

Primordial Magnetic Fields from Pre-inflation

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Inflaton Coupling to Electromagnetism

$$S_{\text{inflaton-photon}} = \int d^4x \quad 4c \left(\phi/M_{\text{Pl}} \right) \mathbf{E} \cdot \mathbf{B}$$

$$\left(\nabla^2 - \frac{\partial^2}{\partial \tau^2} \right) \mathbf{B} = 4c \frac{d\theta}{d\tau} \nabla \times \mathbf{B}$$

$$\begin{aligned} \tau &= \eta / H \\ d\eta &= H dt / a(t) \\ \theta &= \phi / M_{\text{Pl}} \end{aligned}$$

η conformal time
 a cosmic scale factor
 H Hubble parameter

$$\mathbf{B} = \nabla \times \mathbf{A}_T$$

$$\begin{aligned} \mathbf{A}_T &= \int \frac{d^3\mathbf{k}}{\sqrt{2(2\pi)^3 k}} \\ &\times \left[e^{i\mathbf{k} \cdot \mathbf{x}} \sum_{\lambda=\pm} b_{\lambda\mathbf{k}} V_{\lambda\mathbf{k}}(\tau) \boldsymbol{\epsilon}_{\lambda\mathbf{k}} + \text{h.c.} \right], \end{aligned}$$

$$\frac{d^2}{d\eta^2} V_{\pm q} + \left(q^2 \mp 4cq \frac{d\theta}{d\eta} \right) V_{\pm q} = 0,$$

with initial conditions at early epoch given by

$$V_{\pm q} = 1, \quad \frac{dV_{\pm q}}{d\eta} = -iq, \quad \mathbf{q} = \mathbf{k}/H$$

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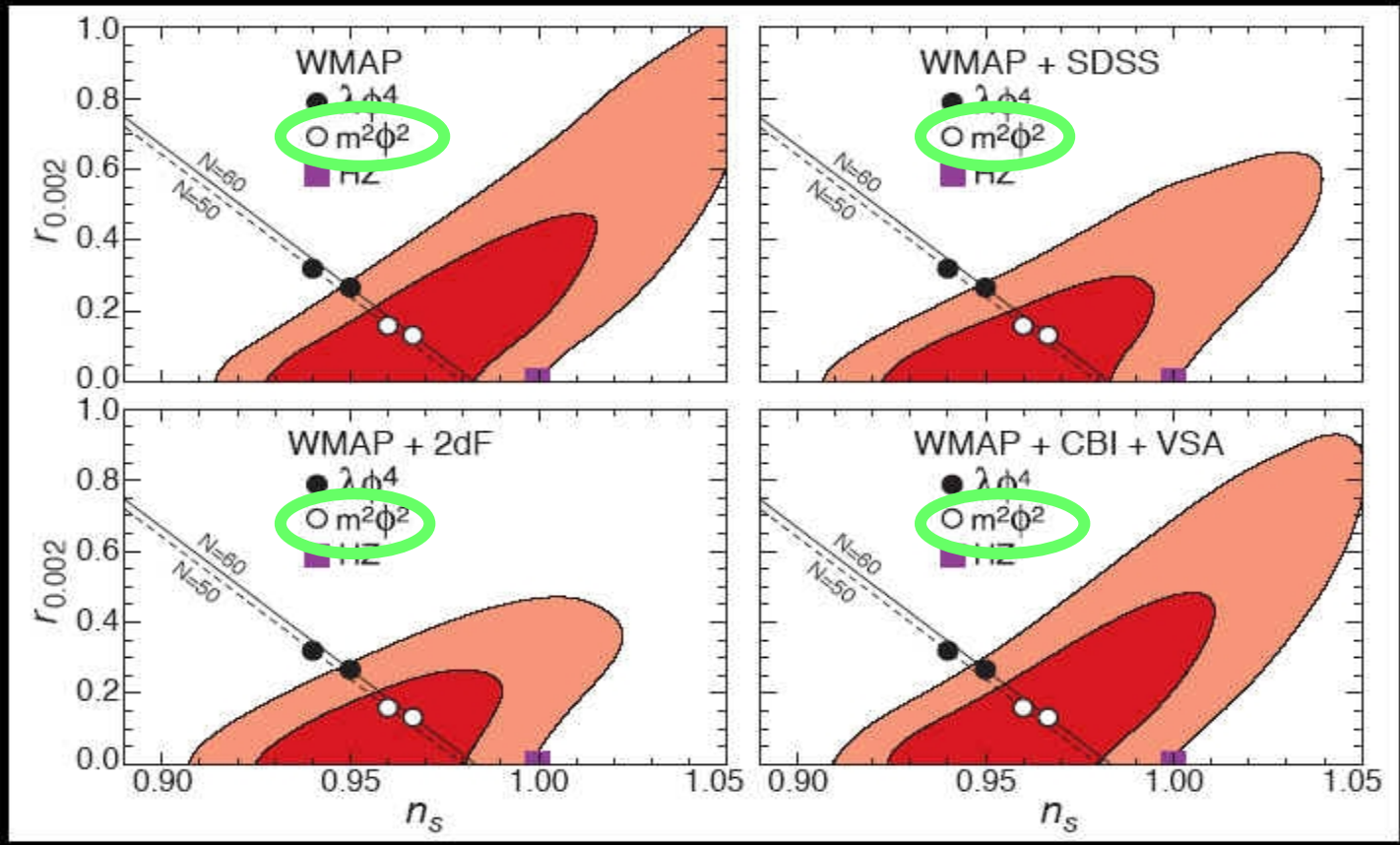
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- Spinoidal instability for long-wavelength + helicity photons $q < 4c d\theta/d\eta$
- negative effective mass
- modes grow exponentially
- helical B fields

WMAP data and chaotic inflation

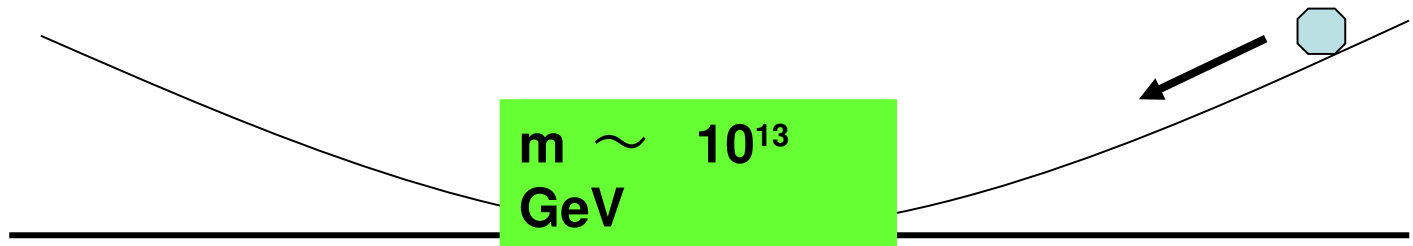
r : tensor/scalar



Spectral index

$$n(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$$

Spergel et al (2006)

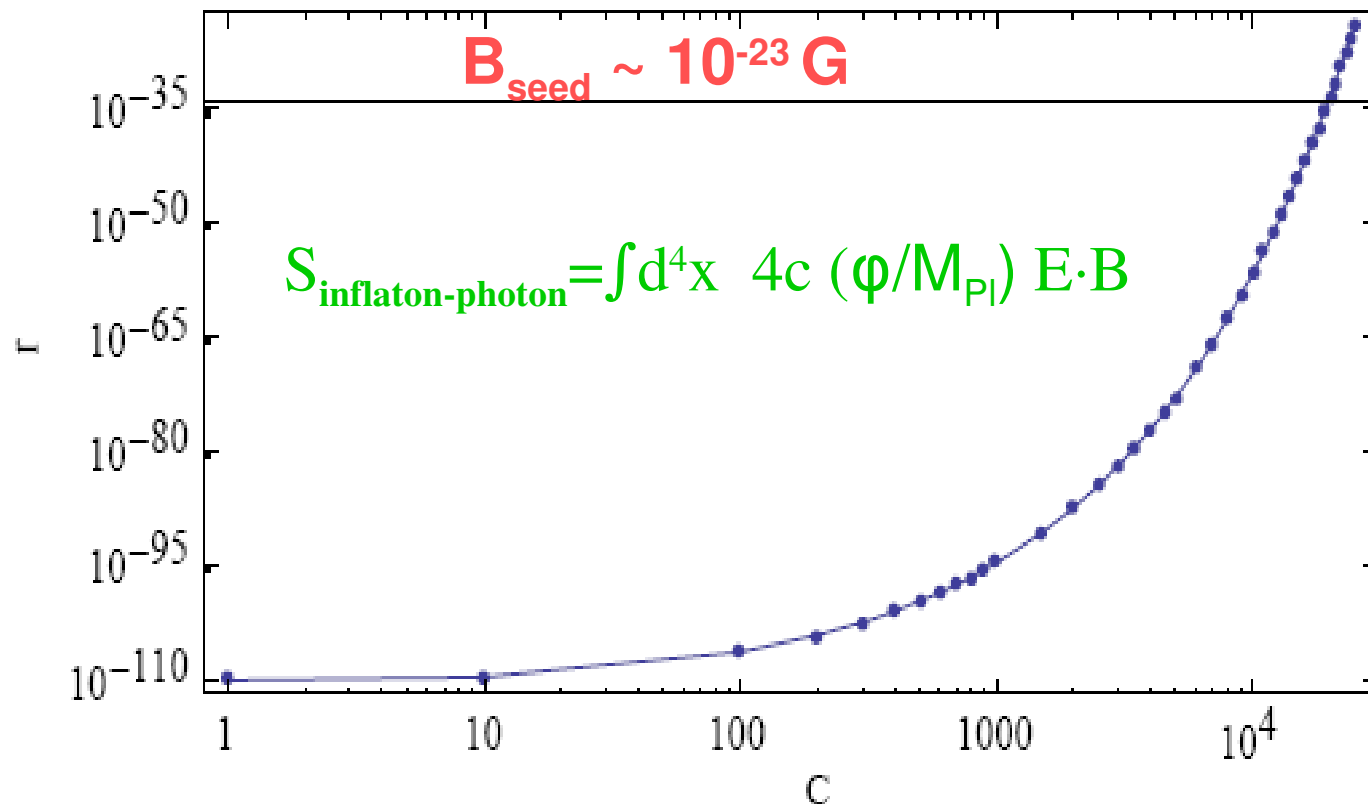


B fields from slow-roll inflation

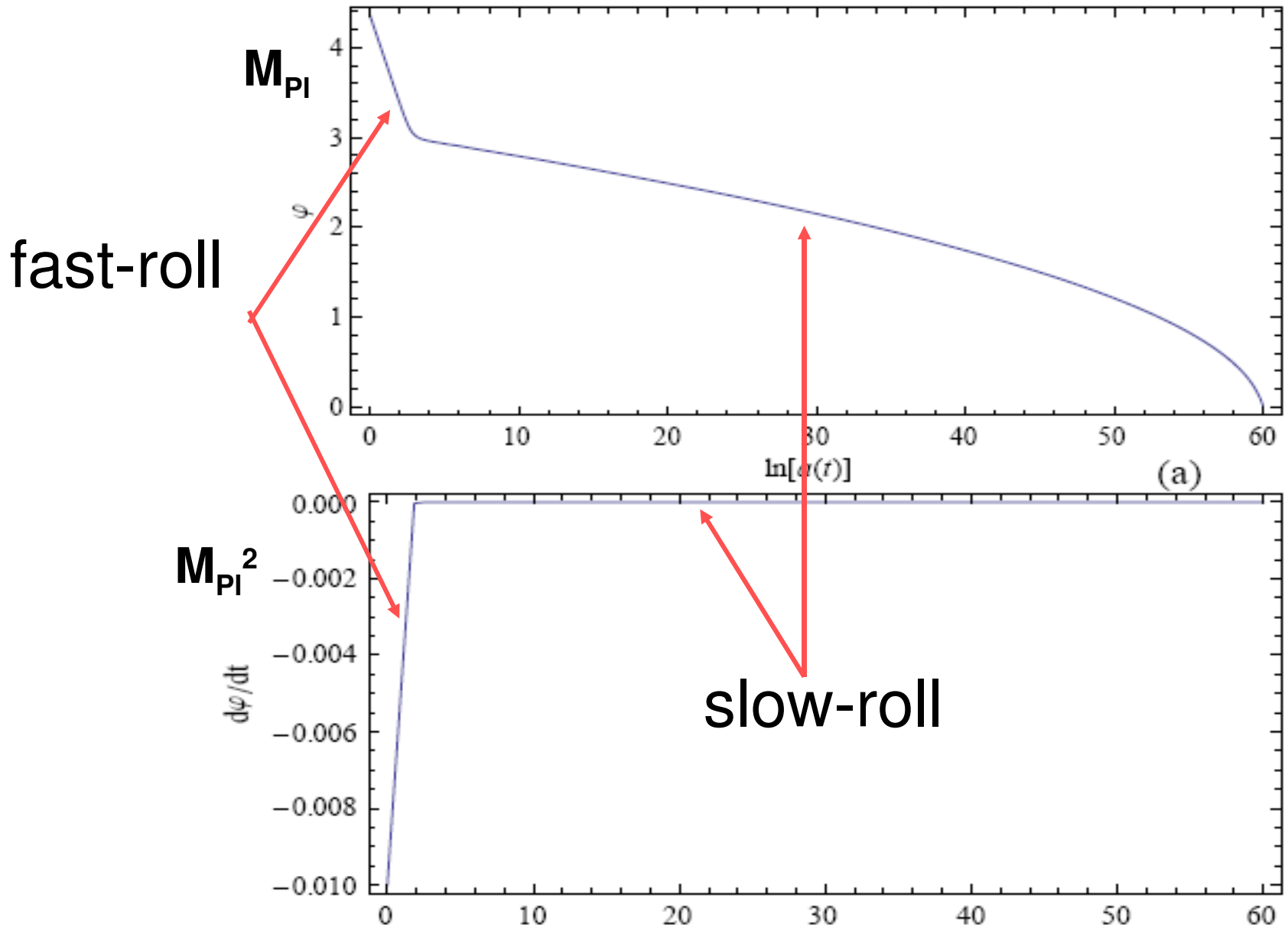
$$\frac{d\rho_B}{dq} = \frac{H^4}{32\pi^3 a_{end}^4} q^3 \sum_{i=\pm} |V_{iq}|^2$$

a_{end} end of inflation
 $q=1$ present horizon

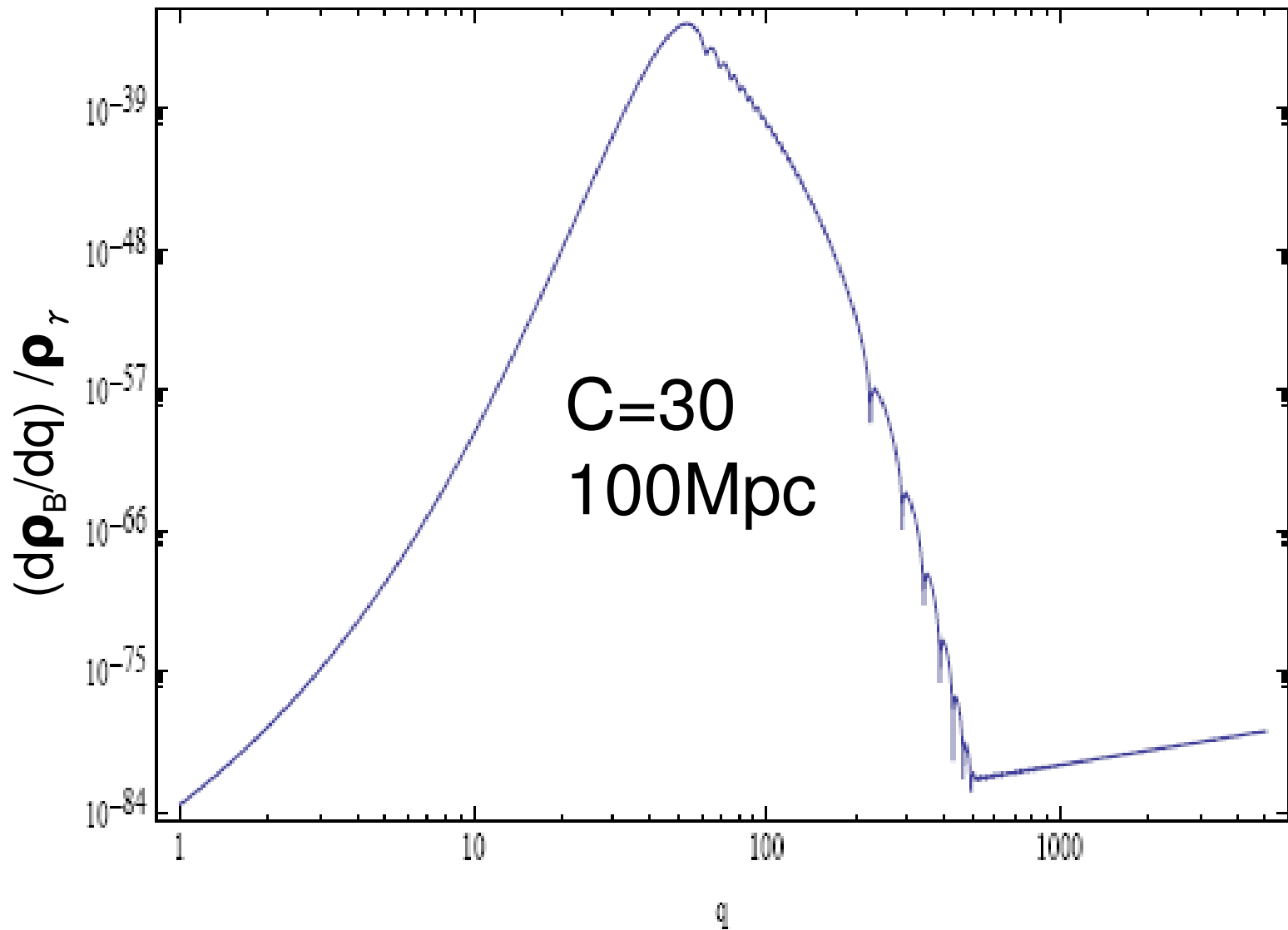
ρ_γ radiation density



Fast-roll stage + slow-roll inflation



B fields from fast-roll stage



Conclusions

- inflaton coupled to E·B ϕ/f string axiverse 2010
- helical B fields
- production of B fields in slow-roll inflation with low efficiency in many papers
- fast-roll stage significantly enhances the production Cheng, Lee, Ng 2010
- while suppressing large-scale inflaton fluctuations Linde et al. 2003
- hence resulting in a lower large-scale CMB anisotropy