Cosmological Seed Magnetic Field From Inflation

Theory and Observations of Extragalactic Magnetic Fields   Paris

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Until recently, only upper bounds on the larger-scale intergalactic magnetic field (IGMF), < few nG now.

Within last year groups claimed IGMF was ≥ few fG (10^{-15} G!) now, so as to deflect e^-e^+ pairs from blazar TeV γs interacting with IR background to agree with data, or to explain γ-ray halos around Fermi AGNs.

Neronov & Vovk Science 328, 73 (2010); Tavecchio et al MNRAS 406, L70 (2010); Ando & Kusenko ApJ 722, L39 (2010);Tavecchio et al 1009.1048; Dolag et al 1009.1782; but see Dermer et al 1011.6660

Further confirmation will make the study of cosmological magnetic fields an even more interesting and active research area.

This is almost certainly beyond Standard Model physics.

Need a primordial "seed" field

Universe is a good conductor now so MHD is a good approx:

$$\partial_t \mathbf{B} = \boldsymbol{\partial}_\mathbf{x} \times (\mathbf{v} \times \mathbf{B}) + \partial_x^{\,2} \mathbf{B}/(4\pi\sigma)$$

No source: $\mathbf{B}(t_i) = 0 \Rightarrow \mathbf{B}(t_i + \Delta t) = 0$ => Initial (non-MHD) seed field needed if we are to explain current B fields.

Galactic seed field might be:

Small-scale, "bottom-up" (stars, star clusters, galactic nuclei, …); can only use in dynamo model

Large-scale, "top-down" (cosmological)

Phenomenological Amplification Models

Anisotropic protogalactic collapse and differential rotation model (less effective amplifier, but does most of it quickly). Piddington (1970); Kulsrud (1986). Needs $10^{-13} – 10^{-11}$ G seed field.

Galactic dynamo model (no consistent model; amplifies exponentially on a rotation time scale; enough rotations result in a large field; high-z objects have many fewer rotations). Parker (1971); Vainshtein & Ruzmaikin (1971). Needs $10^{-24} – 10^{-16}$ G seed field.

Some cosmological seed field models:

If there is large-scale vorticity in the radiation epoch, ions spin down relative to the Thomson coupled e's and CMB γ's resulting in an IGMF of 10^{-23} G. Harrison(1969) Barely large enough for the dynamo; no vorticity in inflation-based models.

Early universe phase transitions (PTs):

QCD PT: 10^{-38} G on 1 Mpc Hogan (1983); Quashnock et al (1989)

Electroweak 2nd order PT: 10^{-31} G on 1 Mpc Vachaspati (1989)

Too small, but can get variants to give larger values, but almost certainly not large enough.

Try inflation …

Cosmological Seed Magnetic Field From Inflation

Exponential potential scalar field inflation

Maxwell $L_{U(1)} \sim \sqrt{-g}\, F^{\mu\nu} F_{\mu\nu}$ does not work

Modifying $L_{U(1)}$ (toy model)

Inflation, radiation, and baryon epoch computations

Results, numbers, consequences

What remains to be done, generalizations

BR ApJL 391 (1992) (rejected by PLB)

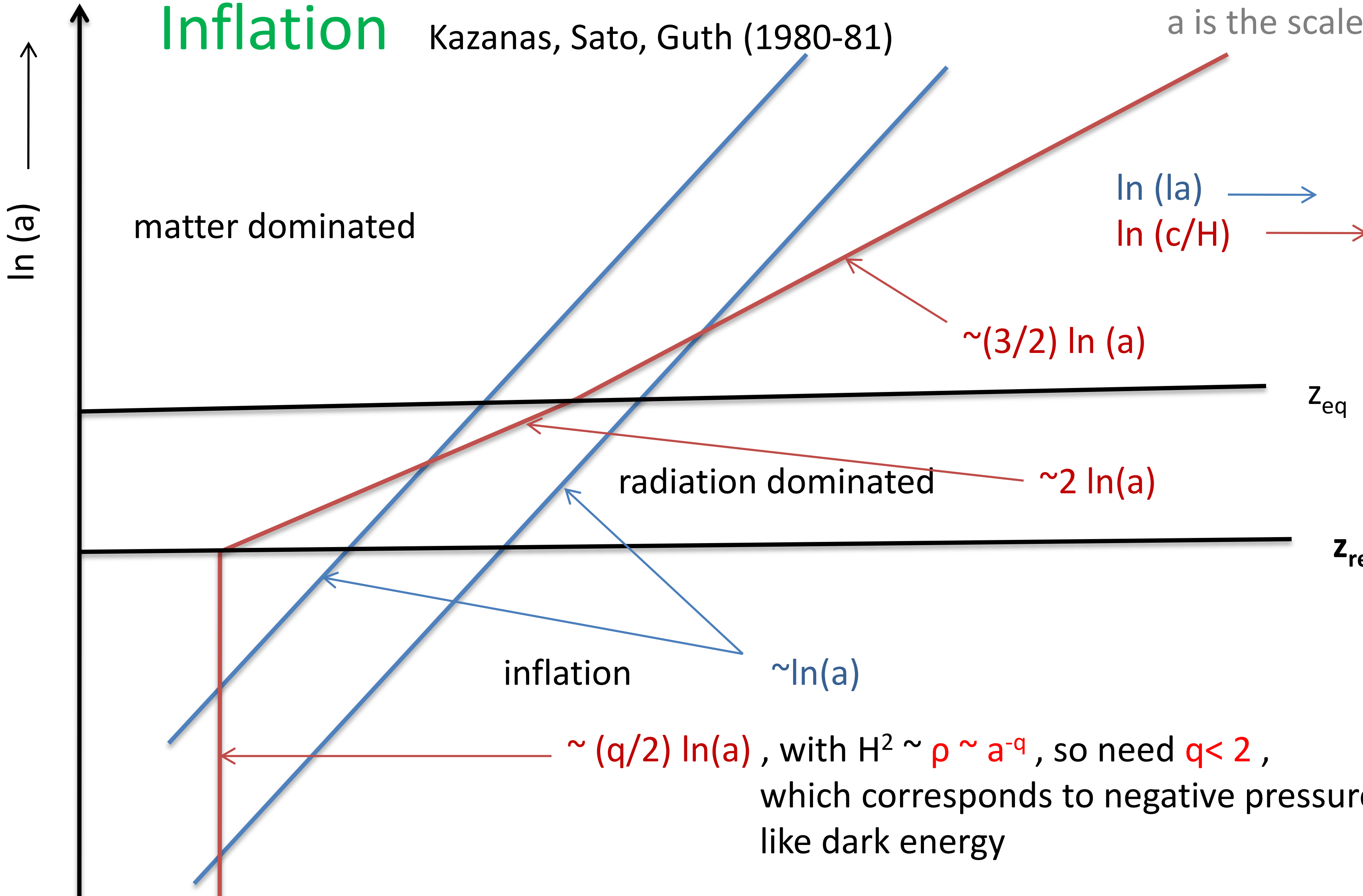
+ Caltech preprint (rejected by PRD) at www.phys.ksu.edu/personal/ratra +

(Backreaction discussed in detail in Caltech preprint; it is not a problem.)

Inflation

Kazanas, Sato, Guth (1980-81)

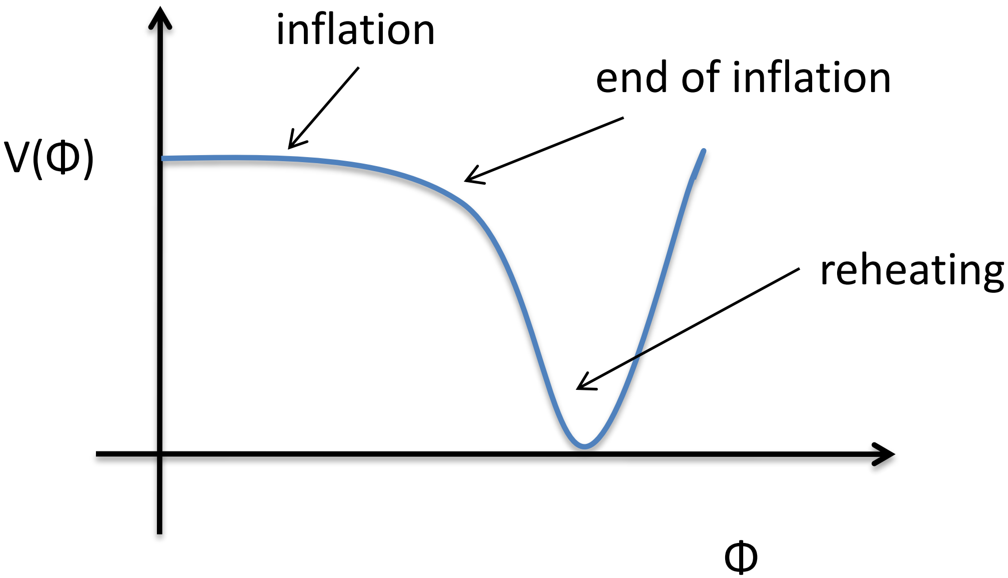
a is the scale factor



~ (q/2) ln(a) , with $H^2 \sim \rho \sim a^{-q}$, so need $q < 2$, which corresponds to negative pressure like dark energy

More precisely, conservation of energy, $\dot{\rho} = -3\,(\dot{a}/a)\,(\rho + p)$ => $p = (1 - q/3)\,\rho$

So, to model inflation use a scalar field ….



Spatially homogeneous background $\Phi(t)$

$\rho_\Phi \sim 1/2(d\Phi/dt)^2 + V \sim V$

$p_\Phi \sim 1/2(d\Phi/dt)^2 - V \sim -V$

$\Rightarrow p_\Phi = -\rho_\Phi$ (negative pressure)

$\dot{\rho} = -3\,(\dot{a}/a)\,(\rho + p)$

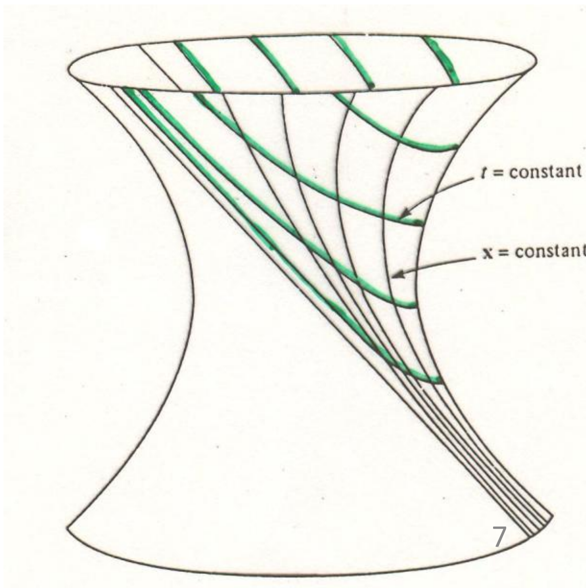
$\Rightarrow \rho_\Phi = \text{constant}$

$\Rightarrow H = \text{constant}$

Friedmann equation $\ddot{a}/a = -(4\pi G/3)(\rho + 3p) > 0$

$\Rightarrow$ accelerated expansion, like dark energy.

Simplest solution is spatially-flat de Sitter with $a(t) \sim \exp(Ht)$.



More generally, instead of a flat potential we can have $V(\phi) \sim \exp(-\sqrt{q/2}\,\phi)$, then free parameters are q and z_{re}, and $a(t) \sim t^{2/q}$.

No direct evidence for the inflaton scalar field, however there is persuasive indirect evidence.

Spatial irregularities

During inflation, quantum mechanics generates small-scale zero-point fluctuations (gravity modifies the usual ground state in a manner similar to what happens for the Casimir effect).

These are stretched by the expansion to cosmological length scales in the late time universe.

Hawking, Starobinsky, Guth & Pi (1982)

This happens for any field present during inflation.

Inflation epoch gravity & scalar field action:

$S = \int dt\, d^3x \sqrt{-g}\, [\, -R + \tfrac{1}{2}\, g^{\mu\nu}\, \partial_\mu\phi\partial_\nu\phi - V(\phi)]$

To this add the electromagnetism action:

$S_{EM} = \int dt\, d^3x \sqrt{-g}\, (1/\hat{e}^2)\, F^{\mu\nu} F_{\mu\nu}$

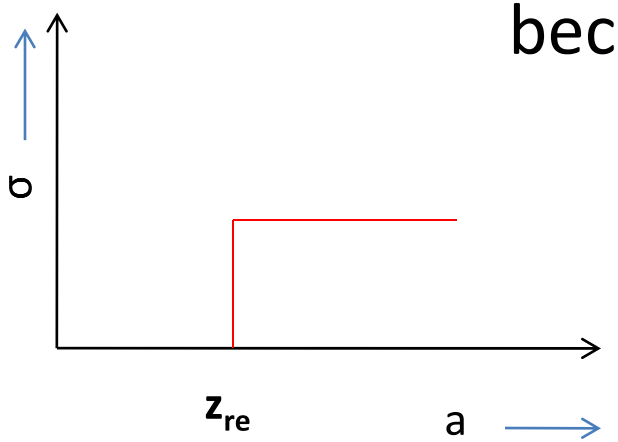
Flux (~ |**B**| X area) is conserved =>

$|\mathbf{B}| \sim 1/\text{area} \sim a^{-2}$

Model charged particles at reheating.

Two free parameters: z_{re} and q.

At z_{re} $E \rightarrow 0$ as the universe becomes a good conductor.

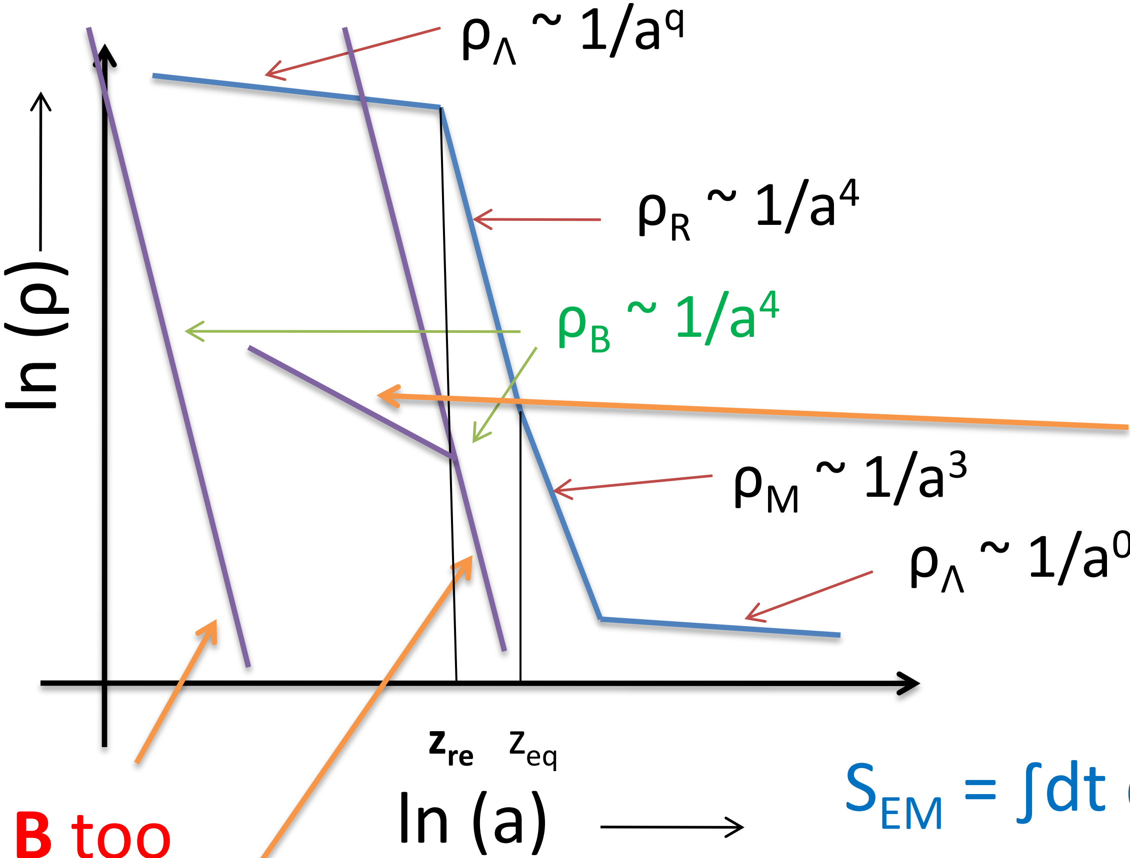


At present, in wavenumber k space $< |B(k)|^2 > \sim k/a^4$,

so $<B^2(r = 10^{-3}/H_0)> = 10^{-59}$ G now.

Too small.

The problem and a solution



So change relation between **B** and ρ during inflation by making coupling constant time variable through a dilaton-like coupling:

B too small.

Increase **B**, but then no inflation.

$$S_{EM} = \int dt\, d^3x \sqrt{-g}\, (1/\hat{e}^2)\, e^{\alpha\phi} F^{\mu\nu} F_{\mu\nu}$$

$\rho_B \sim T_{00} \sim \mathbf{B}^2/\mu$. During inflation: $\mu \sim e^{-\alpha\phi} \sim \varepsilon^{-1}$. After inflation, Φ freezes at VEV, $\mu = 1 = \varepsilon^{-1}$ and standard U(1) is recovered.

Three free parameters: z_{re} , q and α.

Synchronous and temporal gauge: **Inflation Epoch**

$\ddot{A}_i + (\dot{a}/a + \alpha\dot{\phi}_b)\dot{A}_i + (\partial_i\partial_j A_j - \partial_j\partial_j A_i)/a^2 = 0,$ $\partial_i \dot{A}_l = 0.$

Solve for A_i. Initial conditions determine constants of integration. On small scales, inside the Hubble radius, appropriately rescaled A_i modes should be in the harmonic oscillator ground state in terms of conformal time.

Then on large scales magnetic field

$$B \sim \begin{cases} a^{-2}\, k^{1-\nu} & \text{for } \nu \geq 0 \text{ or } \alpha \geq -(2-q)/(2\sqrt{(2q)}) \\ a^{-(6-q+2\alpha\sqrt{(2q)})/2}\, k^{1+\nu} & \text{for } \nu \leq 0 \text{ or } \alpha \leq -(2-q)/(2\sqrt{(2q)}) \end{cases}$$

where $\nu = ½ + \alpha\sqrt{(2q)}/(2-q)$. Compared to ordinary electromagnetism, where $B \sim a^{-2}\, k^{1/2}$, power is shifted to the infrared for $\nu > ½$ & $\nu < -½$.

And on large scales electromagnetic energy density

$$\varepsilon \sim \begin{cases} a^{-(4+\alpha\sqrt{(2q)})}\ k^{2\nu} & \text{for } \alpha < 0 \\ a^{-(4-\alpha\sqrt{(2q)})}\ k^{2(1+\nu)} & \text{for } \alpha > 0 \end{cases}$$

where $\nu = ½ + \alpha\sqrt{(2q)}/(2-q)$. Compared to ordinary electromagnetism where $\alpha = 0$, for $\alpha \neq 0$ the U(1) energy density increases slower than a^{-4} as $a \to 0$.

Radiation, Matter Epochs

Synchronous and temporal gauge and using Ohm's law:

$\ddot{A}_i + (\dot{a}/a + 4\pi\sigma)\dot{A}_i + (\partial_i\partial_j A_j - \partial_j\partial_j A_i)/a^2 = 0,$ $\partial_i \dot{A}_l = 0$;

σ is conductivity. Solve for A_i.

Joining conditions determine constants of integration; at reheating σ jumps, ϕ freezes at VEV: $(1/\hat{e}^2)\, e^{\alpha\phi}(t_{re})=(1/e^2)$ and usual U(1) is recovered. σ does not jump at radiation-matter transition.

Take $\sigma \to \infty$ limit as the late universe is a good conductor.
E vanishes at late times.

(Ignore dark energy.)

Normalize density perturbations to large-scale structure or CMB anisotropy observations. Then at the present time

$<|B(k)|^2> \sim k^{\hat{n}} a^{-4}$ $-3 < \hat{n} < 2$

with amplitude range 10^{-65} G $\leq$ $<\mathbf{B}^2(r=10^{-3}/H_0)>^{1/2} \leq 10^{-9}$ G.

At the upper end, $<|\delta(k)|^2> \sim k$, HPYZ scale-invariant energy-density spectrum, with a scale-invariant magnetic field spectrum, $<|B(k)|^2> \sim k^{-3} a^{-4}$ or $<\mathbf{B}^2(r)>^{1/2} \sim r$ and normalization $<\mathbf{B}^2(r=10^{-3}/H_0)>^{1/2} \sim 10^{-9}$ G.

Close to this limit, the model provides energy-density perturbations that are observationally consistent and seed magnetic field amplitude strong enough even for the collapse and differential rotation amplification scenario!

Extensions and Other (Inflation) Options

Careful confirmation. Bamba & Yokoyama PR D69, 043507 (2004); Martin & Yokoyama JCAP 0801, 025 (2008). (Backreaction is not a problem: Demozzi et al JCAP 0908, 025 (2009).)

Use two scalar fields: an inflaton and dilaton-like one. This is also viable. Lemoine & Lemoine PR D52, 1955 (1995); Gasperini et al PRL 75, 3796 (1995); Bamba & Yokoyama PR D69, 043507 (2004).

Use trace anomaly to break conformal invariance. Might work in models with very many particle species. Dolgov PR D48, 2499 (1993).

Axion electrodynamics. Might be able to generate an interesting amount of small-scale magnetic helicity but not a large-scale magnetic field. Garretson et al PR D46, 5346 (1992); Prokopec astro-ph/0106247; Campanelli IJMP D18, 1395 (2009).

Scalar electrodynamics. Inflation-generated current might produce a (too weak?) large-scale magnetic field. Turner & Widrow PR D37, 2743 (1988); Calzetta et al PR D57, 7139 (1998); Giovannini & Shaposhnikov PR D62, 103512 (2000); Calzetta & Kandus PR D65, 063004 (2002). (Massive fermions not yet studied much.)

Non-Einsteinian gravity or gravitational coupling to U(1) field. Probably can generate a large enough magnetic field but not yet clear this is consistent with general relativity tests. (Are we that desperate?) Turner & Widrow PR D37, 2743 (1988); Mazzitelli & Spedalieri PR D52, 6694 (1995); Lambiase & Prasanna PR D70, 063502 (2004); Akhtari-Zavareh et al PTP 117, 803 (2007); Campanelli et al PR D77, 123002 (2008).

Nonlinear electrodynamics (DBI, Heisenberg-Euler,...). There is probably enough freedom here also. Garousi et al PL B606, 1 (2005); Ganjali JHEP 0509, 004 (2005); Kunze PR D77, 023530 (2008); Campanelli et al PR D77, 043001 (2008).

Can also use extra spatial dimensions or break gauge invariance or Lorentz invariance or do other things that some feel should not be discussed in public.

Summary

Need a seed field, possibly a large-scale cosmological one; this could soon be settled by new data.

It seems we could be seeing the first indications of another beyond Standard Model physics phenomenon, to add to density perturbations, dark matter, neutrino and other masses and couplings, strong CP, matter-antimatter asymmetry, and dark energy. This could have very interesting implications.

There are a number of inflation models that can work. I discussed in some detail the simplest mechanism that can generate a large enough magnetic seed field for probably any amplification model.

Open Questions

In the case of interest, when $<B^2(r=10^{-3}/H_0)>^{1/2} \sim 10^{-9}$ G, the model is strongly coupled early on. OK as is, but can (should) charged particles be present during inflation?

Need account for dark energy. Need improve reheating model.

Need account for electroweak transition. Field during inflation is hypercharge field, if there is no GUT or other model. $\sqrt{2}$ increase in B field amplitude.

Can this phenomenological mechanism work in a more realistic model, inspired by high energy physics?

What is the underlying quantum model that results in this effective semiclassical mechanism?

Work out more observational predictions, derive limits.

We are all agreed that your theory is crazy. The question that divides us is whether it is crazy enough to have a chance of being correct.

Bohr to Pauli (1958)

(Heisenberg-Pauli nonlinear field theory of elementary particles was not crazy enough.)

Better observational data will show if we are crazy enough.