Primordial Magnetic Field Effects on CMB polarization

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Collaboration

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Outline

- General Overview
  - Why Polarization Measurements are Crucial?
- Primordial Magnetic Field Signatures
  - CMB polarization anisotropies
  - Faraday Rotation Effect
- Magnetic Field Limits
  - B-polarization
  - CMB Oddities
By Now We Observe the Universe Mostly Through Electromagnetic Waves

In future: Neutrinos, Gravitational Waves
In astrophysics EM waves polarization measurements are widely used.

In particular, for the magnetic tests.
Polarization of Electromagnetic Waves

\[ E_x(t) = a_x(t) \cos(\omega_0 t + \phi_x), \quad E_y(t) = a_y(t) \sin(\omega_0 t + \phi_y) \]

- **I** – intensity: \( a_x^2 + a_y^2 \)
- **Q** – polarization (linear) \( a_x^2 - a_y^2 \)
- **U** – polarization (linear) \( 2a_x a_y \cos(\phi_x - \phi_y) \)
- **V** – polarization (circular) \( 2a_x a_y \sin(\phi_x - \phi_y) \)

- **I** and **V** – invariants under rotation
- **Q** exchange **U**, **U** exchange **Q**: 
  - \( Q^2 + U^2 \) invariant
E and B polarization

- $E_l$ – electric $(-1)^l$
  Pure Q
  • North/South
  • East/West

- $B_{l+1}$ – magnetic $(-1)^{l+1}$
  Pure U
  • Northeast/Southwest
  • Northwest/Southeast
Magnetic Field on the Universe

- Observations:
  - Magnetic field in galaxies and clusters, \(10^{-6}-10^{-5}\) Gauss
  - Cosmic rays propagation, \(10^{-11}\) Gauss on 1 Mpc

- Numerical simulations

- Theoretical models
  - Nonlinear process, magnetic field amplification, MHD
  - Cosmological magnetic field

- E. Fermi “On the origin of the cosmic radiation”, PRD, 75, 1169 (1949)
- F. Hoyle in Proc. “La structure et l’évolution de l’Univers” (1958)
Magnetic Helicity

- Astrophysical Observations (Mirror symmetry breaking)
  - Sun magnetic field
  - Active galactic nuclei
  - Jets

- How we observe magnetic helicity
  - The polarization of emitted synchrotron radiation

T.A. Ensslin, 2003; J. P. Valee, 2004

\[ \mathcal{H} = \frac{1}{V} \int_V d^3 x \mathbf{A} \cdot \mathbf{B} \]
Fig. 2: Light, medium and dark grey: known observational bounds on the strength and correlation length of EGMF, summarized in the Ref. (25). The bound from Big Bang Nucleosynthesis marked “BBN” is from the Ref. (2). The black hatched region shows the lower bound on the EGMF derived in this paper. Orange hatched regions show the allowed ranges of $B, \lambda_B$ for magnetic fields generated at the epoch of Inflation (horizontal hatching) the electroweak phase transition (dense vertical hatching), QCD phase transition (medium vertical hatching), epoch of recombination (rear vertical hatching) (25). White ellipses show the range of measured magnetic field strengths and correlation lengths in galaxies and galaxy clusters.
Why Cosmological Magnetic Fields are attractive?

- Might serve as seeds for the observed fields in galaxies and clusters
- Might be responsible for large scale correlated magnetic fields in the voids
- Might explain some cosmological observations
Generation Mechanisms

- Inflation
- Phase transitions
- Supersymmetry
- String Cosmology
- Topological defects
The EM Waves for the past - CMB

CMB allows us “see” until last scattering surface

The cosmic microwave background Radiation’s “surface of last scatter” is analogous to the light coming through the clouds to our eye on a cloudy day.
CMB Polarization


- Generation of Polarization anisotropy
  - Boltzmann equation
  - Scalar mode – only E-polarization

- Propagation effects
  - Birefrigence
  - Lensing
  - Lorentz symmetry
    - Input: E-polarization
    - Output: B-polarization

Wayne Hu Web-page
SEVEN-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE (WMAP\textsuperscript{1}) OBSERVATIONS: COSMOLOGICAL INTERPRETATION

E. Komatsu\textsuperscript{2}, K. M. Smith\textsuperscript{3}, J. Dunkley\textsuperscript{4}, C. L. Bennett\textsuperscript{5}, B. Gold\textsuperscript{5}, G. Hinshaw\textsuperscript{6}, N. Jarosik\textsuperscript{7}, D. Larson\textsuperscript{5}, M. R. Nolta\textsuperscript{8}, L. Page\textsuperscript{7}, D. N. Spergel\textsuperscript{3,9}, M. Halpern\textsuperscript{10}, R. S. Hill\textsuperscript{11}, A. Kogut\textsuperscript{6}, M. Limon\textsuperscript{12}, S. S. Meyer\textsuperscript{13}, N. Odegard\textsuperscript{11}, G. S. Tucker\textsuperscript{14}, J. L. Weiland\textsuperscript{11}, E. Wollack\textsuperscript{6}, and E. L. Wright\textsuperscript{15}
PLANCK
First Images

The Planck one-year all-sky survey
Fig. 10.— Constraint on the polarization rotation angle, $\Delta \alpha$, due to a parity-violating interaction that rotates the polarization angle of CMB (§ 4.3). We have used the polarization spectra (TE/TB/EE/BB/EB at $l \leq 23$, and TE/TB at $l \geq 24$), and did not use the TT power spectrum. (Left) One-dimensional marginalized constraint on $\Delta \alpha$ in units of degrees. The dark blue, light blue, and red curves show the limits from the low-$l$ ($2 \leq l \leq 23$), high-$l$ ($24 \leq l \leq 450$), and combined ($2 \leq l \leq 450$) analysis of the polarization data, respectively. (Right) Joint two-dimensional marginalized constraint on $\tau$ and $\Delta \alpha$ (68% and 95% CL). The bigger contours are from the low-$l$ analysis, while the smaller ones are from the combined analysis. The vertical dotted line shows the best-fitting optical depth in the absence of parity violation ($\tau = 0.086$), whereas the horizontal dotted line shows $\Delta \alpha = 0$ to guide eyes.
Magnetic Field Spectrum

- Two point correlation function Fourier space

- The averaged helicity spectrum amplitude $H^M(k,t)$

- The averaged magnetic field energy spectrum amplitude $E^M(k,t)$

- Schwartz’s inequality

$$|H^M(k,t)| \cdot \frac{2}{k} E^M(k,t)$$

Isotropic & helical divergence free vector field
Smoothed Magnetic Field

- The smoothed field value

\[ B_{\lambda}^2 = \langle B(x) \cdot B(x) \rangle |_{\lambda}. \]

- The magnetic energy density

\[ \rho_B = \frac{B_{\text{eff}}^2}{8\pi} \]

An important issue to define the magnetic field cut-off scale \( K_D \). We assume that the magnetic field cut-off scale is determined by the Alfvén wave damping scale \( k_D \sim v_A L_S \) with \( v_A \) the Alfvén velocity and \( L_S \) the Silk damping scale [16]. Assuming the Alfvén velocity is determined by the \( B_{\text{eff}} \), the simple computations gives the expression of \( k_D \) in terms of \( B_{\text{eff}} \)

\[ \frac{k_D}{1\text{Mpc}^{-1}} = 1.4 \sqrt{\frac{(2\pi)^{n_B+3} h}{\Gamma \left( \frac{n_B+5}{2} \right)}} \left( \frac{10^{-7} \text{G}}{B_{\text{eff}}} \right) \]  

(k_d definition

Subramanian and Barrow 1998
Jedamzik, Katalinic, and Olinto 2000
If the magnetic field is generated during inflation and has a scale invariant spectrum with $n_B \rightarrow -3$ then $B_\lambda = B_{\text{eff}}$ for any value of $\lambda$ and this result does not depend on $n_B$.

For any other fields the difference between $B_{\text{eff}}$ and $B_\lambda$ ($\lambda = 1\text{Mpc}$) might be enormous, while the physical effects depend on $B_{\text{eff}}$, we proposed to derive all effects in terms of the total magnetic energy density present in the Universe.
CMB Faraday rotation

- Kosowsky & Loeb 1996
  CMB polarization rotation angle

\[ \langle \varphi^2 \rangle^{1/2} \approx \frac{e^3 B_0}{2 \sqrt{2} \pi m^2 \sigma_T \nu_0^2} = 1.6^\circ \left( \frac{B_0}{10^{-9} \text{ Gauss}} \right) \left( \frac{30 \text{ GHz}}{\nu_0} \right)^2, \]

- Scannapieco & Ferreira 1997
  CMB temperature and polarization anisotropies cross-correlations

Limits - around $10^{-8}$-$10^{-9}$ Gauss
Faraday Rotation by a Stochastic Magnetic Field

Kosowsky, Kahniashvili, Lavrelashvili, and Ratra 2005

\[
\langle R(n)R(n') \rangle \simeq \frac{9}{128\pi^5 q^2} \sum_l \frac{2l + 1}{4\pi} l(l + 1) P_l(n \cdot n') \int dk k^2 P_B(k) \left( \frac{j_l(k\eta_0)}{k\eta_0} \right)^2,
\]

with corresponding multipole moments

\[
C_l^R \simeq \frac{9l(l + 1)}{(4\pi)^3 q^2} \frac{B^2}{\Gamma(n_B/2 + 3/2)} \left( \frac{\lambda}{\eta_0} \right)^{n_B + 3} \int_0^{x_D} dx x^{n_B} j_l^2(x),
\]

where \( x_D = k_D \eta_0 \) and the multipole moments are defined via

\[
\langle R(n)R(n') \rangle = \sum_l \frac{2l + 1}{4\pi} C_l^R P_l(n \cdot n').
\]

The rotation angle power spectrum is simply given by the rescaling

\[
C_l^\alpha = \nu_0^{-4} C_l^R.
\]
\[
\langle \alpha^2 \rangle^{1/2} \approx 0.14^\circ \left( \frac{B_{\text{eff}}}{10^{-9} \text{G}} \right) \left( \frac{100 \text{GHz}}{\nu_0} \right)^2 \left[ \sum_{l=0}^{\infty} \frac{\sqrt{n_B + 3}}{(k_D \eta_0)^{(n_B+3)/2}} (2l + 1) l(l + 1) \right]^{1/2} \]

(11)

One might be interested to compare Eq. (11) with the corresponding result, Eq. 2 from Ref. [3] derived for an homogeneous magnetic field and the frequency \( \nu_0 = 30 \) GHz,

\[
\langle \alpha^2 \rangle^{1/2} \approx 1, 6^\circ \left( \frac{B_0}{10^{-9} \text{G}} \right) \left( \frac{30 \text{GHz}}{\nu_0} \right)^2 \]

(12)

Both equations are in an agreement for \( n_B \rightarrow -3 \) under accounting for \( \sum_l (2l + 1) j_l^2(x) = 1 \) and the property of Bessel functions that they peak at \( x \sim l \) for given \( l \) (see text).
$B_\lambda$ vs. $B_{\text{eff}}$

- Kahniashvili, Tevzadze, et al. 2010
FIG. 1: The solid line shows the $B$-polarization power spectrum due to Faraday rotation from the WMAP best-fit cosmology, plus a stochastic magnetic field with amplitude $B_\lambda = 0.2 \mu G$, $\lambda = 1$ Mpc, $\nu = 30$ GHz, and power law index $n_B = -2.9$. The data points show binned WMAP 5-year $B$-polarization data with $l < 800$. Note that this magnetic field amplitude and power spectrum is ruled out by the data in the region between $l = 300$ and $l = 500$. Points with $l < 150$ do not contribute significantly to the constraint.
FIG. 2: The C-polarization power spectrum of the microwave background induced by the Faraday rotation field in Fig. 1, again with the magnetic field normalization scale $\lambda = 1$ Mpc.

FIG. 2: The 68% and 95% C.L. limit bands on the $B_\lambda$ as function of $n_B$ for $\lambda = 1$ Mpc.

FIG. 3: 95% C.L. upper limits on $B_\lambda$ for fixed values of $n_B = -2.9, -2.5, -2.0, -1.5,$ and $-1.0$ as functions of $\lambda$. 

T.K., Maravin, and Kosowsky 2008

Kosowsky et al. 2004
Figure 1: The angular power spectra of the B-mode autocorrelations (plot at the left) and the absolute values of the cross-correlations (plot at the right) are reported in the case when the pseudo-scalar background and the magnetized background are simultaneously present. The ΛCDM parameters have been chosen in accordance with the best fit to the WMAP5-yr data alone [1].
CMB polarization rotation angle limits
Kahniashvili, Tevzadze, Sethi, Pandey, and Ratra 2010

FIG. 2: Effective magnetic field limits set by the rotation angles $\alpha$ for different spectral indices ($n_B = -3, -2, -1, 0, 1, 2$). The horizontal solid line shows upper limit set by BBN constraints. Vertical dashed lines correspond to the angles $\alpha = 3.16^\circ$ that is set by the BBN limit on the effective magnetic field with spectral index $n = 2$ and $\alpha = 4.4^\circ$ set by the 7-year ... . The values of the effective magnetic field constraints at $\alpha = 4.4^\circ$ angle are shown on the graph for different spectral indices, respectively.
CMB B-polarization

- Scalar mode does not produce CMB B-polarization signal.
- CMB B-polarization detection is a powerful test to probe inflation (gravitational waves); It will also will significantly improve the existing limits on the primordial magnetic fields ($10^{-10}$-$10^{-11}$ Gauss)
B-polarization Magnetic Sources

- Up to $l \sim 60$ – gravitational waves (tensor mode) generated by the magnetic field anisotropic stress
- Vorticity perturbations from the magnetic field (Alfven waves); Contribution to the CMB anisotropies – in particular B-mode (peaks at $l \sim 2000$)
- Secondary B-polarization arising from the birefrigence (Faraday rotation of the initial E-mode – secondary B-mode); peaks at $l \sim 16000$
B-polarization

\[ \frac{B_l^{(V)}(\eta_0, k)}{2l + 1} = -\sqrt{6} \int_0^{\eta_0} d\tau \hat{\tau} e^{-\tau} P^{(V)} \beta_l^{(V)}[k(\eta_0 - \eta)], \]

\[ P^{(V)} = \frac{\sqrt{3} k}{9} \tilde{v}_b^{(V)} \approx \frac{\sqrt{3} k}{9} \tilde{\Omega} \]

\[ \beta_l^{(V)}(x) = \frac{1}{2} \sqrt{(l-1)(l+2)} \frac{j_l(x)}{x} \]

Hu and White 1997

\[ \frac{B_l^{(T)}(\eta_0, k)}{2l + 1} = -\sqrt{6} \int_0^{\eta_0} d\tau \hat{\tau} e^{-\tau} P^{(T)} \beta_l^{(T)}[k(\eta_0 - \eta)], \]

\[ P^{(T)} = -\frac{1}{3} \frac{\dot{h}}{\tilde{\tau}} \]

\[ \beta_l^{(T)}(x) = \frac{1}{2} \left[ j_l'(x) + 2 \frac{j_l(x)}{x} \right] \]
Metric Perturbations

\[ \dot{\Omega}_{\gamma i} + \dot{\gamma}(v^{(V)}_{\gamma i} - v^{(V)}_{b i}) = 0, \]

\[ \dot{\Omega}_{b i} + \frac{\dot{a}}{a} \Omega_{b i} - \frac{\dot{\gamma}}{R} (v^{(V)}_{\gamma i} - v^{(V)}_{b i}) = \frac{L^{(V)}_{i}(k)}{a^{4}(\rho_{b} + p_{b})}. \]

\[ \Omega_{i}(\eta, k) \approx \frac{k \Pi^{(V)}_{i}(k) \eta}{(1 + R)(\rho_{\gamma 0} + p_{\gamma 0})}. \]

\[ \Omega_{i}(\eta, k) \approx \frac{\Pi^{(V)}_{i}(k)}{(kL_{\gamma}/5)(\rho_{\gamma 0} + p_{\gamma 0})}, \quad k > k_{S}. \]

\[ \ddot{h}_{i j}(\eta, k) + 2\frac{\dot{a}}{a} \dot{h}_{i j}(\eta, k) + k^{2}h_{i j}(\eta, k) = 8\pi G\Pi^{(T)}_{i j}(k)/a^{2}, \]

\[ h(\eta, k) = \frac{2\pi G\Pi^{(T)}(k)z^{2}_{eq} \eta_{eq}^{2}}{(3 - 2\sqrt{2})k \eta} \int_{\eta_{in}}^{\eta} d\eta' \frac{\sin[k(\eta - \eta')]}{\eta'}, \quad \eta < \eta_{eq}. \]

Subramanian and Barrow 1998
Mack, Kahniashvili, and Kosowsky 2002

Deriaigin, Sazhin and Veryaskin 1982;
Durrer, Ferreira, and Kahniashvili 2000
What we should account for?

- Gravitational lensing – \( l \sim 1000 \)
- Lorentz symmetry violation – similar to the Faraday rotation – different frequency dependence
- Any other vector mode (defects, strings, neutrinos...)
Strategy

- The magnetic field induces all kinds of perturbations and so we must cross check
  - Do we observe the Doppler peaks shifts? (magnetosound waves in the Universe)
    * Adams, Danielson, Grasso, and Rubinstein, 1996, Kahniashvili and Ratra 2007
  - Do we observe the CMB non-gaussianity?
  - Do we see tensor mode contribution (additional) at low l-s?
Magnetic Field Limits

- CMB non-gaussianity – few nanoGauss  Trivedi, Subramanian, and Seshadri 2010
- CMB polarization fluctuations – few nanoGauss  Yamazaki et al. 2010
- CMB Faraday rotation – few nanoGauss  T.K., Tevzadze, et al. 2010
- LSS first object formation – few nanoGauss  Sethi and Subramaian 2005
CMB Oddities

- Low multipole anomalies
  - Off-diagonal cross correlations
  - North-South asymmetry

- Parity violation
  - Temperature-B-polarization
  - E- and B-polarization
Magnetic Helicity Generation

- **Cosmological Sources**

- **MHD Processes in Astrophysical Plasma**

- **Turbulence**
Primordial Magnetic Helicity

- If magnetic helicity is present the large-scale properties of the magnetic field are significantly affected.
- Reflects a manifestation of the parity symmetry violation
- Ways to detect
  - Cosmic rays arrival velocities
  - CMB polarization measurements
  - Direct detection of gravitational wave polarizations
CMB anisotropies
parity even & odd power spectra

- **Parity-even power spectra:**
  \[ C_l^{TT}, C_l^{EE}, C_l^{BB}, C_l^{TE} \]

- **Parity-odd power spectra:**
  \[ C_l^{TB}, C_l^{EB} \]
  - Vanishing in the standard model
  - Present if
    - Lorentz symmetry is broken
    - Cosmological helical magnetic field
    - Parity symmetry is violated
Vector – Tensor modes comparison

**Vector mode**

- Surviving up to small angular scales.
  - Subramanian and Barrow, 1998; Lewis, 2004
  - Vanishing E-B polarization cross correlations (with respect of temperature-B-polarization).
  - Kahniashvili and Ratra, 2005

**Tensor mode**

- Gravitational wave source damping after equality!
  - Contribution in CMB for large angular scales ($l < 100$)
  - The same order of magnitude for temperature - B-polarization and E-B polarization cross correlations.
  - Caprini, Durrer, and Kahniashvili, 2004
GWs sourced by a helical magnetic field

CMB anisotropy parity odd power spectra (tensor mode) might reflect the presence of primordial magnetic helicity

$C_l^{TB}/C_l^{TE}$ (black); $C_l^{EB}/C_l^{EE}$ (red)

$l=50$, $n_s=-3$

B-polarization signal: the peak position insures to distinguish the source of the signal

- Zaldariagga and Seljak, 1997
- Kamionkowsky, Kosowsky, & Stebbins, 1997

Caprini, Durrer, and Kahniashvili 2004
Parity symmetry violation in the early Universe

- Gravitational Chern-Simons term
  Lue, Wang, Kamionkowsky, 1999

Specific signatures on CMB - non-zero parity odd cross correlations between temperature & B-polarization; E & B-polarization anisotropies

Lyth, Quimbay, Rodriguez 2005
Satoch, Kanno, Soda 2008
Saito, Ichiki, Taruya 2007
Seto, Taruya 2008

FIG. 1. The dashed curve shows the $C_l^{TE}$ power spectrum induced by rotation of the polarization of an initially P symmetric CMB polarization pattern by $0.05^\circ$. The solid curve shows the $C_l^{TE}$ power spectrum produced by a GW background that consists of only right-handed GWs.
Parity Odd CMB fluctuations

- An crucial test for the fundamental symmetry breakings.
- It is more promising way to test primordial inflation or short-after inflation generated helicity.
- This apply also for the Chern-Simons term induced parity symmetry violation (Lue, Wang, Kamionkowski 1999), but it has been shown that the signal is not observable through current or nearest future CMB missions.
Conclusion

- Cosmological magnetic field order of 0.1 nanoGauss can be detectable by the nearest future CMB polarization measurements.

- On the other hand if the field is significantly smaller – it would satisfy the LOWER limit bound but would not been observable through cosmological observations.
THANKS