplus$ apogee 19.6 $R_\oplus$, visited a wide set of magnetospheric regions.

Inter-spacecraft separation ranging from 10^2 to 10^4 km.





Cluster data analysis 3: the philosophy

- Small mass $\rightarrow$ precise experiment or very large apparatus (Compton wavelength). The largest-scale magnetic field accessible to *in situ* spacecraft measurements, *i.e.* the interplanetary magnetic field carried by the solar wind.
- First time direct computation of 3D quantities as $\nabla \times \vec{B}$ and thus j_B using the curlometer from the 4 fluxgate magnetometers. This method allows to avoid assumptions on the field analytical form (only assuming linear gradients).
- $B_x > 0$, $B_y > 0$ and $B_x, B_y \gg B_z$, as expected for a Parker's spiral configuration close to the ecliptic plane.
- Our analysis does not rely on the Parker's model, since the magnetic field is measured *in situ*. The conditions are similar to those presented by Ryutov (1997, 2007), PDG limit, for comparison.
- Cluster carries also particle detectors.

Cluster data analysis 4: the philosophy

Since we are interested in the large-scale steady components of the magnetic field, *i.e.* to very low frequencies, the displacement current density in Eq. (3) can be dropped: indeed

$$\epsilon_0\mu_0\frac{\partial E}{\partial t}\sim\epsilon_0\mu_0\frac{Ev_{sw}}{L_B}\sim\epsilon_0\mu_0\frac{Bv_{sw}^2}{L_B}\sim 2\times 10^{-22}\ \mathrm{Am}^{-2}\ ,$$

being $v_{sw} = 4\times 10^2$ km s^{-1} the typical solar wind velocity, and L_B the caracteristic length of the magnetic field.
The dBP modified Ampère's law reads

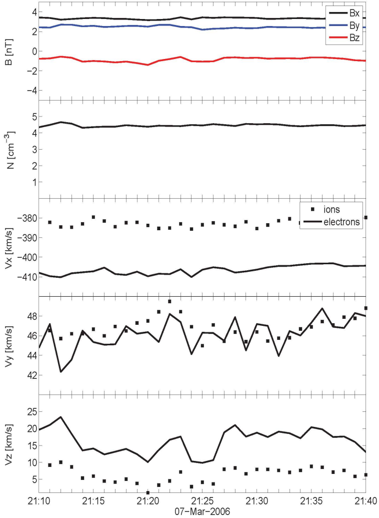
$$\nabla\times\vec{B} = \mu_0\vec{j} - \mathcal{M}^2\vec{A}\ . \tag{6}$$

Cluster data analysis 5: the philosophy

- For $\vec{j}_B = \nabla \times \vec{B}/\mu_0$ and $\vec{j} = \vec{j}_P = ne(\vec{v}_i - \vec{v}_e)$, n the number density, e the electron charge, $\vec{v}_i$, $\vec{v}_e$ the velocity of the ions and electrons, respectively, the dBP photon mass is

$$m_\gamma = \frac{k}{\left|\vec{A}_H\right|^{\frac{1}{2}}} \left| ne(\vec{v}_i - \vec{v}_e) - \frac{\nabla \times \vec{B}}{\mu_0} \right|^{\frac{1}{2}} = \frac{k \left|\vec{j}_P - \vec{j}_B\right|^{\frac{1}{2}}}{\left|\vec{A}_H\right|^{\frac{1}{2}}} , \qquad (7)$$

  where $k = \hbar \mu_0^{\frac{1}{2}} c^{-1}$, and $\vec{A}_H$ is the vector potential from the interplanetary magnetic field.
- Event selection to compare with PDG (1 AU) limit: (i) an *undisturbed* solar wind, *i.e.* disconnected from the terrestrial bow shock, far from the terrestrial H; (ii) the closest location of the spacecraft to the equatorial plane; (iii) the widest inter-spacecraft separation, 10^4 km, assuring the largest differences in H among the spacecraft; (iv) the configuration best approaching the tetrahedron; (v) the availability of good quality particle currents.



Panel (a). The three components of the magnetic field for Cluster 3 in the GSE (Geocentric Solar Ecliptic) coordinate system. Panel (b). The average plasma density. Panels (c,d,e).

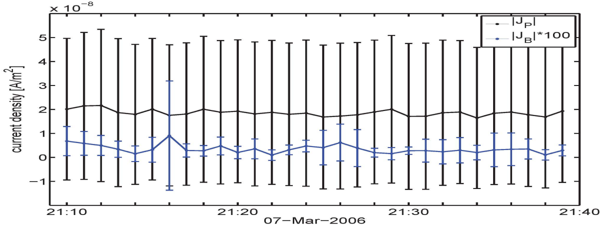
The v_x, v_x, v_x velocity components of ions (dotted line) and electrons (full line).

Cluster data analysis 7: vector potential 3 methods

- $\vec{\nabla} \times \vec{B} = \vec{A}$ holds in dBP theory, but the Ampére eq. depends on $\vec{A}$ (not Lorenz gauge invariant). A_H is measurable and implies a change in the field. Three estimates for A_H.
- Four point estimate: for $L_B \sim B/\mu_0 j_B \approx 9.6 \times 10^7$ m, then $A_H \sim B \times L_B \approx 4.1 \times 10^{-1}$ T m. Advantages: no assumptions on the structure of B and on the lack of steadiness of the solar wind. Disadvantage: based on first derivatives of B, not suited for small volumes.
- Single spacecraft estimate: a single spacecraft monitors B (scalar field) as it advects past the spacecraft transported by the temporal variations in the solar wind.
$A_H^Z \sim \int B^Y(t)dx \sim v_{sw} \int B^Y(t)dt \approx 29$ T m.
- Parker's model estimate: $A_H = 637$ T m . The vector potential is computed in Coulomb's gauge $\nabla \cdot \vec{A} = 0$. The dBP equations are not Lorenz gauge invariant but automatically satisfy the Lorenz gauge, that is $\nabla \cdot \vec{A} + 1/c^2 \partial\phi/\partial t = 0$. Thus, in our Coulomb gauge case, the scalar potential ϕ must be constant in time. This latter condition inserted in the time derivative of $\vec{E} = \nabla\phi - \partial\vec{A}/\partial t$, and recalling that we deal with a static case for which $\partial\vec{E}/\partial t = 0$, implies that A_H varies at most linearly in time. Indeed, the event under scrutiny

Cluster data analysis 8: particle current

- The particle current density $\vec{j} = \vec{j}_P = ne(\vec{v}_i - \vec{v}_e)$ from ion and electron currents; n is the number density, e the electron charge and $\vec{v}_i$, $\vec{v}_e$ the velocity of the ions and electrons, respectively.
- An accurate assessment of the particle current density in the solar wind is difficult due to inherent instrument limitations.
- $j_P >> j_B$ (up to four orders of magnitude), mostly due to the differences in the i, e velocities, while the estimate of density is reasonable. While we can't exclude that this difference is due to the dBP massive photon, the large uncertainties related to particle measurements hint to instrumental limits.



Cluster data analysis 9: our mass limit

- $j_P = 1.86 \cdot 10^{-7} \pm 3 \cdot 10^{-8}$ A m^{-2}, while $j_B = |\nabla \times \vec{B}|/\mu_0$ is $3.5 \pm 4.7 \cdot 10^{-11}$ A m^{-2}. A_H is an estimate, not a measurement.

$$A_H{}^{\frac{1}{2}}(m_\gamma + \Delta m_\gamma) = A_H{}^{\frac{1}{2}}\left(m_\gamma + \left|\frac{\partial m_\gamma}{\partial j_P}\right|\Delta j_P + \left|\frac{\partial m_\gamma}{\partial j_B}\right|\Delta j_B\right) = k\left[(j_P - j_B)^{\frac{1}{2}} + \frac{\Delta j_P + \Delta j_B}{2(j_P - j_B)^{\frac{1}{2}}}\right] . \qquad (8)$$

Considering j_P and Δj_P of the same order, $j_P = 0.62\ \Delta j_P$, and both much larger than j_B and Δj_B, Eq. (8), after squaring, leads to

$$A_H{}^{\frac{1}{2}}(m_\gamma + \Delta m_\gamma) \sim k\,(j_P + \Delta j_P)^{1/2} . \qquad (9)$$

Table: The values of m_γ (according to the estimate on A_H).

A_H [T m]	0.4	29 (Z)	637
m_γ [kg]	1.4×10^{-49}	1.6×10^{-50}	3.4×10^{-51}

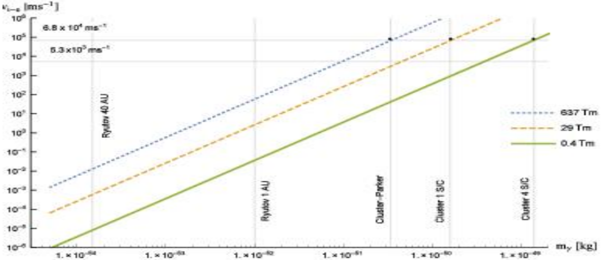
Cluster data analysis 11: the technology

Most stringent limitation comes from particle detectors. The difference between ion and electron velocities is (Cluster performance)

$$v_{i-e} \sim \frac{(j_P + \Delta j_P)}{ne} \approx 6.8 \times 10^4 \ \mathrm{m\ s^{-1}} \tag{10}$$

$n = 4.46 \times 10^6 \ \mathrm{m}^{-3}$ electron density. Writing $v_{i-e}(m_\gamma)$, we have v_{i-e} that particle detectors should measure to resolve an upper bound for m_γ.

The upper limit 10^{-52} kg (Ryutov PDG) requires v_{i-e} 3.6×10^{-2}, 2.6, 5.7×10^1 m s^{-1} (for the three values of A_H) not possible with current technology. Else, $A_H = 7.5 \times 10^5$ T m for $v_{i-e} \approx 6.8 \times 10^4$ m s^{-1}, but a value of 10^3 T m is stated.



Ryutov analysis 1

Ryutov [1997] first refers to the limit of 10^{-51} kg, relative to the Jupiter magnetic field, obtained by others [Davis, Goldhaber, Nieto, 1975]. Then he discusses that an imbalance of the magnetic forces in the neighbouring areas of the solar wind would have caused violent plasma motions with an average energy exceeding the energy of the ions by three orders of magnitude; since such motions are not observed in the solar wind, the author lowers the estimate in [Davis, Goldhaber, Nieto, 1975] of one order of magnitude, and sets the mass upper limit at 1 AU.
Ryutov [2007] refers to Voyager 1 and Voyager 2 data appeared in previous work to justify strict use of Parker's model, and to adopt the *reductio ad absurdum* approach already used in [Ryutov, 1997]. Indeed, on the basis that the magnetic field is almost entirely azimuthal, Ryutov considers that the Lorentz force would be increased of a factor $(2\pi L_B/\lambda)^2$, where L_B is the magnetic field characteristic length, with respect to the Maxwellian case. Since the deviations from the observed flow structure would become grossly incompatible with the real situation, the mass upper limit is lowered to 1.5×10^{-54} kg at 40 AU. A margin of a factor three constitutes the error budget.
Finally, the authors of [Liu, Shao, 2012], again without presenting data, argue that the mass upper limit could be lowered of a factor two.

For checking such solar wind estimates, we therefore attempt a more experimental approach.

Cluster data analysis 12: improvements, conclusions, perspectives

- Consider only the z component.
- Set artificially but justifiably $j_p = j_B$. Why? a) Confidence on previous literature results; b) difference between ion and electron velocities cannot be very large. Consequence If A_H Parker, then PDG limit.
- Ultimately, a technological revolution for particle detectors.
- A zero cost experiment based a non-dedicated mission leads to a result just one order of magnitude worse than ground experiment.
- Only solar wind test considering in detail the experimental errors.
- The domain between our findings ($m_\gamma < 1.4 \times 10^{-49}$ kg) and the results from *ad-hoc* model in the solar wind ($m_\gamma < 1.5 \times 10^{-54}$ kg) is still subjected to assumptions and conjectures, though fewer now, and not to clear-cutting outcomes from experiments. Our experiment is limited by the resolution of the velocity difference between ions and electrons.

Retinò A., SPALLICCI A., Vaivads A., 2016. Solar wind test of the de Broglie-Proca's massive photon with Cluster

de Broglie-Proca (dBP) theory 3: dispersion relations

From the Lagrangian we get $\partial_\alpha F^{\alpha\beta} + \mathcal{M}^2 A^\beta = \mu j^\beta$. With the Lorentz subsidiary condition $\partial_\gamma A^\gamma = 0$,

$$\left[\partial_\mu \partial^\mu + \mathcal{M}^2\right] A^\nu = 0 \tag{11}$$

Through Fourier transform, at high frequencies (photon rest energy ¡ the total energy; $\nu \gg 1$ Hz), the positive difference in velocity for two different frequencies ($\nu_2 > \nu_1$) is

$$\Delta v_g = v_{\rm g2} - v_{\rm g1} = \frac{c^3 \mathcal{M}^2}{8\pi^2}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) , \tag{12}$$

being v_g the group velocity. For a single source at distance d, the difference in the time of arrival of the two photons is

$$\Delta t = \frac{d}{v_{\rm g1}} - \frac{d}{v_{\rm g2}} \simeq \frac{\Delta v_g d}{c^2} = \frac{dc\mathcal{M}^2}{8\pi^2}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)$$
$$\simeq \frac{d}{c}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) 10^{100} m_\gamma^2 \, . \tag{13}$$

Entanglement plasma and photon mass, FRBs

- Such behaviour reproduces interstellar dispersion the delay in pulse arrival times across a finite bandwidth. Dispersion occurs due to the frequency dependence of the group velocity of the pulsed radiation through the ionised components of the interstellar medium. Pulses emitted at lower radio frequencies travel slower through the interstellar medium, arriving later than those emitted at higher frequencies.
- In absence of an alternative way to measure plasma dispersion, there is no way to disentangle plasma effects from a dBP photon.
- Data on FRB 150418 indicate $m_\gamma \lesssim 1.8 \times 10^{-14}$ eV c^{-2} (3.2×10^{-50} kg), if FRB 150418 has a redshift $z = 0.492$. In the future, the different redshift dependences of the plasma and photon mass contributions to DM can be used to improve the sensitivity to m_γ. Bonetti L., Ellis J., Mavromatos N.E., Sakharov A.S., Sarkisyan-Grinbaum E.K.G., SPALLICCI A., 2016. Photon mass limits from Fast Radio Bursts, Phys. Lett. B, 757, 548. arXiv:1602.09135 [astro-ph.HE].

Other investigations

- MMS four satellite data for a Cluster-like data analysis
- Heisenberg-Euler on magnetars overcritical magnetic field : Luca Bonetti poster.
- International collaboration for OLFAR proposed to ESA: a swarm of nano-satellites opening the 100 KHz-30 MHz window.
- Extensions of the Standard Model (SM) address issues like the Higgs boson mass discrepancy, the dark universe, neutrino oscillations and their mass. We focus on models involving Super and Lorentz symmetries breaking and analyse four general classes of such models in the photon sector. All dispersion relations show a non-Maxwellian behaviour for the, phenomenologically both present, CPT (Charge-Parity-Time reversal symmetry) even and odd sectors. In the latter, a massive photon behaviour in the group velocities emerges. Then, we extract a massive and gauge invariant Carroll-Field-Jackiw term in the Lagrangian and show that the photon mass is proportional to the background vector. The mass is lower than 10^{-26} eV or 10^{-62} kg. Finally, we discuss other extensions to the SM and comment on the likelihood of a massive photons being inherent in such formulations Luca Bonetti poster.

Mersi (Piemontèis)

Other non-Maxwellian (nM) theories 1: Stueckelberg

- The Stueckelberg Lagrangian

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + m^2\left(A_\mu - \frac{\partial_\mu B}{m}\right)^2 - (\partial^\mu A_\mu + mB)^2 \qquad (14)$$

  where B is a scalar field to render the dBP *manifestly* gauge invariant.

- We have two fields and two equations of motion. The wave equations are

$$\partial_\mu\partial^\mu A^\nu + m^2 A^\nu = 0 \qquad (15)$$

$$\partial_\mu\partial^\mu B + m^2 B = 0 \qquad (16)$$

- First massive photon theory, gauge invariant

$$A_\mu \rightarrow A_\mu + \partial_\mu\Lambda \qquad B \rightarrow B + m\Lambda \qquad (\partial^2 + m^2)\Lambda = 0$$

- Used as alternative to dark energy, Akarsu et al., 2014 arXiv:1404.0892.

Other non-Maxwellian (nM) theories 2: Podolsky

- The Podolsky Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{b^2}{4}\left(\partial^\nu F^{\mu\nu}\right)\partial_\nu F_{\mu\nu} + j^\mu A_\mu \qquad (17)$$

where b has the dimension of m^{-1}.

- The equations are

$$-b^2\partial_\mu\partial^\mu\left(\vec{\nabla}\cdot\vec{E}\right) + \vec{\nabla}\cdot\vec{E} - \rho = 0 \qquad (18)$$

$$-b^2\partial_\mu\partial^\mu\left[\frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B}\right] + \frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B} + \vec{j} = 0 \qquad (19)$$

- Gauge invariant $A_\mu \rightarrow A_\mu + \partial_\mu\Lambda$
- Magnetic monopoles? and massive photons.
- Cut-off for short distances $\phi = \frac{e}{4e\pi}(1 - e^{-\frac{r}{b}})$

Other non-Maxwellian (nM) theories 3: Born-Infeld

- The Born-Infeld Lagrangian

$$\mathcal{L} = \sqrt{1+F} - 1 + j^\mu A_\mu \tag{20}$$

- The equations are

$$\partial_\mu \left(\frac{F^{\mu\nu} (1+F)^{-\frac{1}{2}}}{2} \right) = j^\nu \tag{21}$$

- Electromagnetic mass. The mass is derived from the field energy.
- Avoidance of infinities out of self-energy $\phi = \frac{e}{r_0} f\left(\frac{r}{r_0}\right)$
- The parameter b poses a limit to the electric field (to be understood).

Other non-Maxwellian (nM) theories 4: Euler-Heisenberg

- The Euler-Heisenberg Lagrangian

$$\mathcal{L} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \frac{e^2}{\hbar c}\int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3}\cdot\left\{ i\frac{\eta^2}{2}F^{\mu\nu}F^*_{\mu\nu}\cdot \right.$$

$$\cdot\frac{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}+iF^{\mu\nu}F^*_{\mu\nu}}\right]+\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}-iF^{\mu\nu}F^*_{\mu\nu}}\right]}{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}+iF^{\mu\nu}F^*_{\mu\nu}}\right]-\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}-iF^{\mu\nu}F^*_{\mu\nu}}\right]}$$

$$\left. + |\mathfrak{E}_k|^2 + \frac{\eta^3}{6}\cdot F_{\mu\nu}F^{\mu\nu}\right\} \qquad (22)$$

$$F^*_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \qquad \mathfrak{E}_k = \frac{m^2c^3}{e\hbar} \sim 10^{16}\frac{V}{m} \qquad (23)$$

$\mathfrak{E}_k$ critical field for creating electron-positron pairs from vacuum.

- Light-Light scattering.
- Particle creation on cosmological scale (Starobinsky and others).
- Photon splitting.
  http://www.nature.com/news/2010/100728/full/news.2010.381.html