The Evolution of Cosmic Magnetic Fields: From the Very Early Universe, to Recombination, to the Present

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Goal of Study I

- magnetic fields almots surely exist in the very early Universe
- mostly prior philosophy: take field on ~ 100 kpc – 10 Mpc - evolution determined by flux freezing
- dynamic evolution on scales of where most magnetic energy resides is ignored
- obtain a holistic picture of the gross features of growth of magnetic coherence length and decay of magnetic energy density during the expansion of the Universe

Banerjee & Jedamzik, PRL 2003, PRD 2004

Prior studies:

- Jedamzik, Katalinic, Olinto 98, Subramanian & Barrow 98 - only linear analysis
- Dimopoulos & Davis 97 - obtain result which is not supported by numerical simulation
- Son 99 - effect of viscosities not treated well
- Christensson, Hindmarsh, Brandenburg 01 - no viscosity, only maximal helical fields

Goal of Study II

- “Causually” produced magnetic fields, e.g. QCD- or electroweak- transition
- $B(\lambda) \sim \lambda^{-n/2}$ - the likely (minimal) n ?
- important for magnetic field strength on large scales

Jedamzik & Sigl, in preparation

Prior studies:

- Hogan 83, ... - $n = 3$ uncorrelated superposition of dipoles
- Caprini & Durrer 01,03,05 - $n \geq 5 \rightarrow$ very strong limits on magnetic fields on large scales

Outline of Talk

I. Some MHD Basics

II. Turbulent MHD evolution

(a) Non-helical fields

(b) Helical fields

III. Viscous MHD evolution

IV. Evolution of Cosmic Magnetic Fields

(a) General Features

(b) Results

V. The Large Scale Tail of Primordial Magnetic Fields

VI. Conclusions

General Features

- incompressible MHD

  $v_A \ll v_s$ in very early Universe;

  $v_A \lesssim v_s$ at/after recomination for $B \lesssim 5 \times 10^{-12}$Gauss

- phases with large viscosity

- large Prandtl number $Pr = R_k / R_m$

  -> magnetic diffusion unimportant

  -> helicity conserved

The MHD equations

$$\text{Euler}: \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} - (\mathbf{v}_{\rm A}\cdot\nabla)\,\mathbf{v}_{\rm A} = \mathbf{f} \ ,$$

$$\text{MHD}: \quad \frac{\partial \mathbf{v}_{\rm A}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v}_{\rm A} - (\mathbf{v}_{\rm A}\cdot\nabla)\,\mathbf{v} = \nu\nabla^2\,\mathbf{v}_{\rm A} \ , \mathbf{v}_{\rm A}(x) = \frac{\mathbf{B}(x)}{\sqrt{4\pi(\varrho+p)}} \ ,$$

$$\text{Dissipation}: \quad \mathbf{f} = \begin{cases} \eta\nabla^2\,\mathbf{v} & l_{\rm mfp} \ll l \\ -\alpha\mathbf{v} & l_{\rm mfp} \gg l \end{cases} \ ,$$

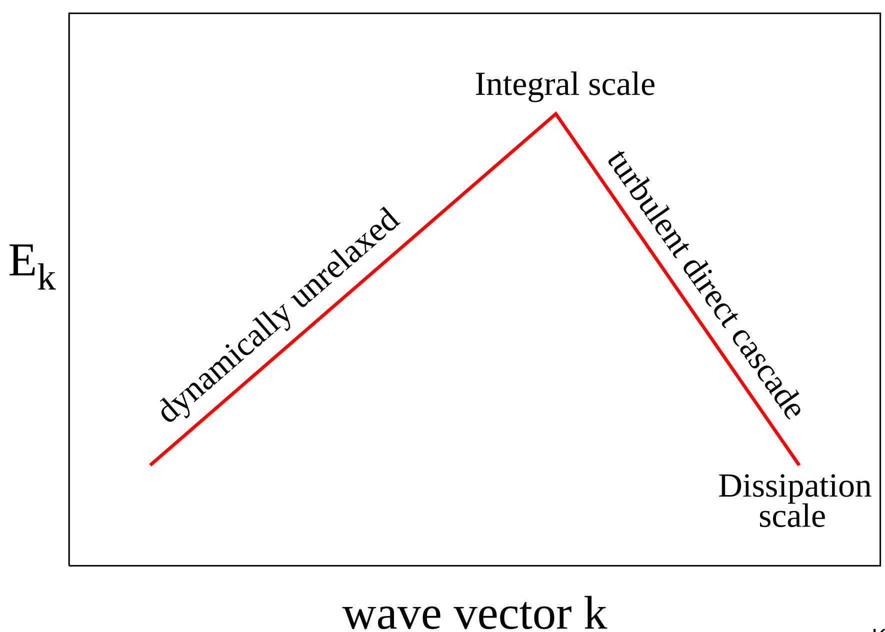
$$\text{Reynolds number}: \quad R_e(l) = \frac{v^2/l}{|\mathbf{f}|} = \begin{cases} \dfrac{v\,l}{\eta} & l_{\rm mfp} \ll l \\ \dfrac{v}{\alpha\,l} & l_{\rm mfp} \gg l \end{cases} \ ,$$

MHD Cascades in Fourier Space

$$E_B = \frac{\varrho}{2}\frac{1}{V}\int \mathrm{d}^3x\, \mathbf{v_A}^2 = \int \mathrm{d}^3k \langle |v_{A,k}|^2\rangle \equiv \int \mathrm{d}\ln\, k\, E_k\,, \qquad (1)$$

quasi-stationary state (Kolmogoroff, Iroshnikov-Kraichnan)

$$\frac{\mathrm{d}E_k}{\mathrm{d}t} \approx \frac{E_k}{\tau_k} \approx \mathrm{const}(k)\ , \qquad (2)$$

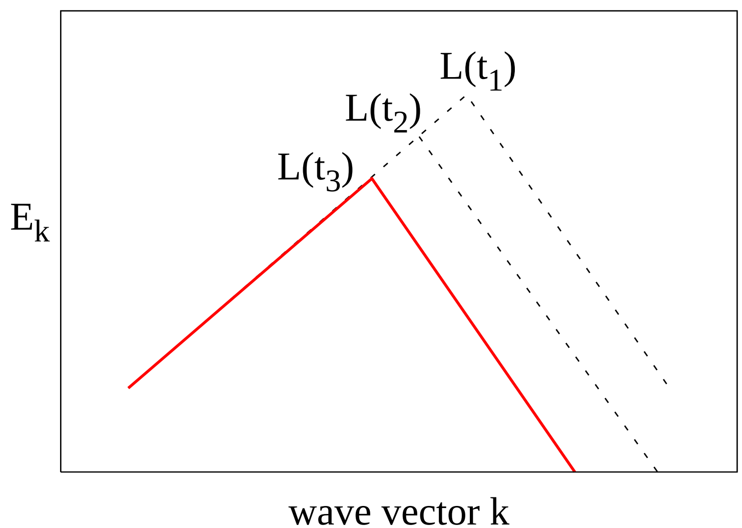


Decay of magnetic energy in MHD

Assume initial small-k spectrum:

$$E_l(t_0) = E_0 \left(\frac{l}{L_0}\right)^{-n} \quad \text{for } l > L_0$$

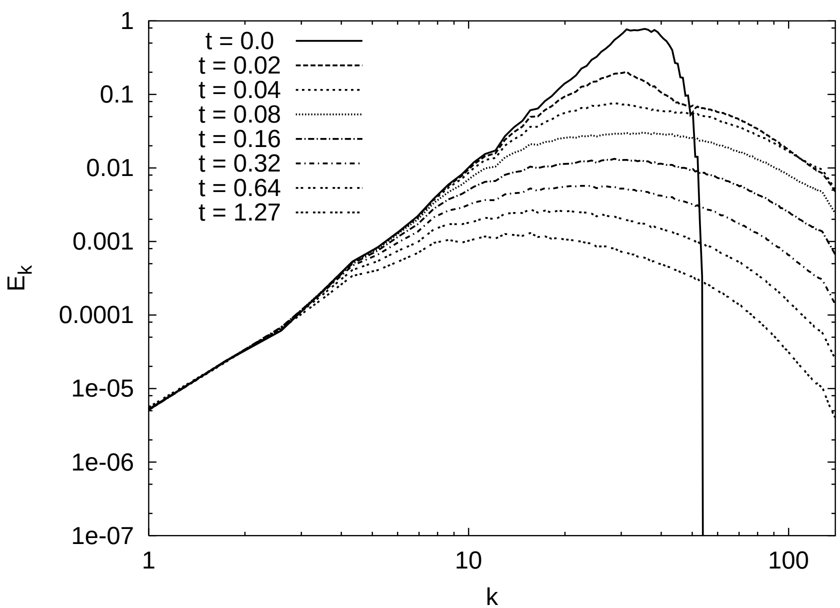
Processing on Integral Scale by development of fluid eddies and cascade of energy to dissipation scale



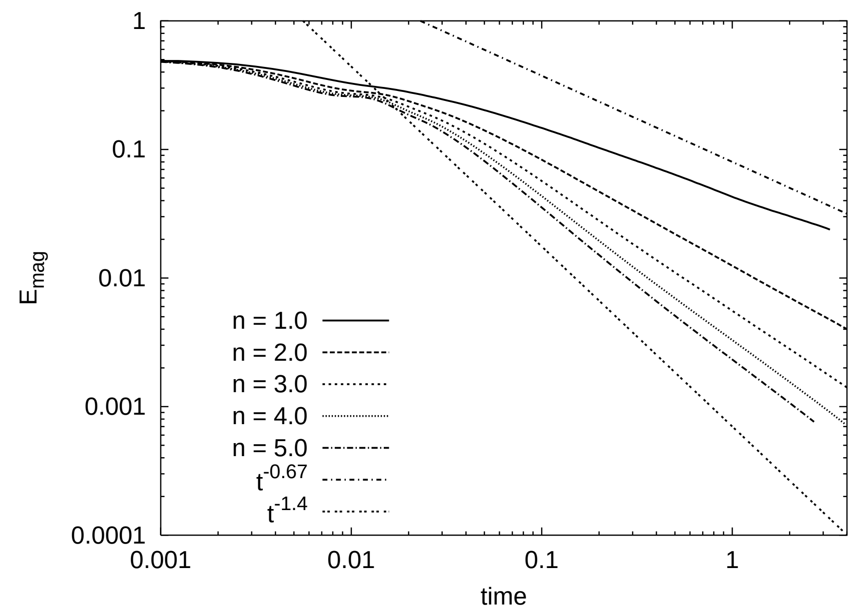
relaxation time: $\tau_L \simeq t \sim L/v_L \simeq L/\sqrt{E_L} \simeq \tau_{Hubble} \rightarrow$ simple prediction for $L(t)$ and $E(t)$ possible

Comparison to numerical simulations

Spectrum with time



Total energy with time



decay slower than predicted ! e.g. $E \propto t^{-\gamma}; \quad n = 3$

theory $\gamma = 1.2$

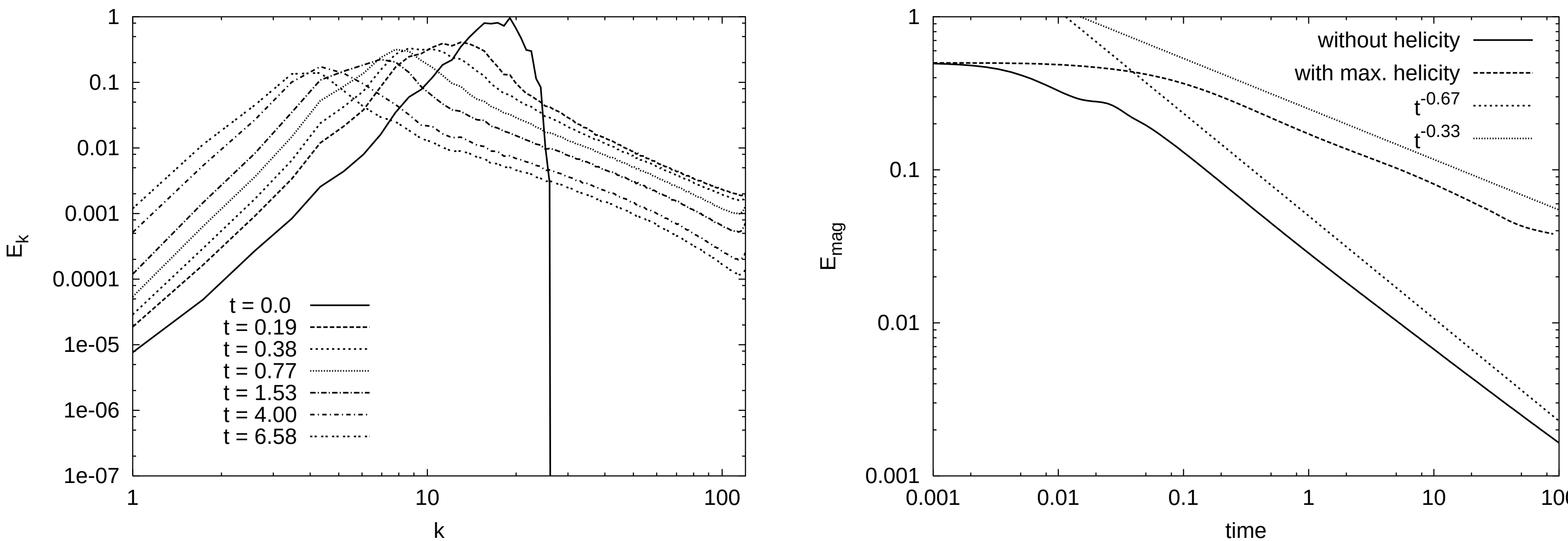
experiment $\gamma = 1.05$

artifact of simulations !? ΔL not resolved

Viscous Magnetohydrodynamics I

- $\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\, \mathbf{v} - (\mathbf{v}_{\mathrm{A}} \cdot \nabla)\, \mathbf{v}_{\mathrm{A}} = -\alpha \mathbf{v};$ e.g. photons before recombination $\alpha = \frac{4}{3} \frac{\varrho_\gamma}{\varrho_b} \frac{1}{l_\gamma}$
- terminal velocity: $\mathbf{v} \approx \frac{1}{\alpha} (\mathbf{v}_{\mathrm{A}} \cdot \nabla)\, \mathbf{v}_{\mathrm{A}} \ll \mathbf{v}_{\mathrm{A}}$
- a system of $\Gamma = \frac{E_{kin}}{E_{mag}} \ll 1$ results
- strong drag $\rightarrow$ dissipation of magnetic fields delayed (counterintuitive)
- no more cascade, rather, energy is "burned" on integral scale itself
- though less intuitive, dissipation time again given by $\tau_L \approx \frac{L}{v_L} \approx \frac{L^2 \alpha}{E}$

Viscous Magnetohydrodynamics II



Condition for relaxation of Integral Scale:

$$t_{\mathrm{eddy}} \approx \frac{L_p(T)}{v(L)} \approx \frac{1}{H(T)} \approx t_H$$

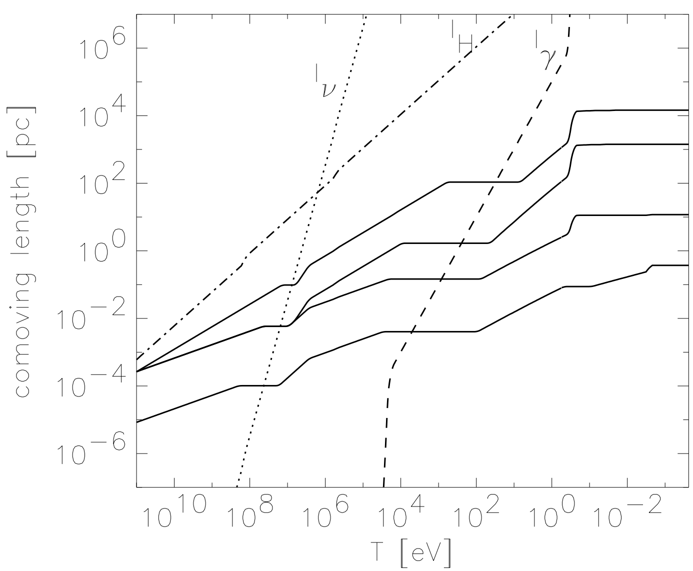
$$v(L) \approx v_{\mathsf{A}}(L)\,; \qquad R_e > 1$$

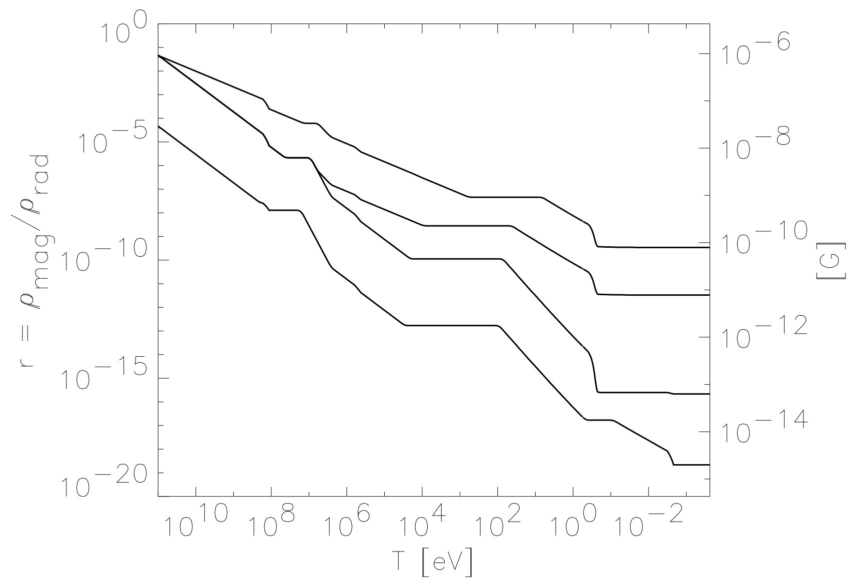
in the turbulent case and

$$v(L) \approx \frac{{v_{\mathsf{A}}}^2(L)\, L}{\eta}\,, \quad v(L) \approx \frac{{v_{\mathsf{A}}}^2(L)}{\alpha\, L}\,, \qquad R_e < 1$$

in the viscous case

Evolution: The Global Picture





from top to bottom: (a) $h_{\rm g} = 1$, $r_{\rm g} = 0.01$, (b) $h_{\rm g} = 10^{-3}$, $n = 3$, $r_{\rm g} = 0.01$, (c) $h_{\rm g} = 0$, $n = 3$, $r_{\rm g} = 0.01$, (d) $h_{\rm g} = 0$, $n = 3$, $r_{\rm g} = 10^{-5}$

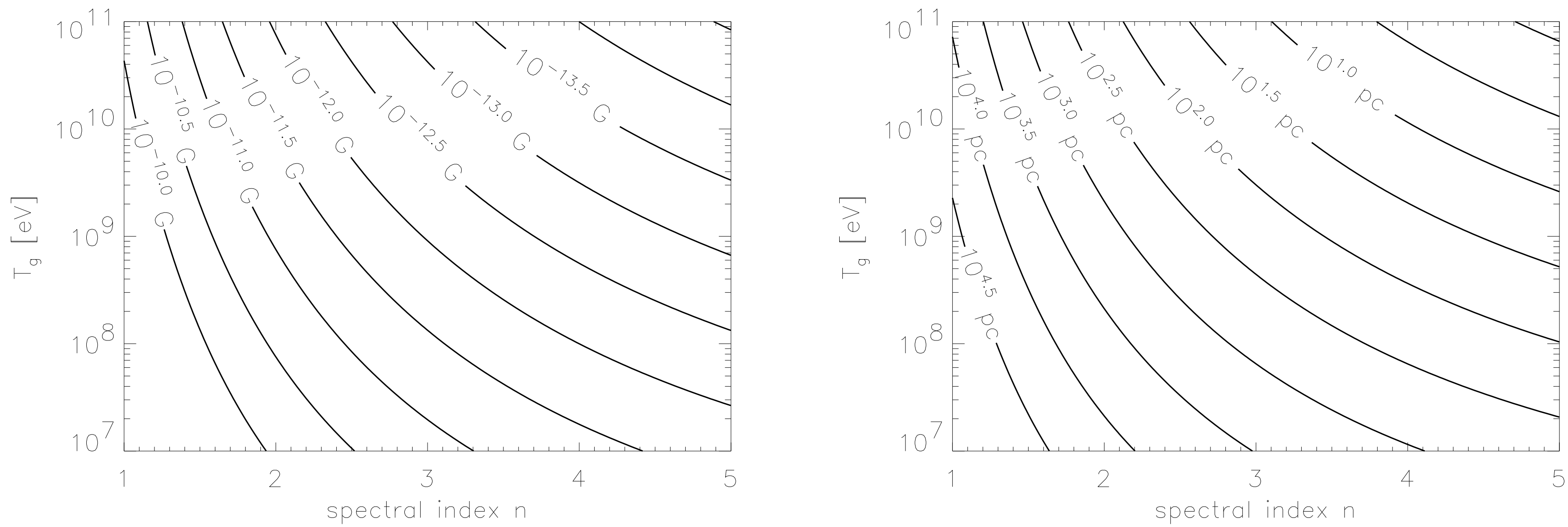
Dissipation scale before recombination

linear limit: $L_{diss} \sim v_A d_{Silk} \approx v_A \sqrt{l_\gamma t_H}$ Jedamzik, Katalinic, Olinto 98; Subramanian & Barrow 98

non-linear limit: $t_H = \tau \sim L/v \approx L/(v_A^2/L\alpha_\gamma)$ and with $\alpha_\gamma \sim 1/l_\gamma \rightarrow L_{diss} \sim v_A \sqrt{l_\gamma t_H} = v_A d_{Silk}$ Banerjee & Jedamzik 04

essentially same result

Final field Properties:



assumed $\varrho_B^i/\varrho_{rad}^i = 0.01$

A Correlation for Primordial Cosmic Magnetic Fields

$$B_0 \lesssim 5 \times 10^{-12}\,\text{Gauss}\left(\frac{L_c}{\mathrm{kpc}}\right)$$

limit saturated when dynamically relaxed, i.e. $v_A(L)/L \approx H_0$

Cluster Magnetic Fields Primordial ?

Observations of cluster magnetic fields seem be be reproduced independent (!!!) of the initial correlation length of the magnetic fields (shear flows, ...) Dolag, Bartelmann, & Lesch 99, 02
– $B_c \simeq 4 \times 10^{-12}$ is required to reproduce the rotation measure

this may be used (in principal) to place upper limits on magnetic field strength on ~ 1 kpc – ~ 100 Mpc scales (Banerjee & Jedamzik 04)

maybe more importantly: very efficient transfer of magnetic energy from small scales to large scales due to large scale shear flows during gravitational collapse ?!

gravitational inverse cascade, dynamo in hierarchical structure formation

The Large-Scale Tail of Primordial Magnetic Fields: Neglecting Velocities

- external currents $\mathbf{j}_{\rm ex} = \boldsymbol{\nabla} \times \mathbf{M}$ imposed by the dynamcis of the phase transition
- evolution equation for B: $\partial_t \mathbf{B} = \eta \left[\Delta \mathbf{B} - 4\pi \left(\Delta \mathbf{M} - \boldsymbol{\nabla} \left(\boldsymbol{\nabla} \cdot \mathbf{M} \right) \right) \right]$
- high conductivity, i.e. $\eta = 1/(4\pi\sigma)$ is small
- solution in Fourrier space: $\mathbf{B}_{\mathbf{k}}(z) \simeq 4\pi \left[\frac{k}{k_r(z)} \right]^2 \mathbf{M}_{\mathbf{k}}^{\perp}(z)$ for $k < k_r(z)$

$\rightarrow$ penetration of B field into plasma is very inefficient due to the high conductivity and screening, in particular $n = 7$

- MHD is non-linear: small q magnetic modes maybe excited due to interactions between large k magnetic modes and large k_1 velocity modes

with $k + k_1 = q$

$$\frac{\partial M_q}{\partial t} \simeq \mathcal{C} \int \mathrm{d}k\mathrm{d}\theta \, \frac{q^4}{k\, k_1^3} \, \frac{v_f^2(L_{k_1})}{v_f(L_k)} f(q/k,\theta) M_k$$

..., Vainshtein 70, ..., Kulsrud & Anderson 92

and in expanding Universe

$$\frac{\partial M_q}{\partial \ln a} \simeq \mathcal{C} \frac{a}{H_0} \int_q^{k_I(a)} \mathrm{d}\ln k \, k \left(\frac{q}{k}\right)^4 v_{f,0} \left(\frac{k}{k_0}\right)^{\alpha/2} M_k$$

where we defined

$$v_f^2(k) = v_{f,0}^2 \left(\frac{k}{H_0}\right)^{\alpha}$$

with the crucial constant $\mathcal{C} \simeq (2\pi)^2$

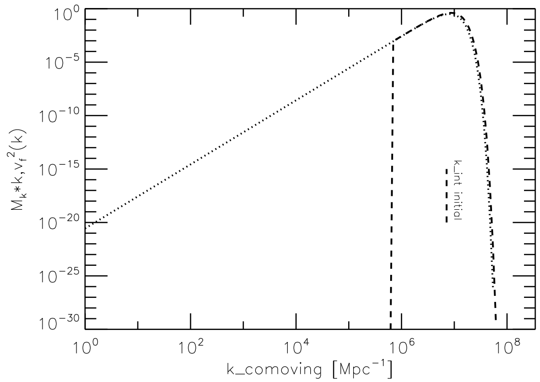
Question: For which small magnetic wave vector modes q is there enough time to come into equipartition with the kinetic modes ?

Answer: For all modes which have

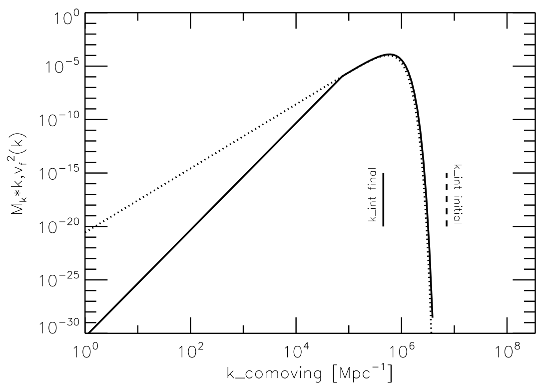
$$q > k_{eq}(a) \simeq (2\pi\mathcal{C})^{1/(\alpha-5)} k_I(a) < k_I(a)$$

independant (!) of epoch and turbulent energy density

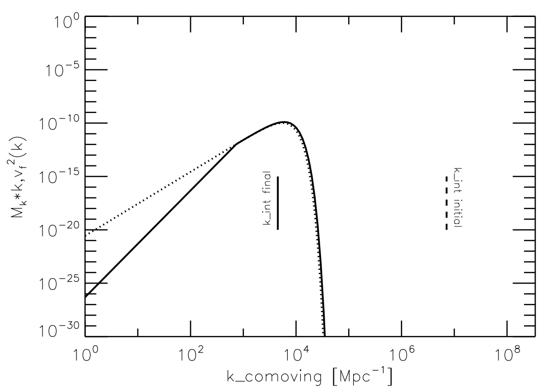
The Large-Scale Tail of Primordial Magnetic Fields:Turbulent Phases



$T = 100$ MeV

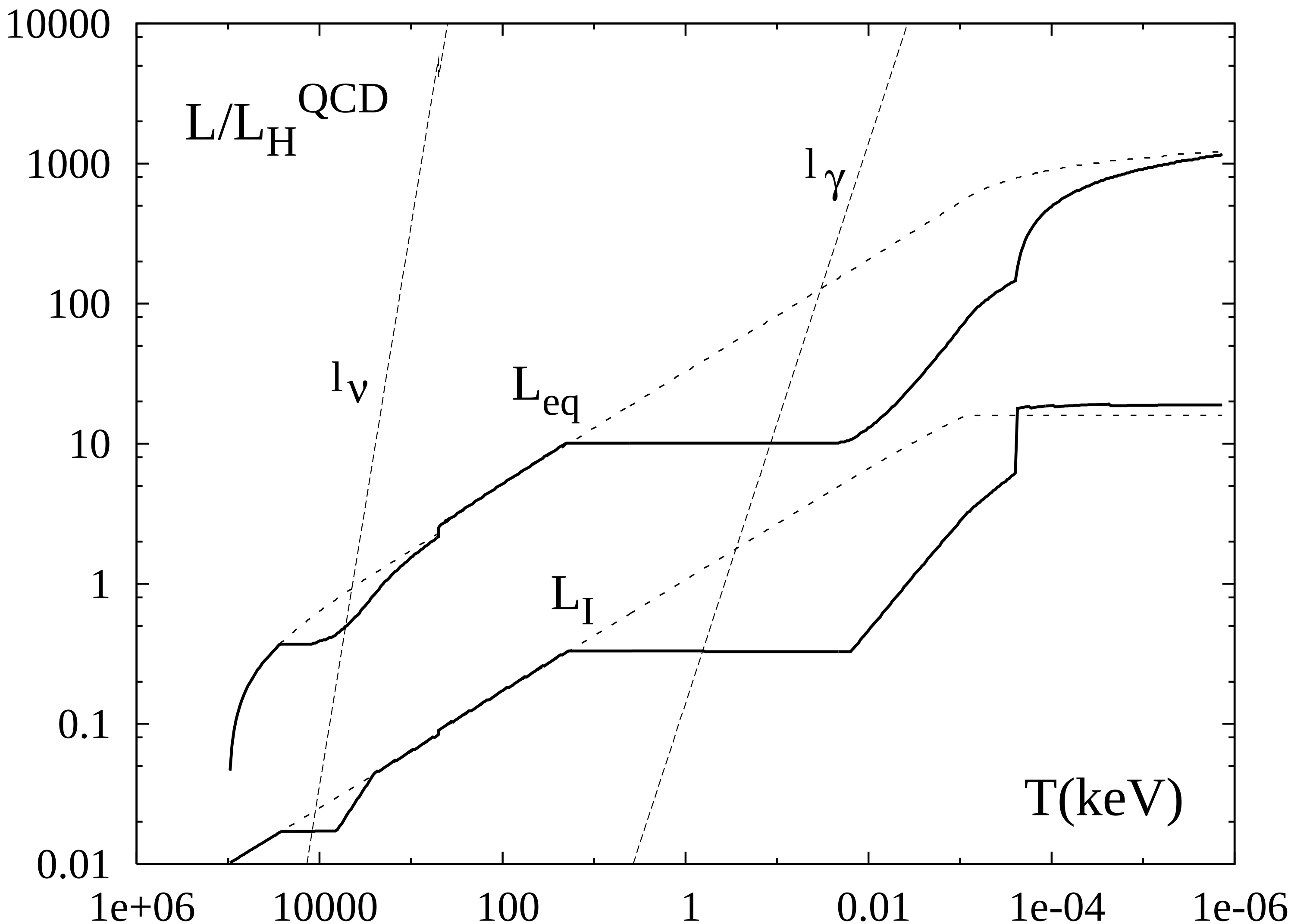


$T = 100$ keV



$T = 1$ eV

The Large-Scale Tail of Primordial Magnetic Fields:Viscous Phases Included



Jedamzik $\&$ Sigl, in preparation

- had been believed (in this community) that the large-scale magnetic field is fully produced during a short period of magnetogenesis
- is found that magnetogenesis is a continous process, with small k magnetic modes produced all the time, and large k modes decaying even without helicity !!!

many .... but, maybe most important:

Magnetic energy density realistically surviving from the early Universe is likley appreciable (compared to most batteries), volume filling (as compared to magnetic fields from outflows) and may (easily) satisfy observational lower limits (Neronov & Vovk), plus it seems to be able to get transferred on large scales via (continous) gravitational collapse ...

Final Gross Magnetic Field Properties

non-helical examples: $n = 3$

$$L_c(T) \simeq 12\,\text{pc}\left(\frac{r_g}{0.01}\right)^{1/2}\left(\frac{T_g}{100\,\text{GeV}}\right)^{-3/5}$$

$$B_c(T) \simeq 6.0\times 10^{-14}\,\text{Gauss}\left(\frac{r_g}{0.01}\right)^{1/2}\left(\frac{T_g}{100\,\text{GeV}}\right)^{-3/5}$$

helical

$$L_c(T) \simeq 1.9\,\text{kpc}\,\sqrt{n}\left(\frac{r_g}{0.01}\right)^{1/2}\left(\frac{h_g}{0.01}\right)^{1/3}\left(\frac{T_g}{100\,\text{GeV}}\right)^{-1/3}$$

$$B_c(T) \simeq 1.6\times 10^{-11}\,\text{Gauss}\left(\frac{r_g}{0.01}\right)^{1/2}\left(\frac{h_g}{0.01}\right)^{1/3}\left(\frac{T_g}{100\,\text{GeV}}\right)^{-1/3}$$

Redshifting

turbulent epochs:

$$\frac{t_{\text{eddy}}}{t_H} \approx \frac{L/v_{\text{A}}}{t_H} \propto \frac{a}{a^2} \propto 1/a \quad \text{(RD)} \qquad \propto \frac{a/1/a^{1/2}}{a^{3/2}} \propto a^0 \quad \text{(MD)}$$

$$v_A = \frac{B}{4\pi\sqrt{\varrho_\gamma + p_\gamma}}(\mathrm{RD}) \qquad v_A = \frac{B}{4\pi\sqrt{\varrho_b}}(\mathrm{MD})$$

viscous epochs:

$$\frac{t_{\text{eddy}}}{t_H} \approx \frac{L/v}{t_H} \propto a \quad (\gamma \text{ diffusion}) \qquad \propto a^{-5/2} \quad (\gamma \text{ free-streaming})$$

$$\eta \sim l_\gamma \sim T^{-3} \qquad \alpha \sim \frac{1}{l_\gamma} \sim T^3$$