Cosmic Magnetism with Information Field Theory

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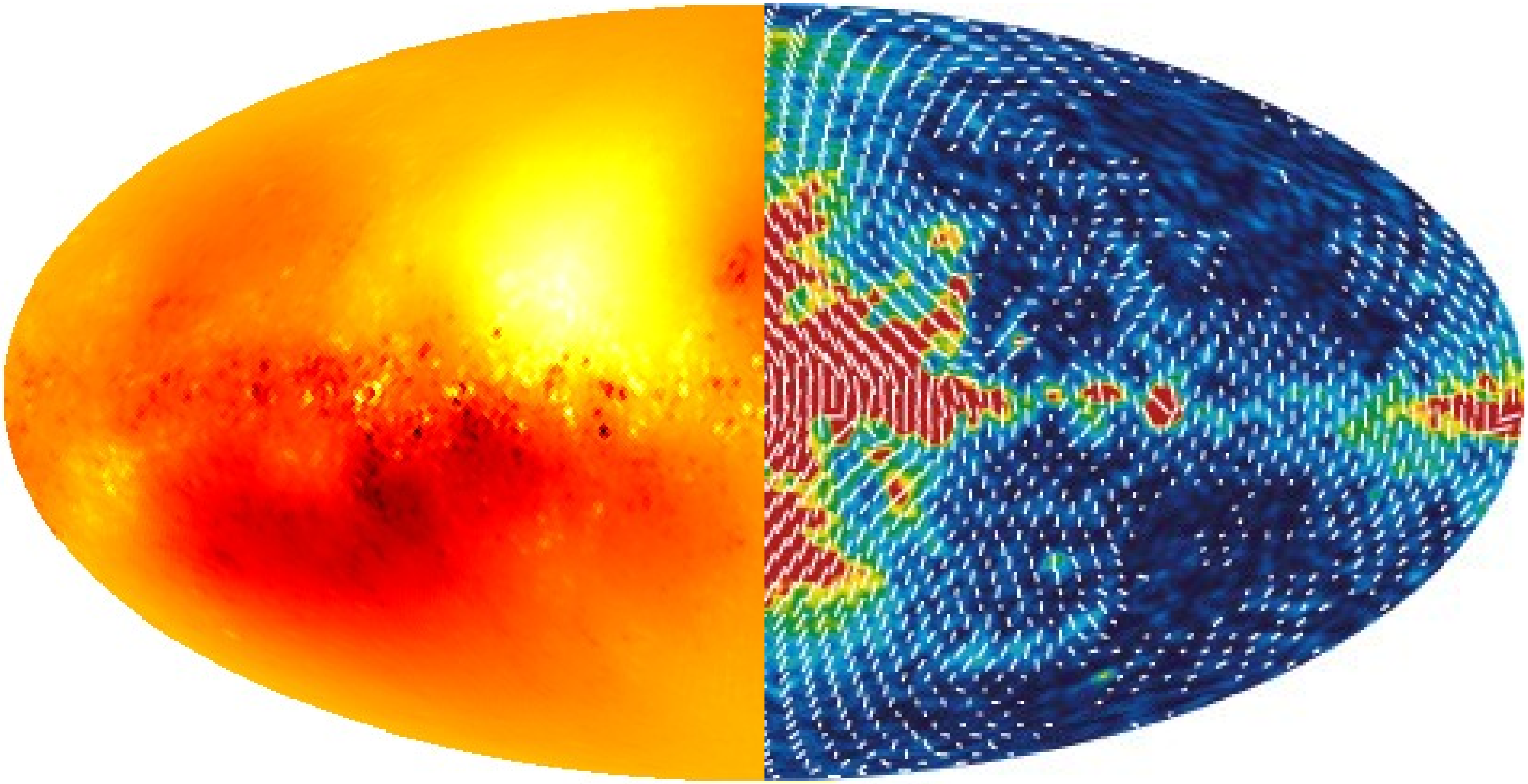
Niels Oppermann, Georg Robbers, Petr Kuchar

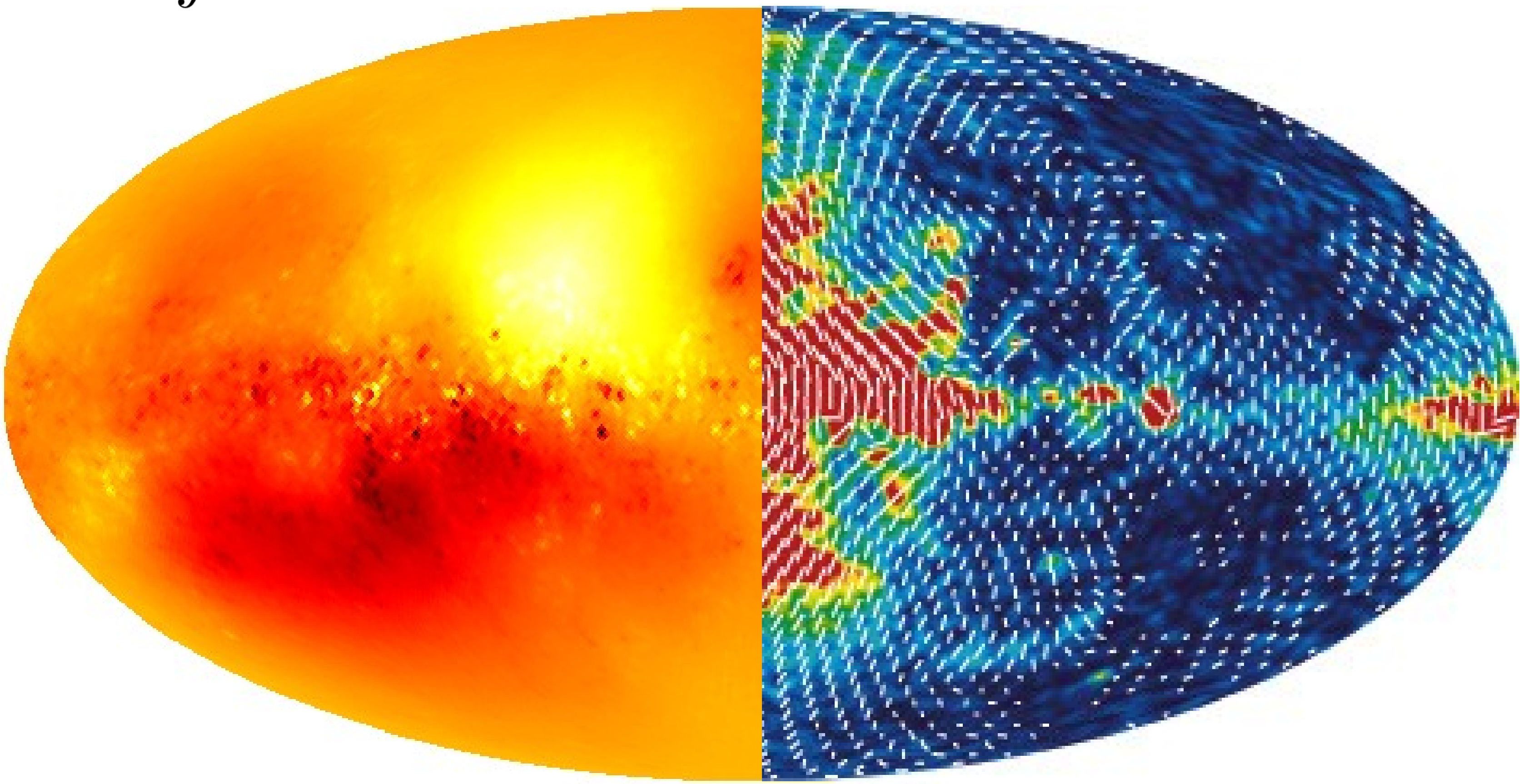
Max-Planck-Institut
für Astrophysik



magnetic field observables

$$\phi \propto \int dz\, B_z$$

$$P \propto \int dz\, (B_x^2 - B_y^2 + 2\,i\,B_x B_y)$$

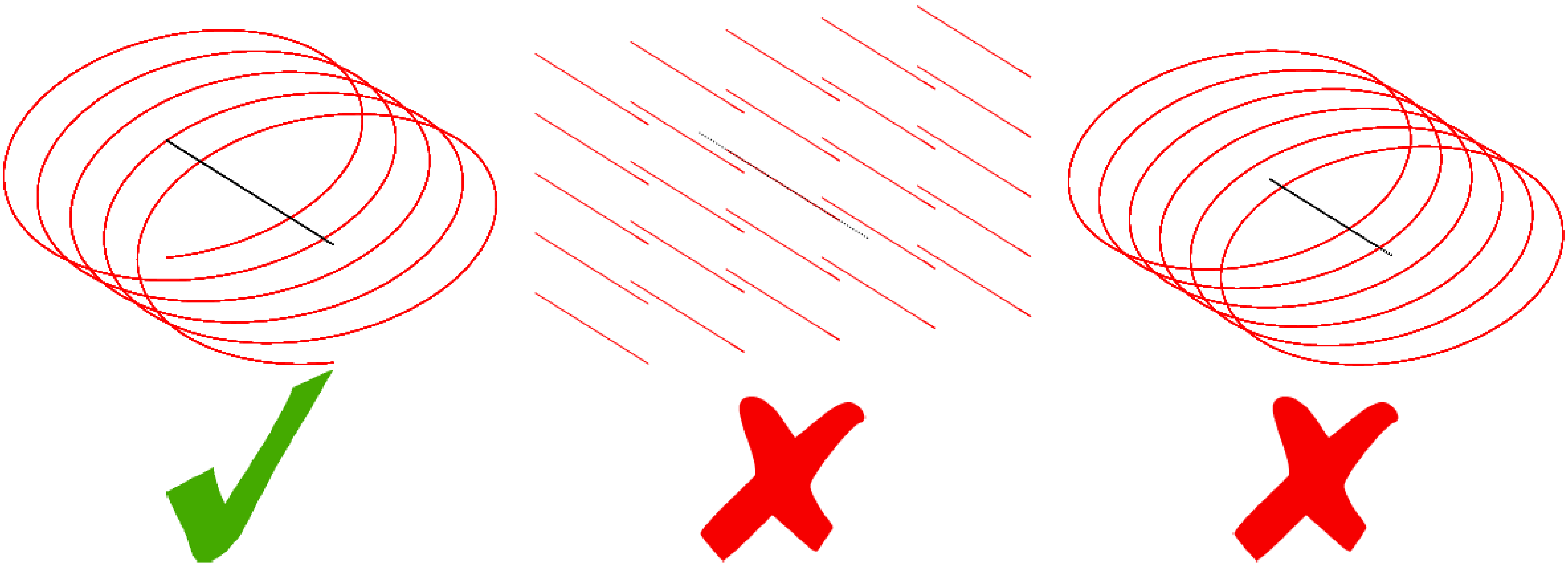


data: NVSS, Taylor et al. 2009
map: Oppermann et al., arXiv:1008.1243

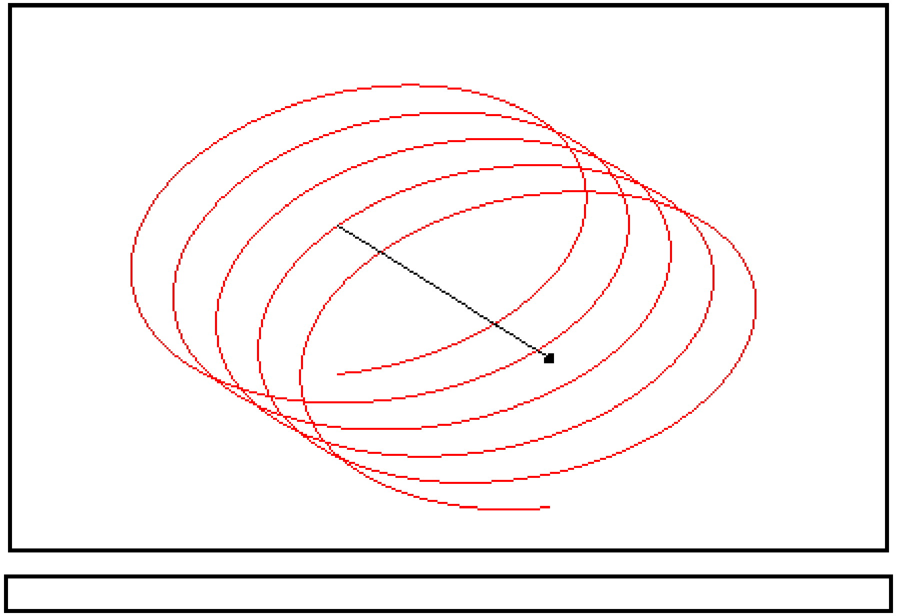
data: WMAP, WMAP-team
map: Page et al.

magnetic helicity

$$H = \int A \cdot B \, \mathrm{d}V$$



Probing Helicity by LITMUS



Local
Inference
Test for
Magnetic fields,
which **U**ncovers
helice**S**

Junklewitz & Enßlin (2010), arXiv:1008.1243

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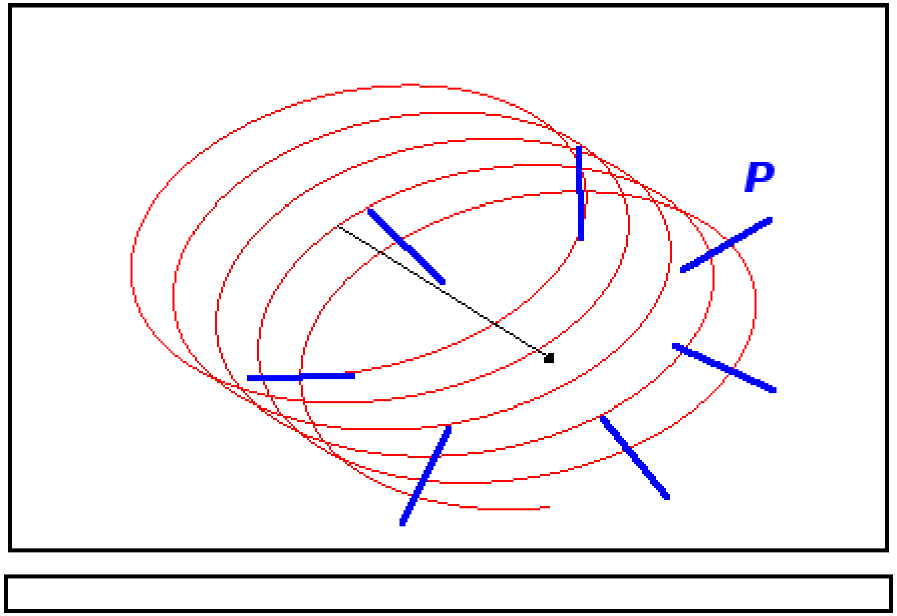

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$$P = |P|e^{2\,i\,\alpha}$$

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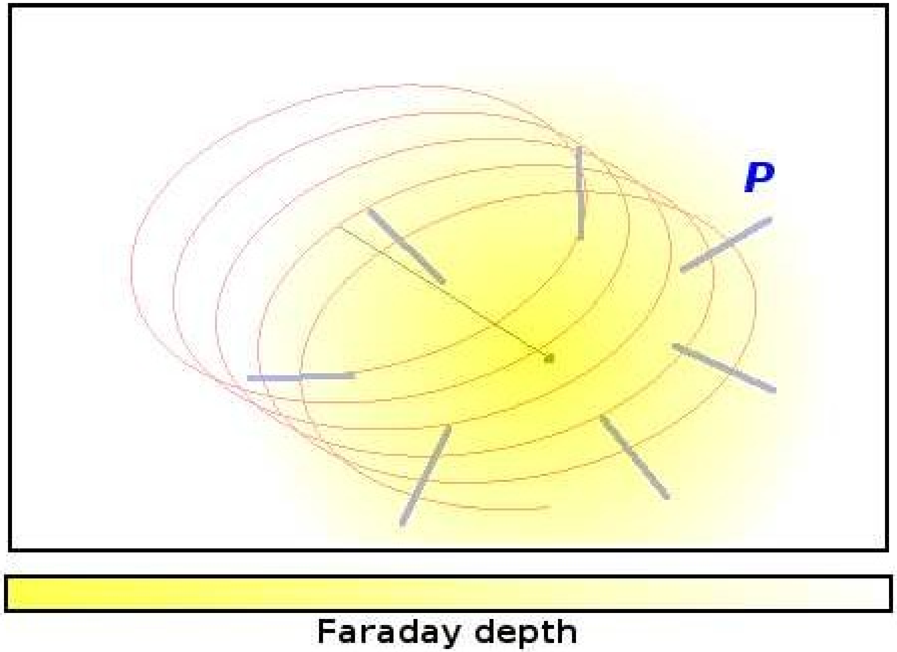

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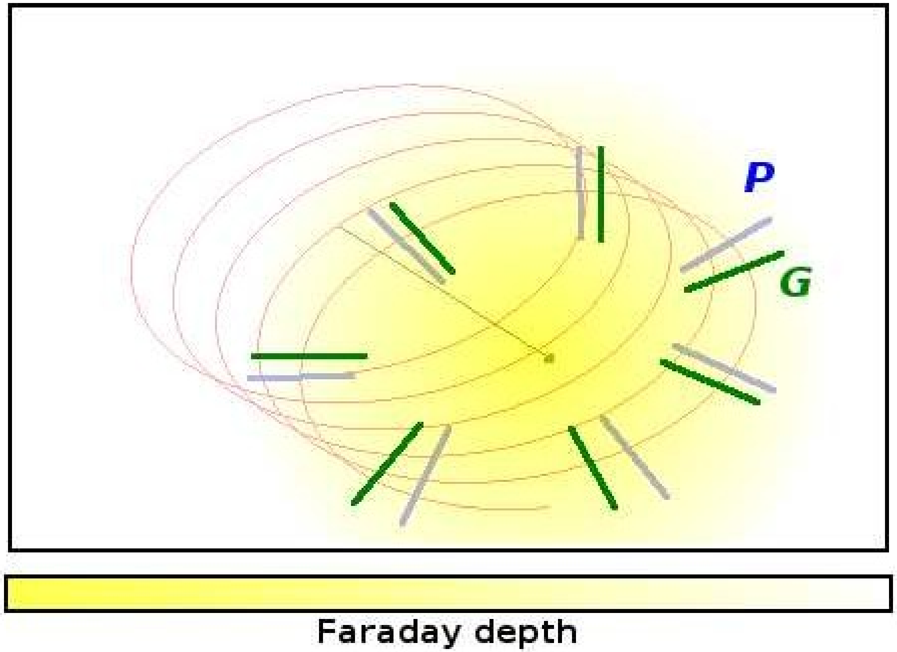

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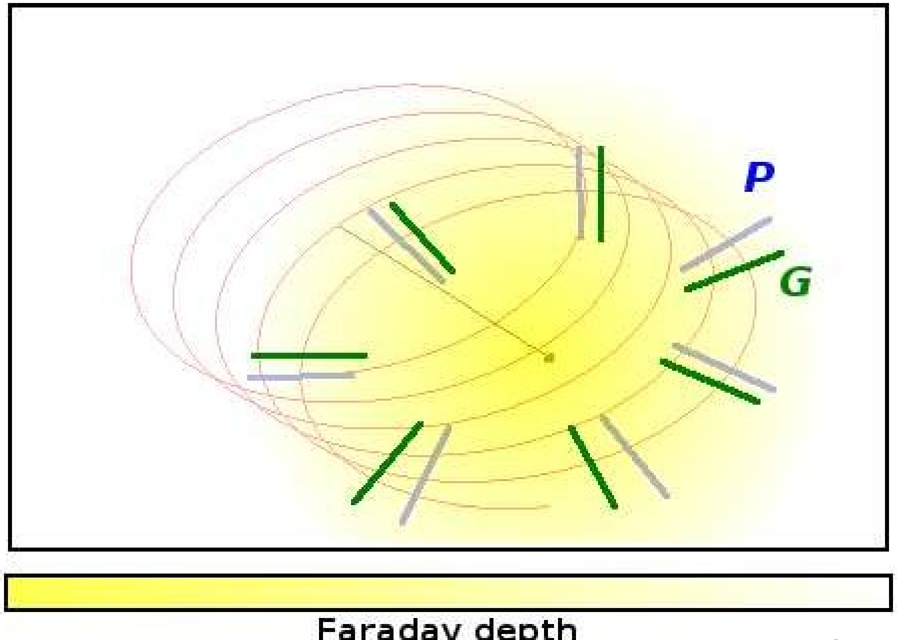
$$
\begin{aligned}
P &= |P| e^{2\,i\,\alpha} \\
G &= T_2(\nabla\,\phi) \\
&= (\partial_x \phi + i\,\partial_y \phi)^2 \\
&= |G| e^{2\,i\,\gamma}
\end{aligned}
$$

$$T_2 : \; \mathbb{R}^2 \rightarrow \mathbb{C}$$

Probing Helicity by LITMUS



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helicity: $\mathrm{Re}(G\,P^*) > 0$

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$$G = T_2(\nabla\,\phi) = (\partial_x \phi + i\,\partial_y \phi)^2 = |G| e^{2\,i\,\gamma}$$

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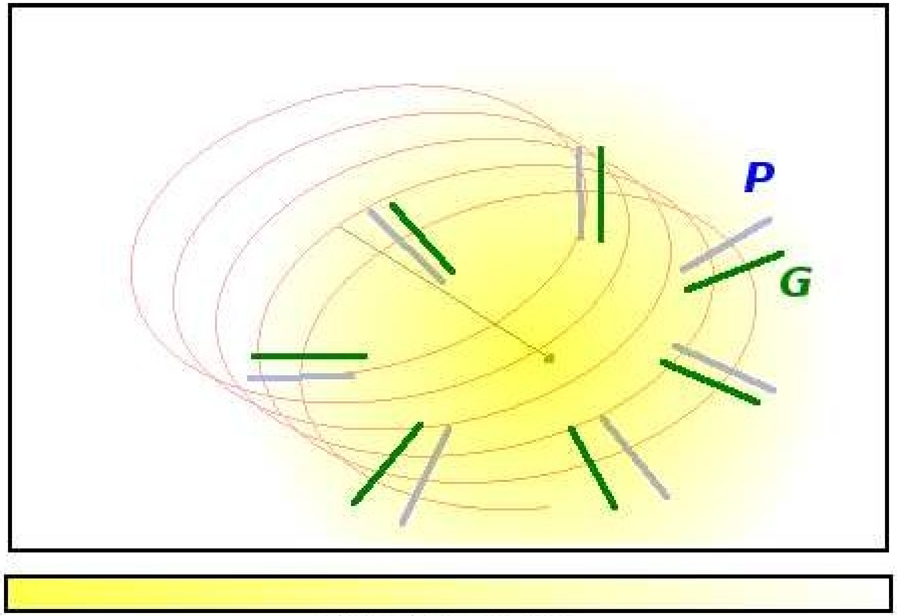

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$$P = |P| e^{2\,i\,\alpha}$$

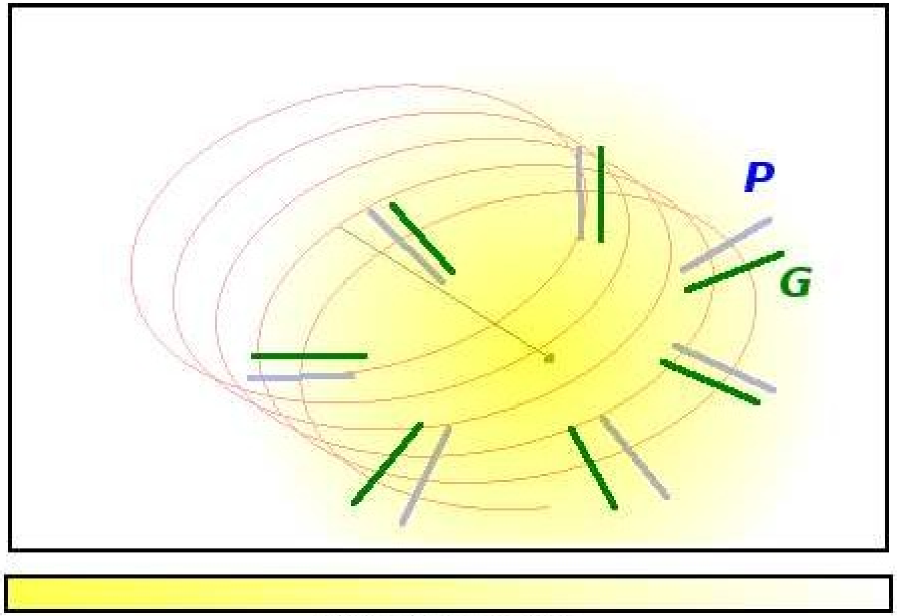
$$G = T_2(\nabla\,\phi)$$

helicity: $\mathrm{Re}(G\,P^*) > 0$

$$\langle G\,P^* \rangle = 2 L_z \pi^4 \left[\int_0^\infty dk\, \frac{\epsilon_\mathrm{H}(k)}{k} \right]^2$$

$$\langle B_i\, B_j^* \rangle(\vec{k}) = \frac{\epsilon(k)}{8\,\pi k^2} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) + \epsilon_\mathrm{H}(k)\,\pi^2\, i\,\varepsilon_{ijk}\,\frac{k_k}{k^4}$$

Probing Helicity by LITMUS



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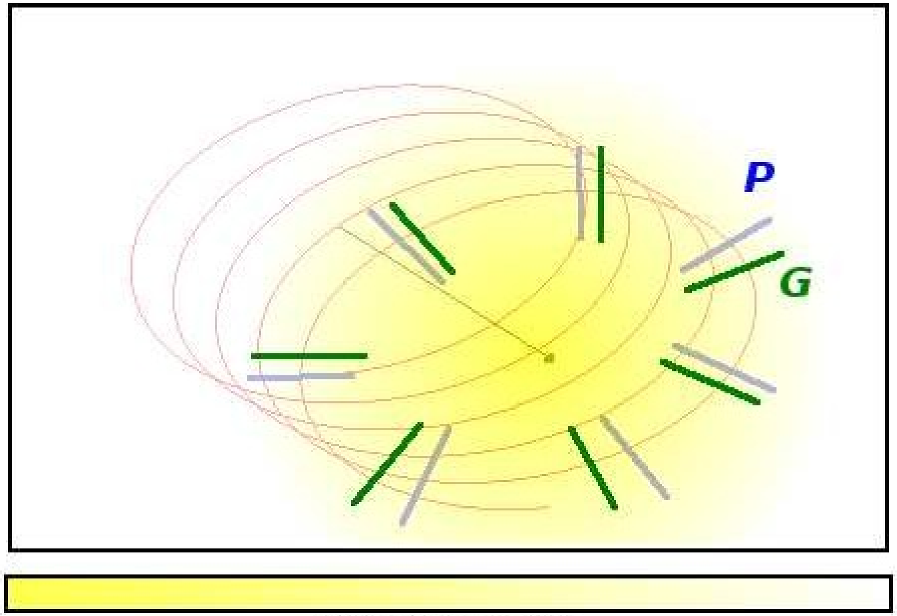

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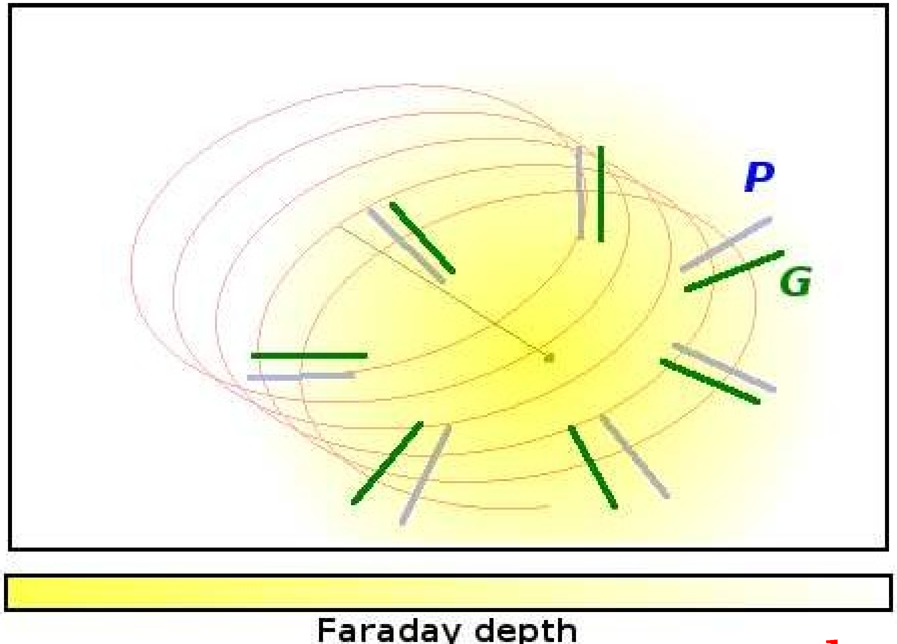

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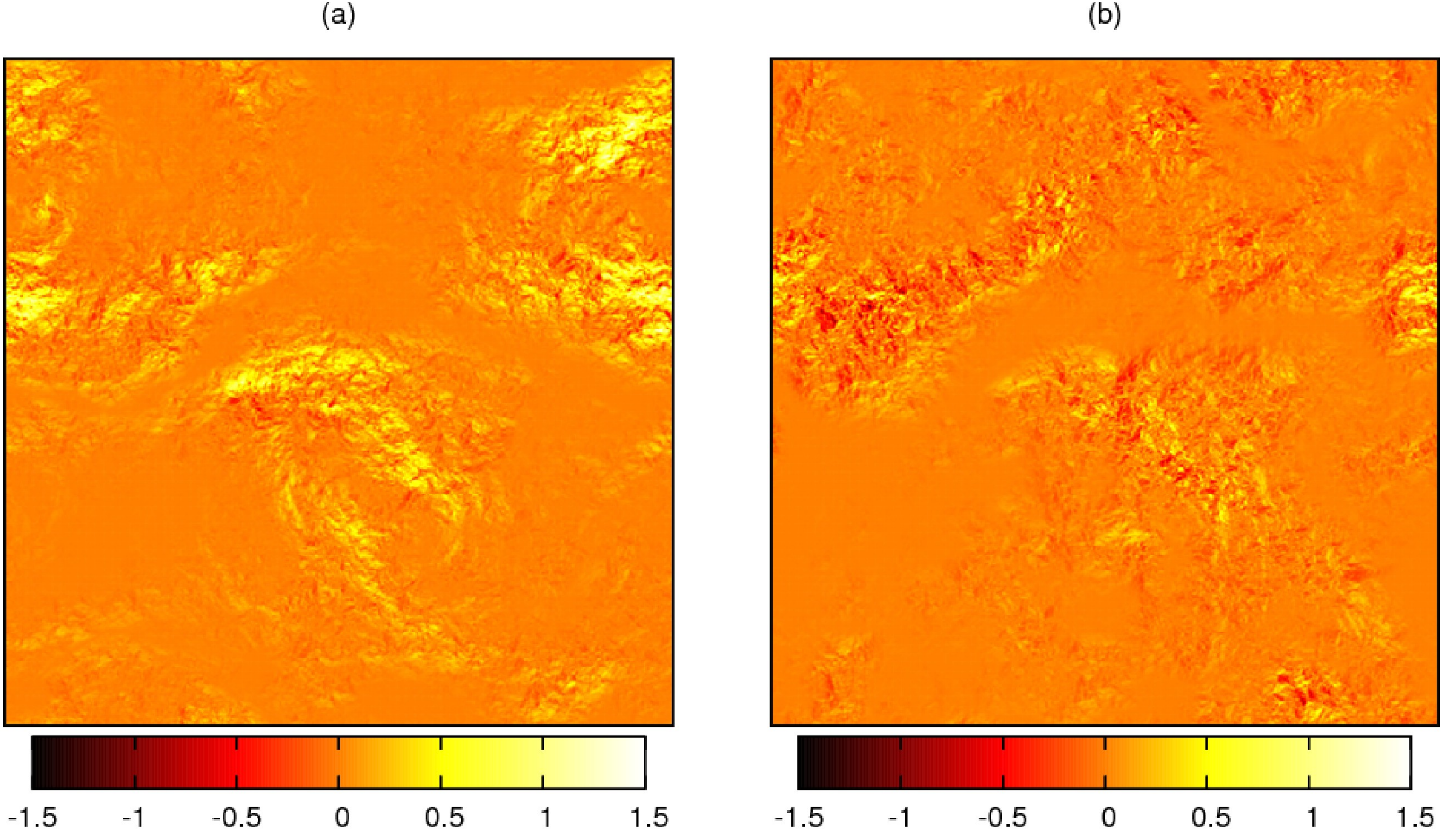
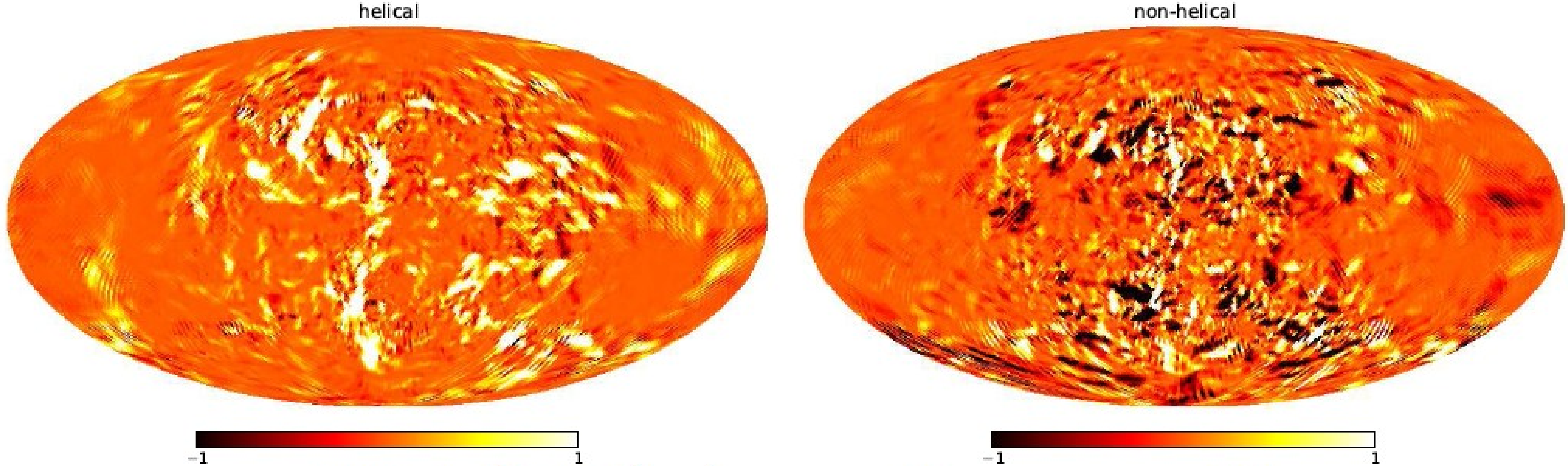
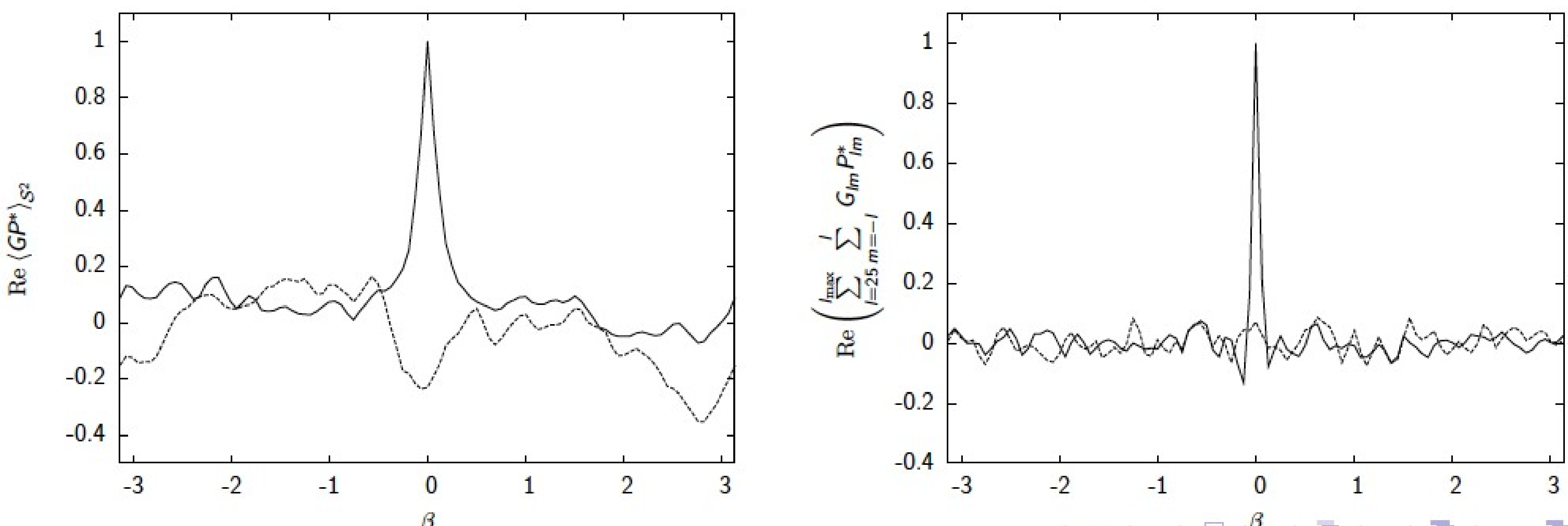
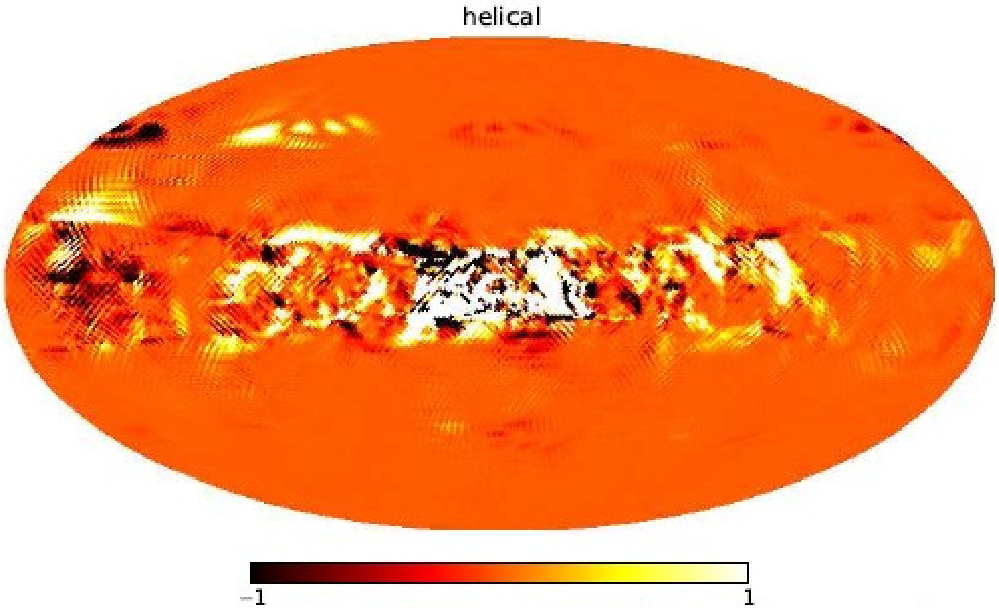


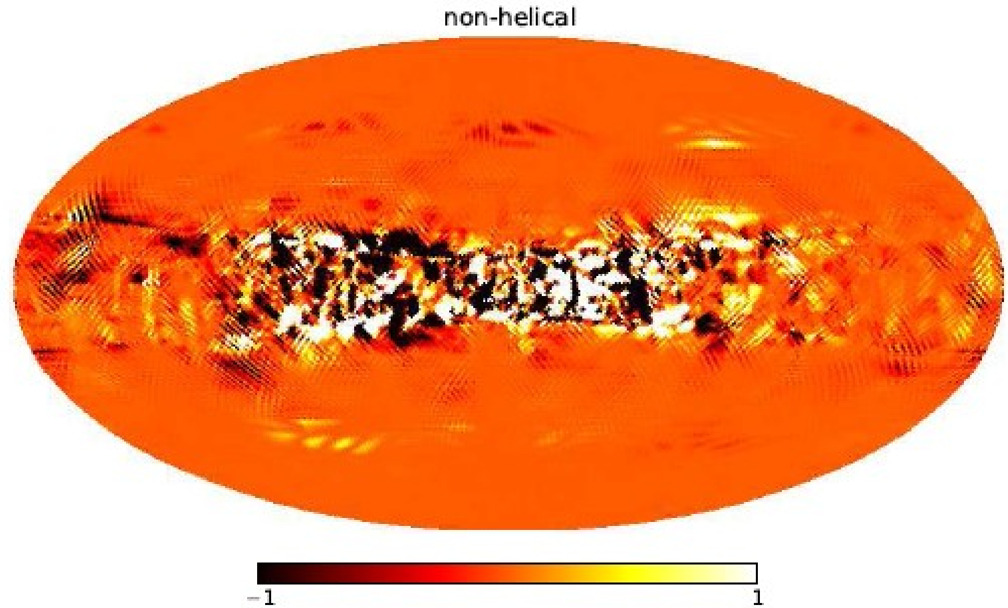


with helicity: $\left\langle \mathrm{Re}\left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = 1.0, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_{\square}} = 0.25$

without helicity: $\left\langle \mathrm{Re}\left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = -0.27, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_{\square}} = 0.23$

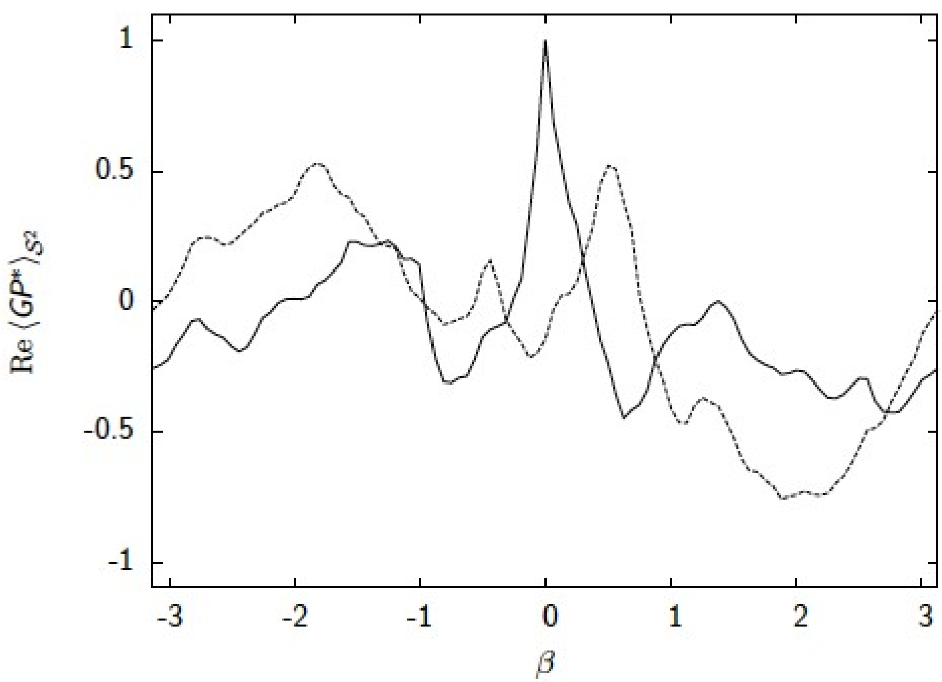


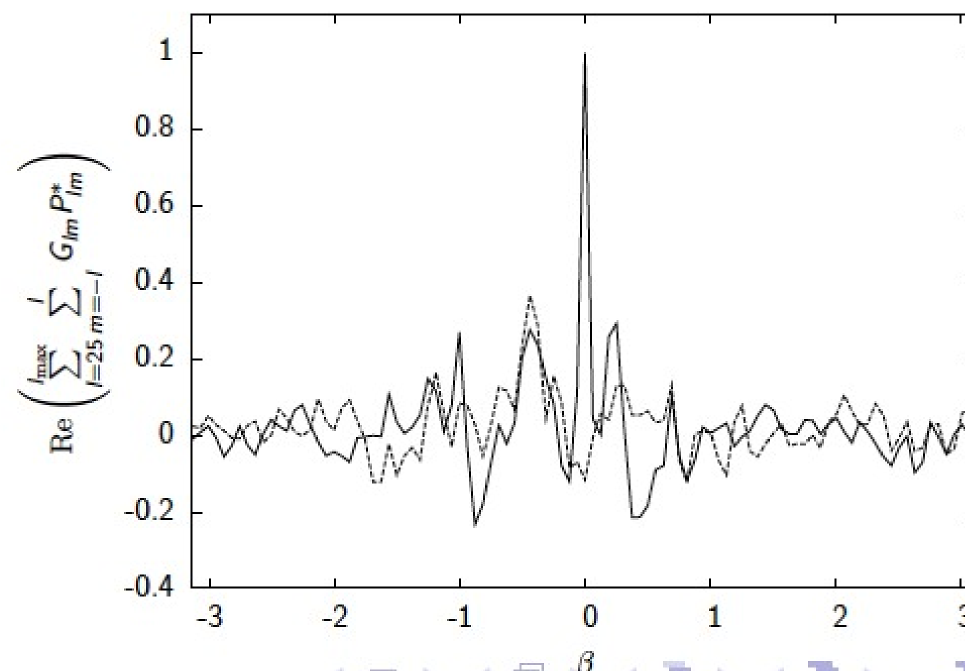


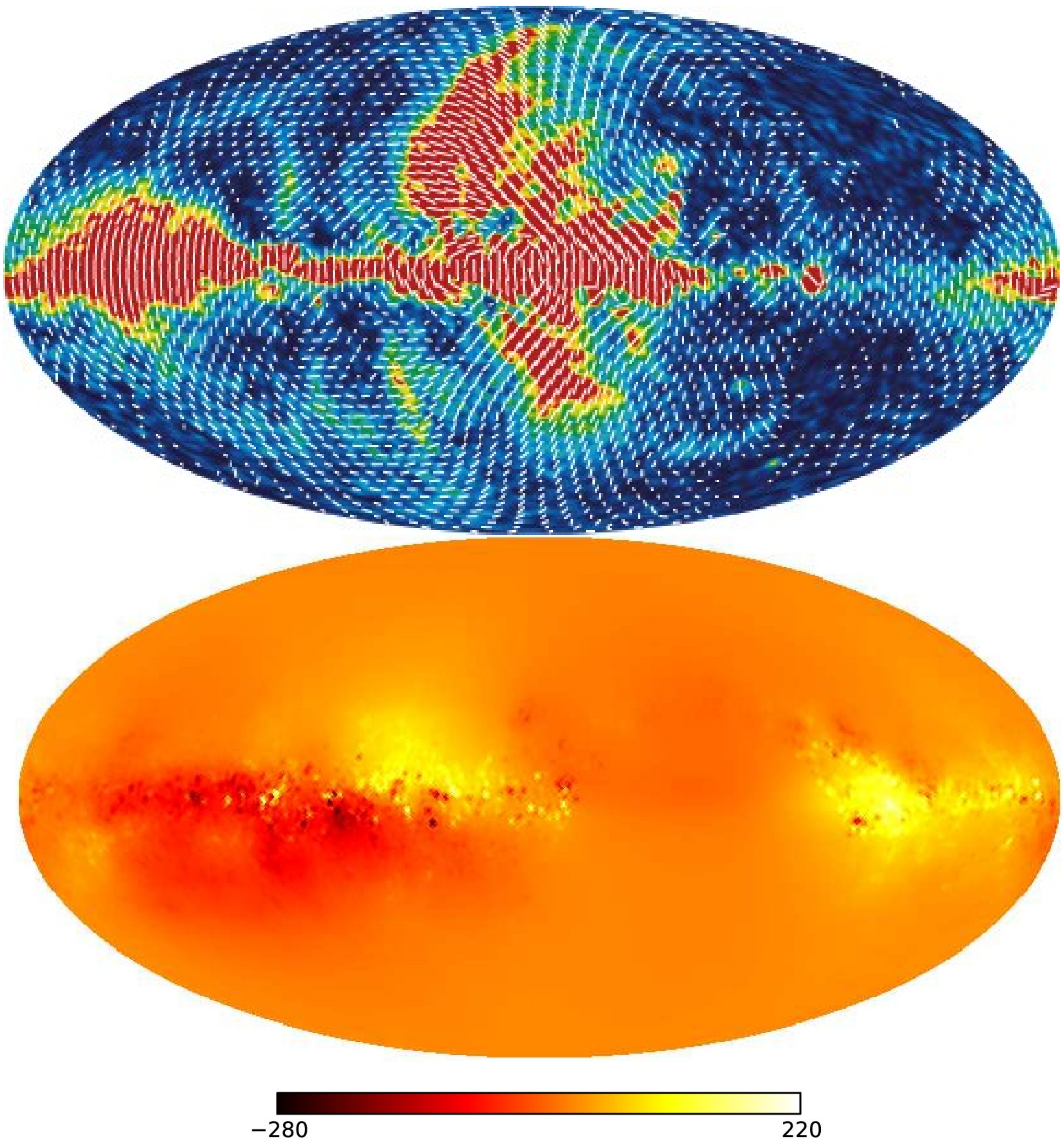


with helicity: $\left\langle \mathrm{Re} \left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = 1.0, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_\square} = 0.57$

without helicity: $\left\langle \mathrm{Re} \left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = -0.43, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_\square} = 0.74$

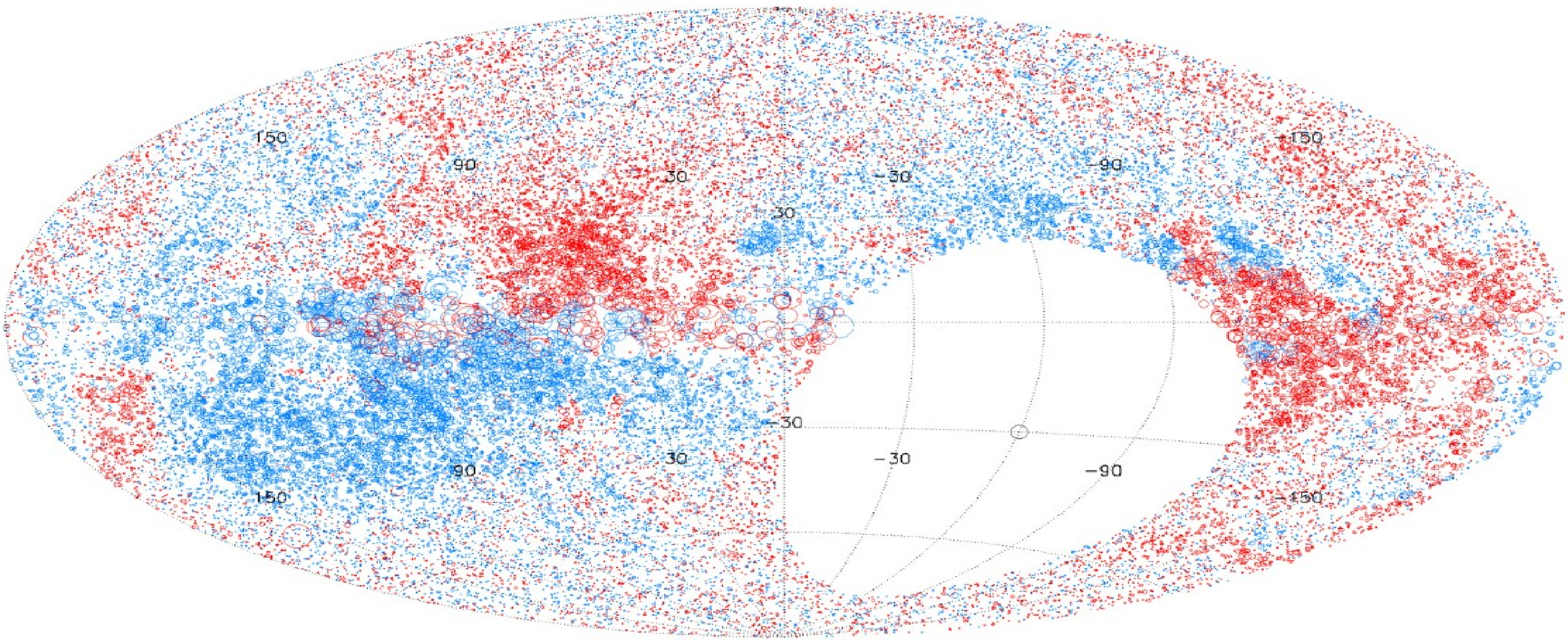






−280
220

The Faraday rotation sky



Taylor,Stil,Sunstrum 2009

37,534 RM extragalactic RM-sources in the northern sky

Information Field Theory

{

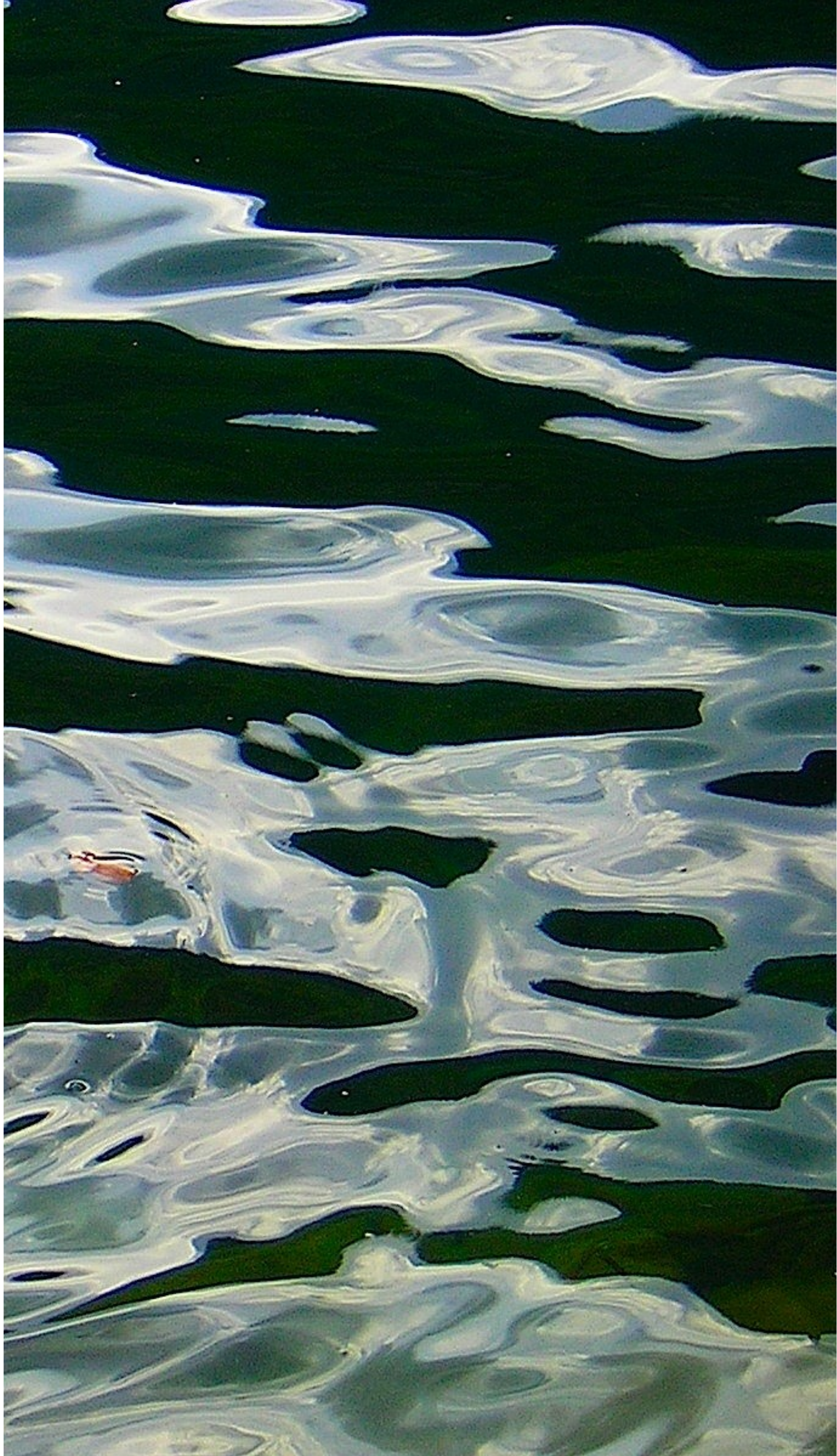
Why Information Field Theory ?

inverse problem => Information Theory

spatially distributed quantity => Field Theory

Information fields of interest:

- magnetic field in 3d
- Faraday rotation map
- polarized emissivity per Faraday depth
- cosmic matter distribution
- magnetic power spectra, ...



What is information theory ?

What is information?
knowledge about

the set of possibilities Ω
their individual probabilities $P:\Omega \rightarrow [0,1]$

knowledge state (Ω, P)

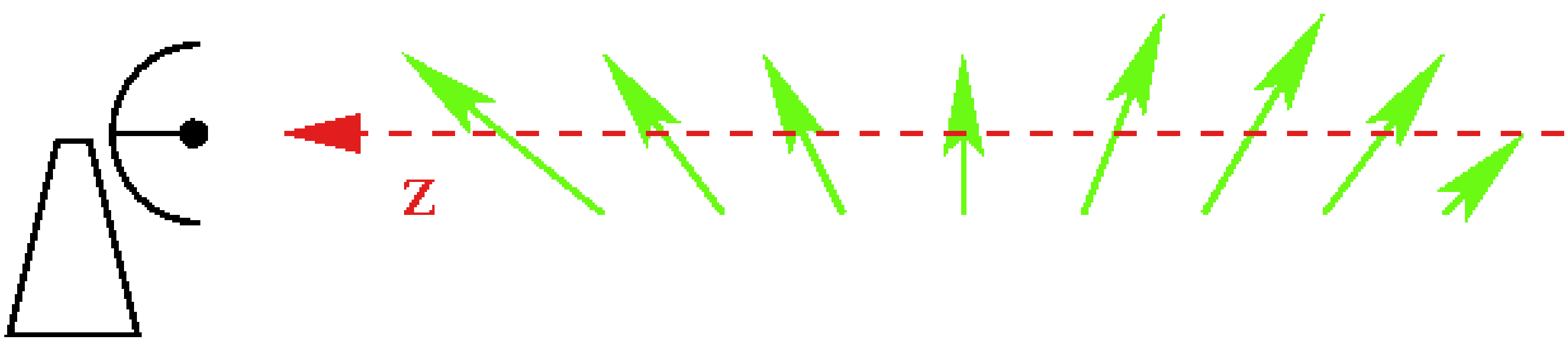
information theory is the math of knowledge states
it is simply probability theory applied to reasoning

How to obtain information ?

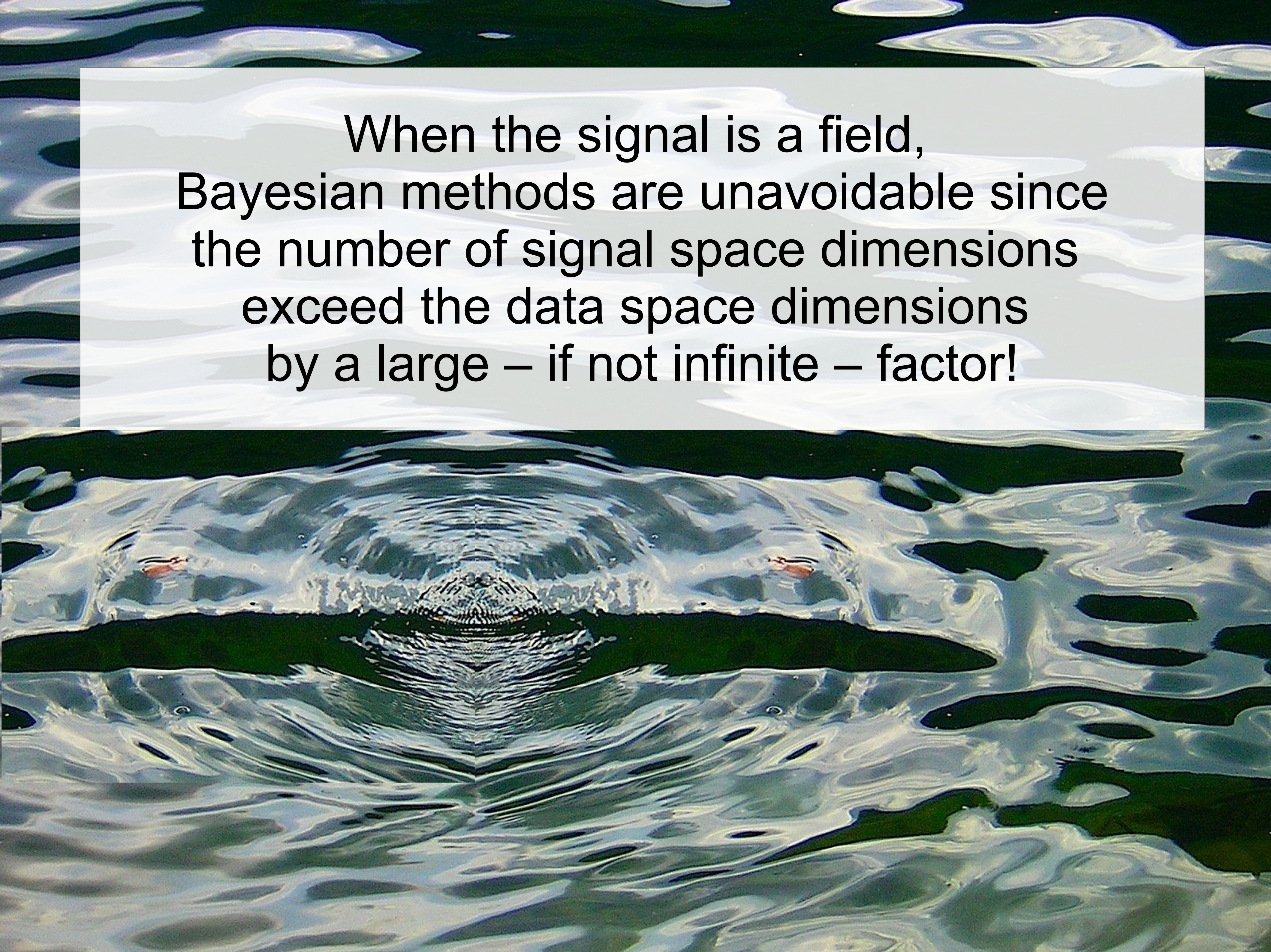
physical signal s

1) prior knowledge: P(s)

2) measurement: d = R(s) + n, P(d|s)

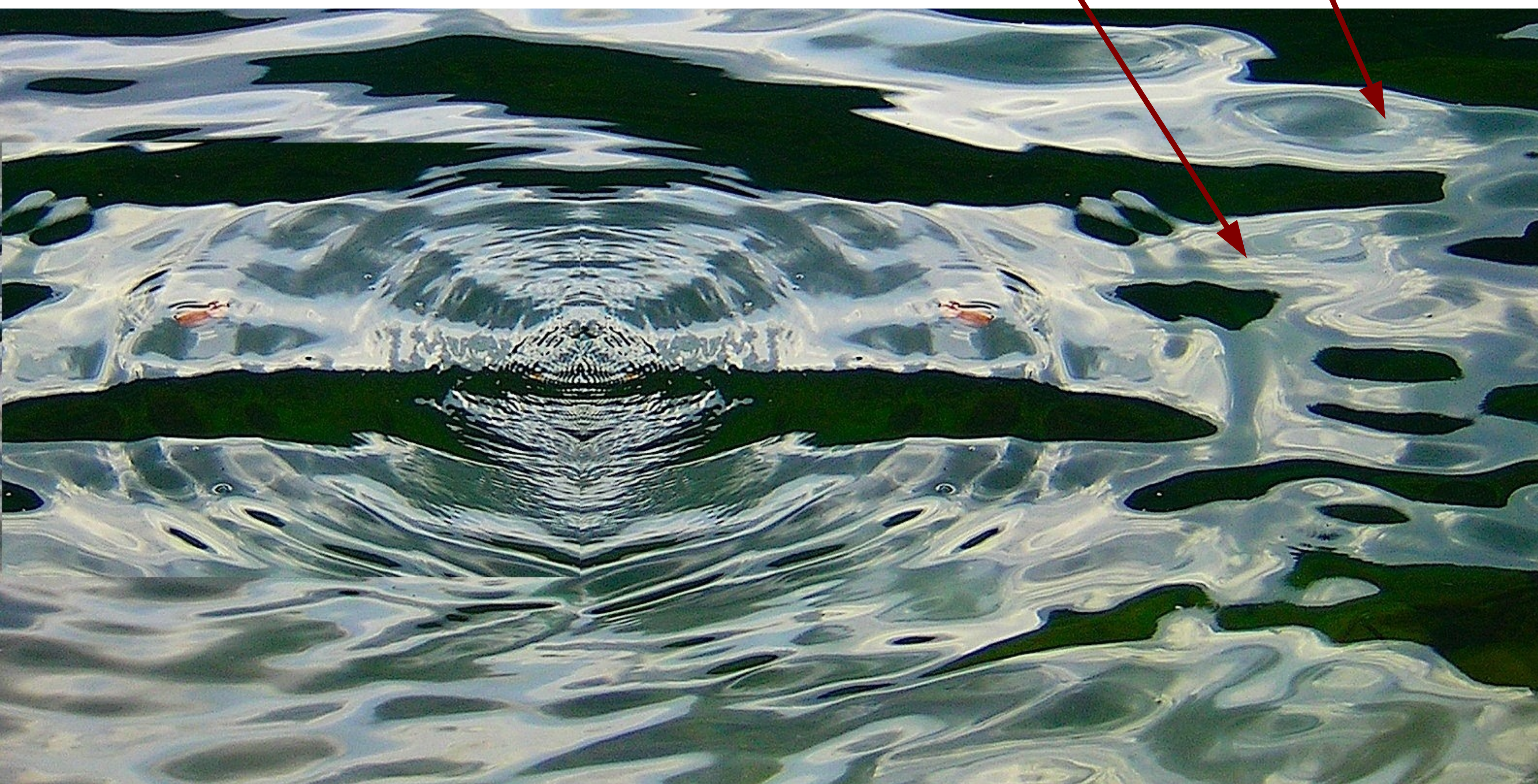


3) inference: P(s|d)=P(d|s) P(s)/P(d), d → s


When the signal is a field,
Bayesian methods are unavoidable since
the number of signal space dimensions
exceed the data space dimensions
by a large – if not infinite – factor!

$$\langle s(x_1)\cdots s(x_n)\rangle_d \equiv \langle s(x_1)\cdots s(x_n)\rangle_{(s|d)}$$

$$\equiv \int \mathcal{D}s\, s(x_1)\cdots s(x_n)\, P(s|d)$$



$$\begin{aligned} \langle s(x_1)\cdots s(x_n)\rangle_d &\equiv \langle s(x_1)\cdots s(x_n)\rangle_{(s|d)} \\ &\equiv \int \mathcal{D}s\, s(x_1)\cdots s(x_n)\, P(s|d) \end{aligned}$$

$$\int \mathcal{D}f\, F[f] \equiv \left(\prod_{i=1}^{N_{\rm pix}} \int df_i\right) F(f_1,\ldots,f_{N_{\rm pix}})$$

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\end{aligned}
$$

$$
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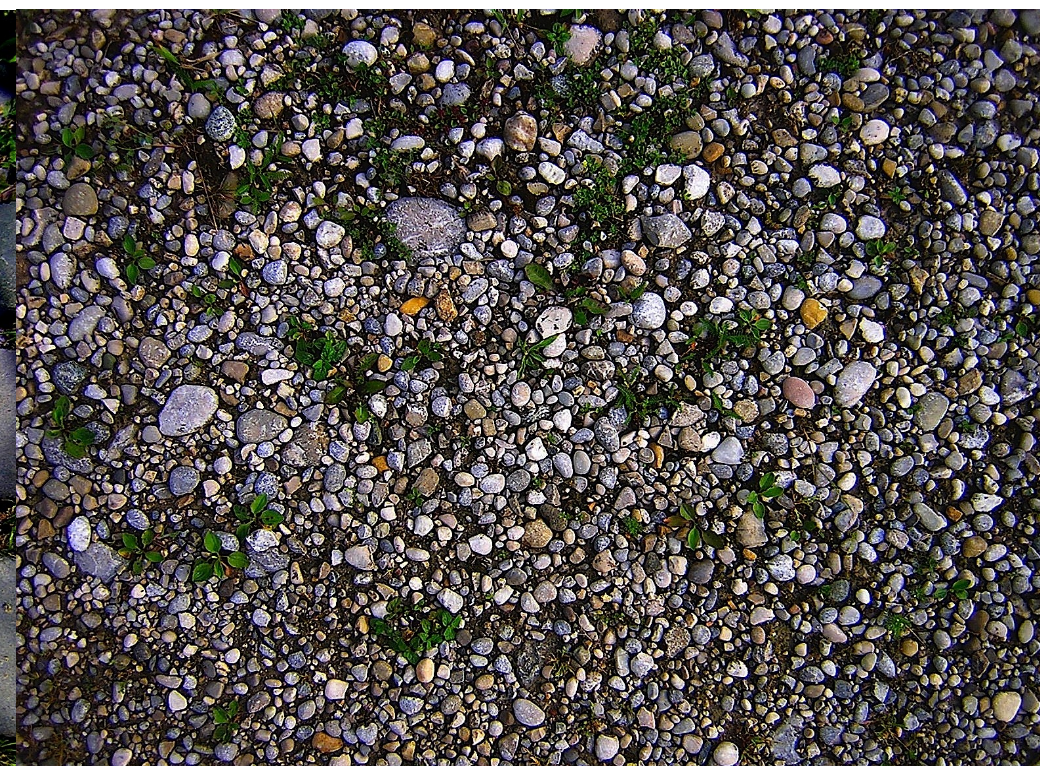
$$



Information Hamiltonian

s = signal
d = data

posterior

likelihood

prior

$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)}$$

evidence

Basic idea: expand posterior around Gaussian

Free Theory

Gaussian signal & noise, linear response

signal :

$$
\begin{aligned}
P(s) &= \mathcal{G}(s,S) = \frac{1}{|2\pi\, S|^{1/2}} \exp\left(-\frac{1}{2}\, s^\dagger S^{-1} s\right) \\
j^\dagger s &= \int dx\, j^*(x) s(x) \\
S &= \langle s\, s^\dagger \rangle_{(s)}
\end{aligned}
$$

data :

$$
d = R\, s + n,\ P(d|s) = P(n = d - R\, s)
$$

noise :

$$
P(n) = \mathcal{G}(n, N),\ N = \langle n\, n^\dagger \rangle_{(n)}
$$

Wiener filter theory

known for 60 years

information source

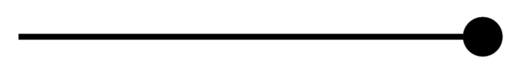
$$H(s) = -\log P(d,s) = -\log P(d|s) - \log P(s)$$

$$= \frac{1}{2}(d - R\,s)^\dagger N^{-1}(d - R\,s) + \frac{1}{2}\,s^\dagger S^{-1} s + \text{const}$$

$$= \frac{1}{2}\,s^\dagger \underbrace{(S^{-1} + R^\dagger N^{-1} R)}_{\equiv D^{-1}}\,s + s^\dagger \underbrace{R^\dagger N^{-1} d}_{\equiv j} + \text{const}$$

information propagator

$$= \frac{1}{2}\,s^\dagger D^{-1}\,s + s^\dagger j + H_0$$

mean: $m = \langle s \rangle_{(s|d)} = D\,j =$ 

uncertainty: $\langle (s - m)\,(s - m)^\dagger \rangle_{(s|d)} = D$

Interacting Theory

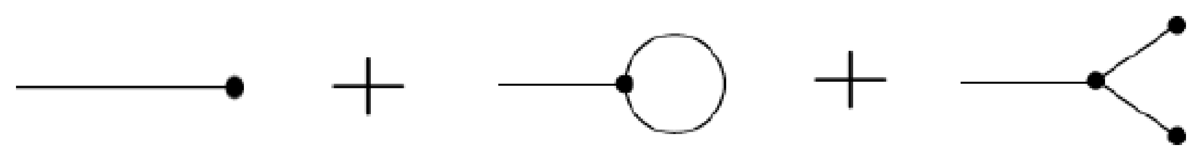
non-Gaussian signal, noise, or non-linear response

$$H[s] = \frac{1}{2}\, s^\dagger D^{-1}\, s - j^\dagger s + H_0 + \sum_{n=3}^{\infty} \frac{1}{n!}\, \Lambda^{(n)}_{x_1 \ldots x_n}\, s_{x_1} \cdots s_{x_n}$$

Taylor-Fréchet expansion of Hamiltonian

Use expansion into Feynman diagrams

$$\langle s \rangle_{(s|d)} =$$



$$+ \quad \ldots$$

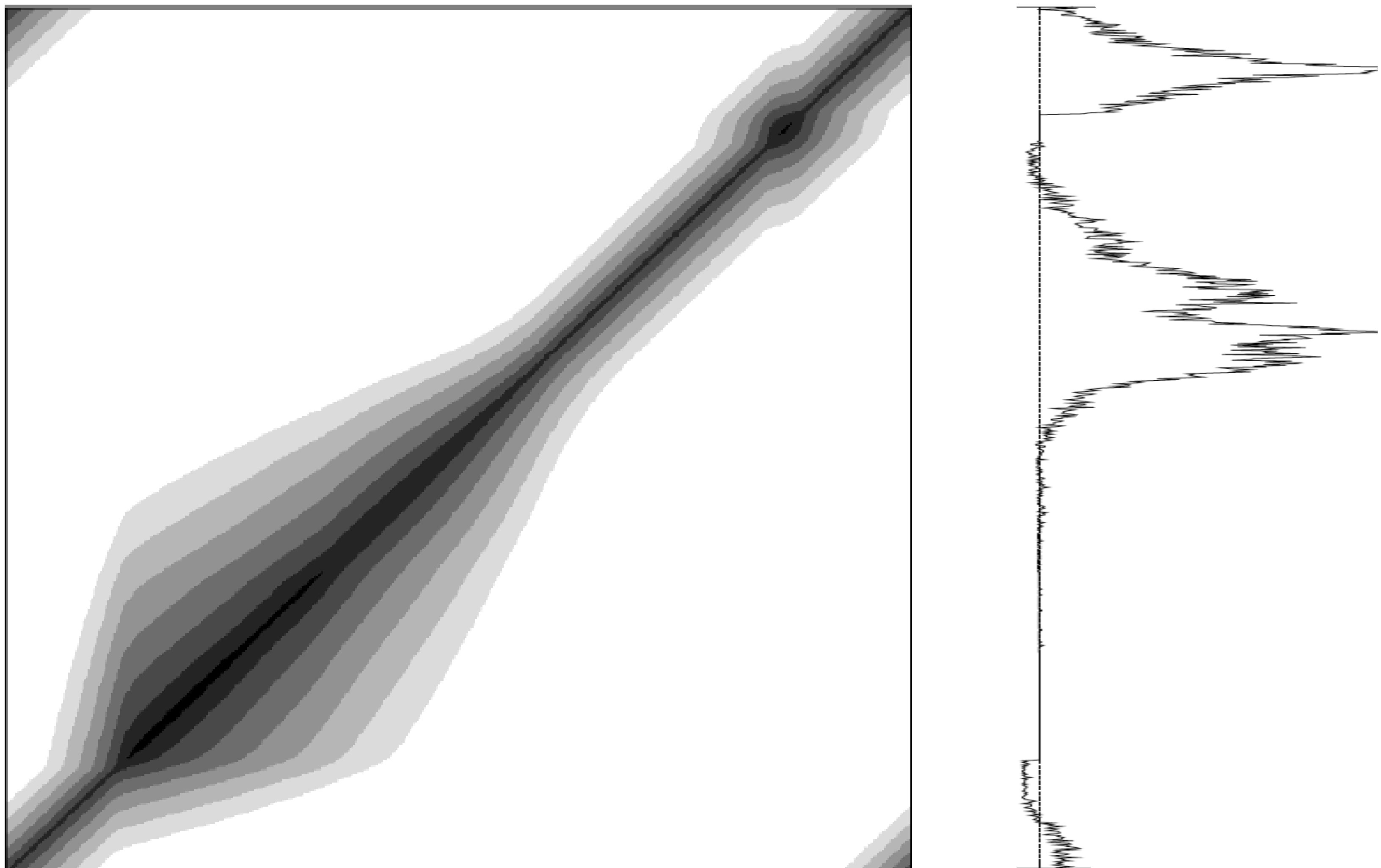
$$= D_{xy}\, j_y \; - \frac{1}{2} D_{xy}\, \Lambda^{(3)}_{yzu}\, D_{zu}$$

$$- \frac{1}{2} D_{xy}\, \Lambda^{(3)}_{yuz}\, D_{zz'}\, j_{z'}\, D_{uu'}\, j_u \quad + \quad \ldots$$

$$m = \langle s \rangle_{(s|d)} = D\, j$$

$$j = R^{\dagger} N^{-1} d \qquad D = \left[S^{-1} + R^{\dagger} N^{-1} R \right]^{-1}$$

$$m = \langle s \rangle_{(s|d)} = D\, j = D_{xy}\, j_y$$



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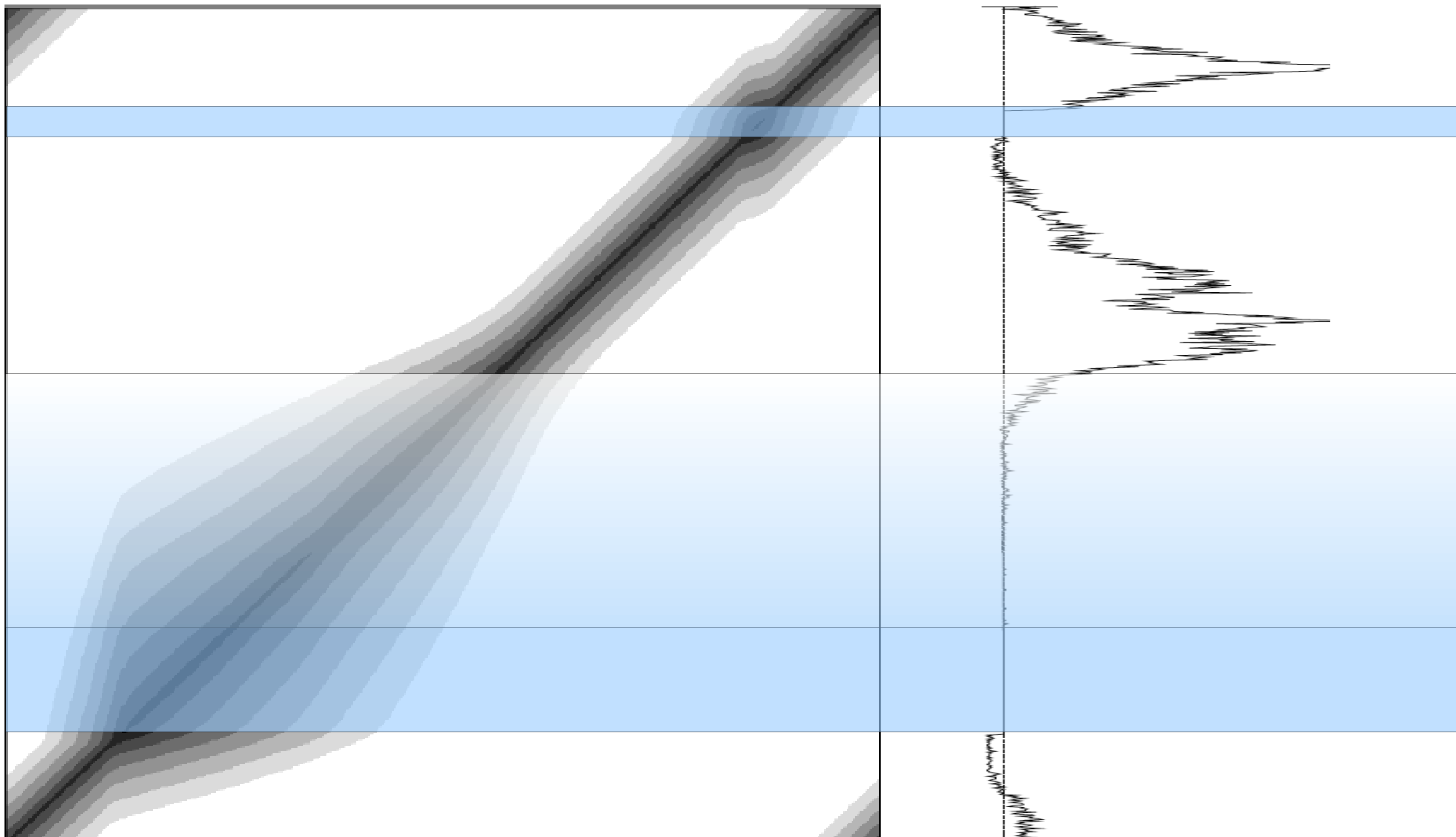

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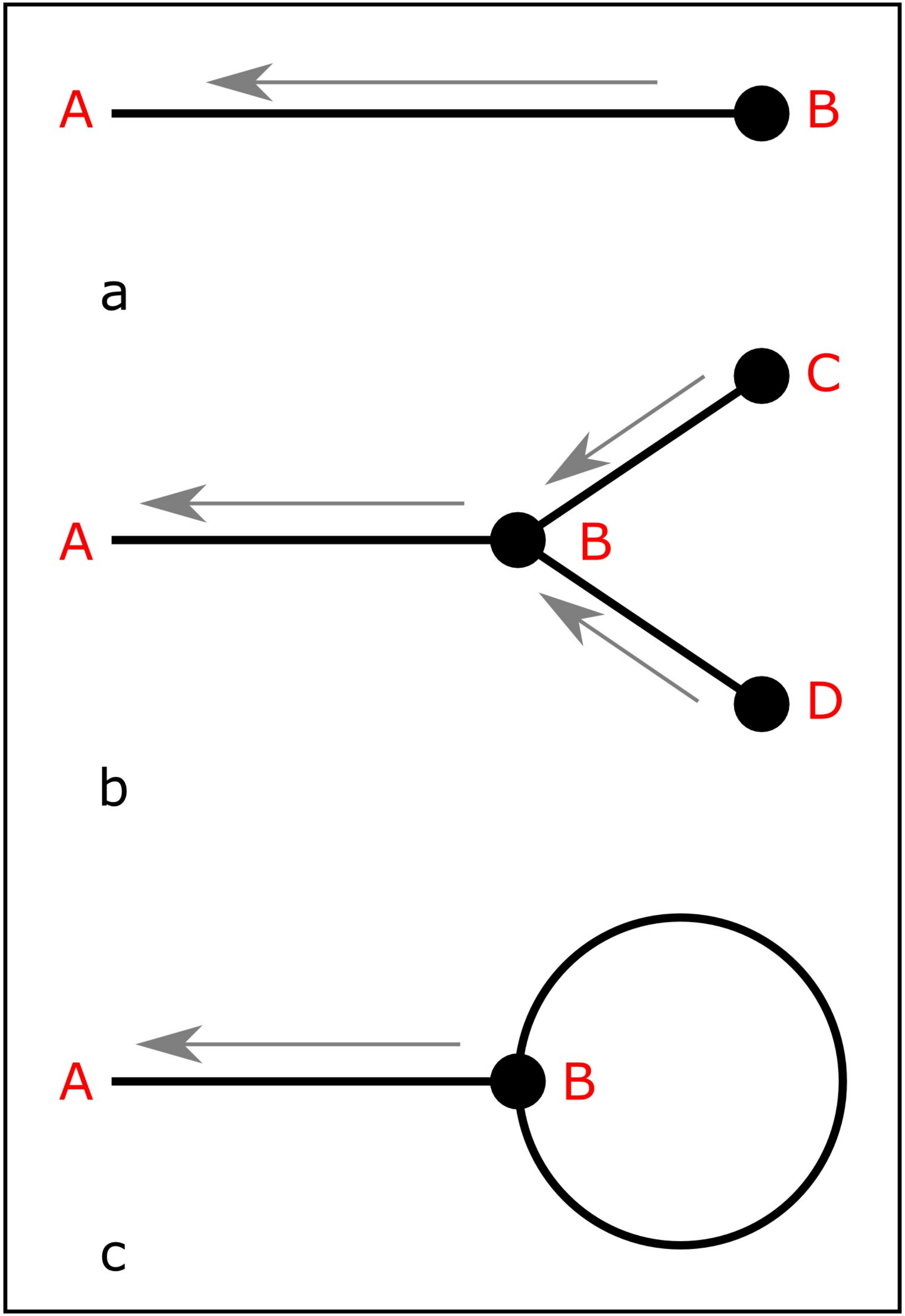

A
B
a
C
A
B
D
b
A
B
c

dictionary

Translation:

inference problem → statistical field theory

$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)} \equiv \frac{1}{Z}\,e^{-H[s]}$$

Dictionary:

log-Posterior	=	negative Hamiltonian
Evidence	=	partition function Z
Wiener variance	=	information propagator
noise weighted data	→	information source
inference algorithms	←	Feynman diagrams
maximum a Posteriori	=	classical solution
uncertainty corrections	=	loop corrections
Shannon information	=	negative entropy

Reconstruction of signals with unknown spectra

Enßlin & Frommert arXiv:1002.2928

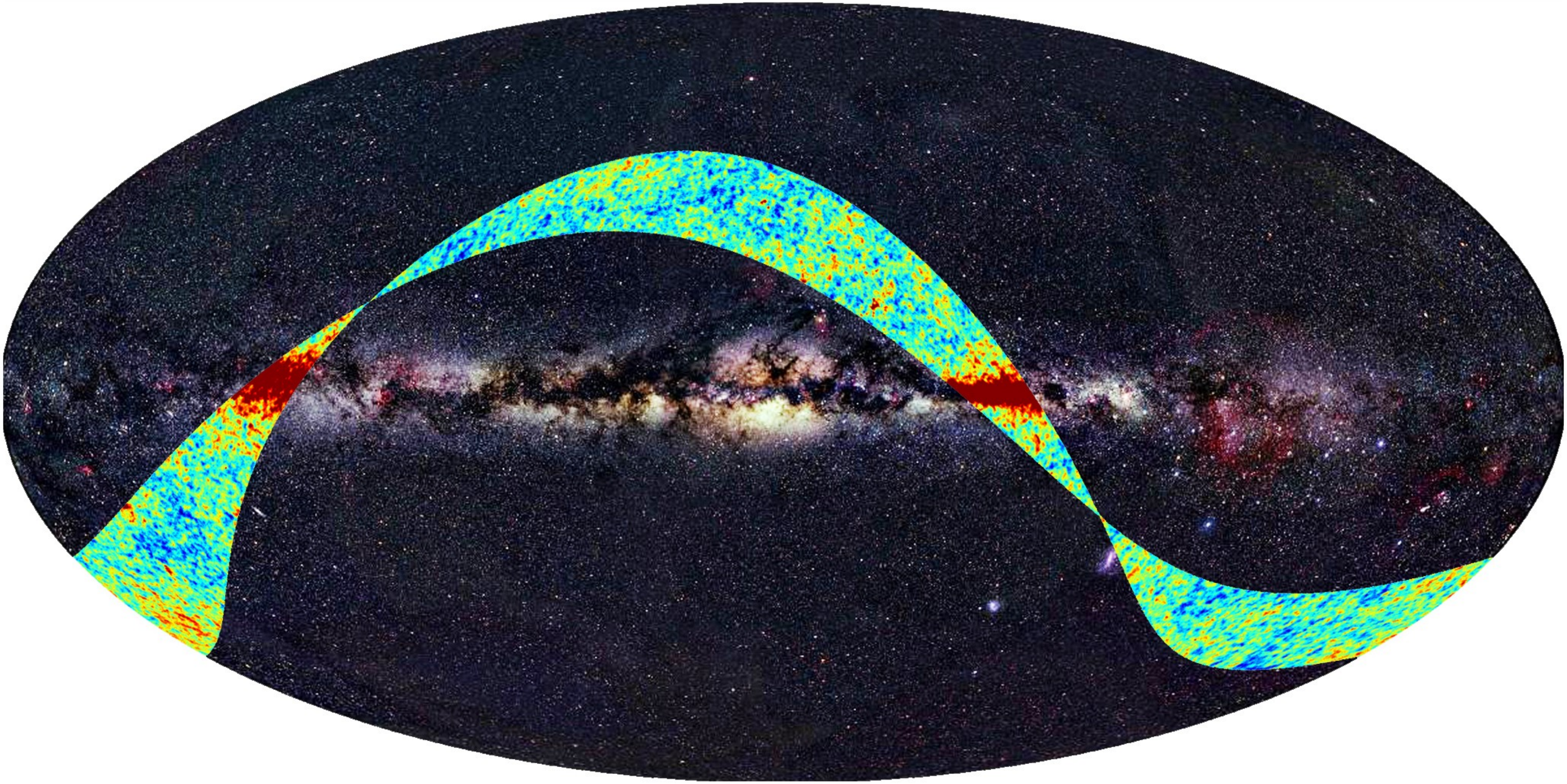
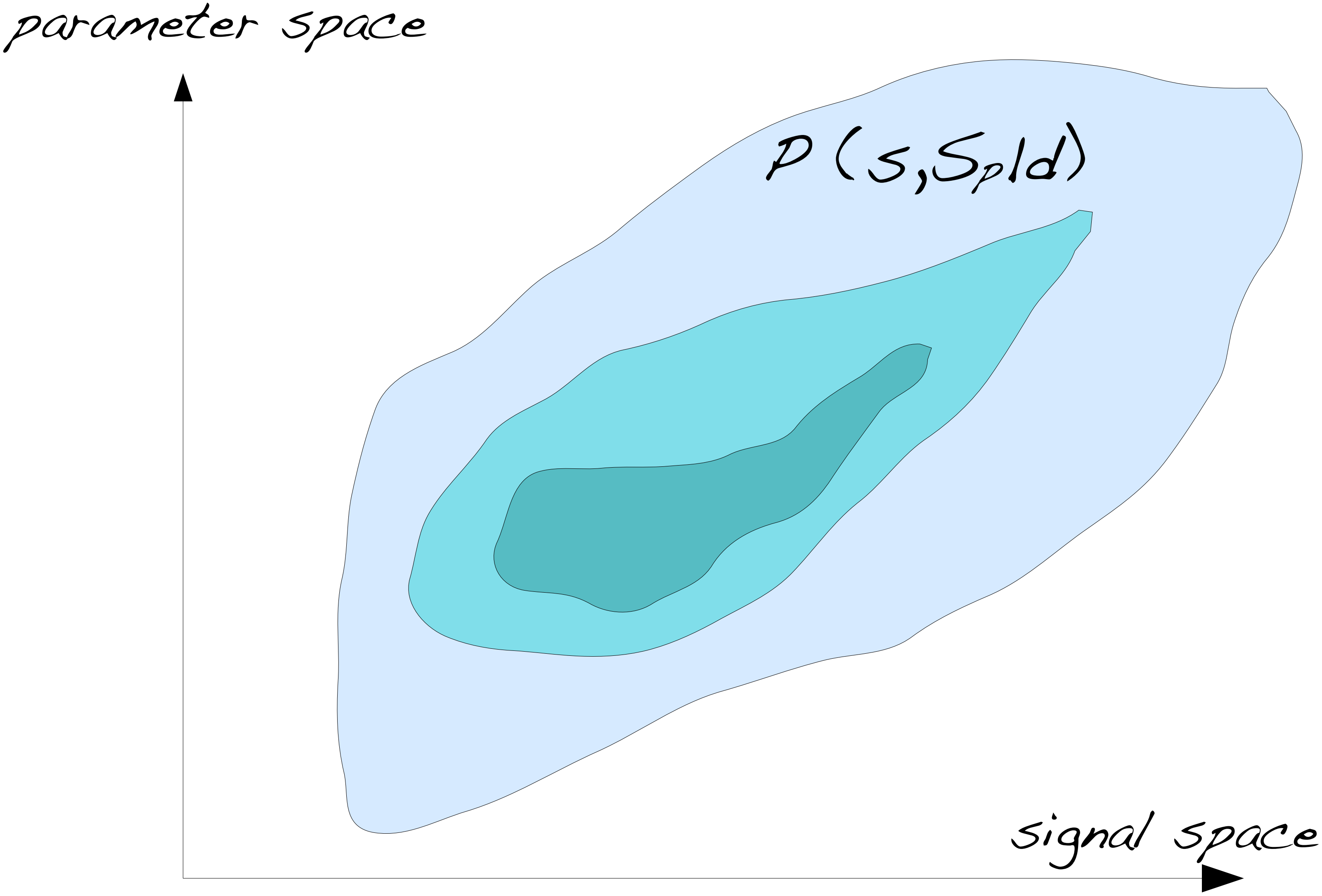


Image credits: Planck team

Joint posterior



Gaussian data model

$$P(s|p) = \mathcal{G}(s, S_p) \equiv \frac{1}{|2\pi\, S_p|^{\frac{1}{2}}} \,\exp\left(-\frac{1}{2} s^\dagger S_p^{-1} s\right)$$

$S_p = \langle s\, s^\dagger \rangle_{(s|p)}$ is the signal covariance

$$j^\dagger s = \int dx\, \overline{j(x)}\, s(x)$$

$$d = R\, s + n \qquad\qquad P(n|s,p) = \mathcal{G}(n, N)$$

$N = \langle n\, n^\dagger \rangle_{(n)}$ is the noise covariance

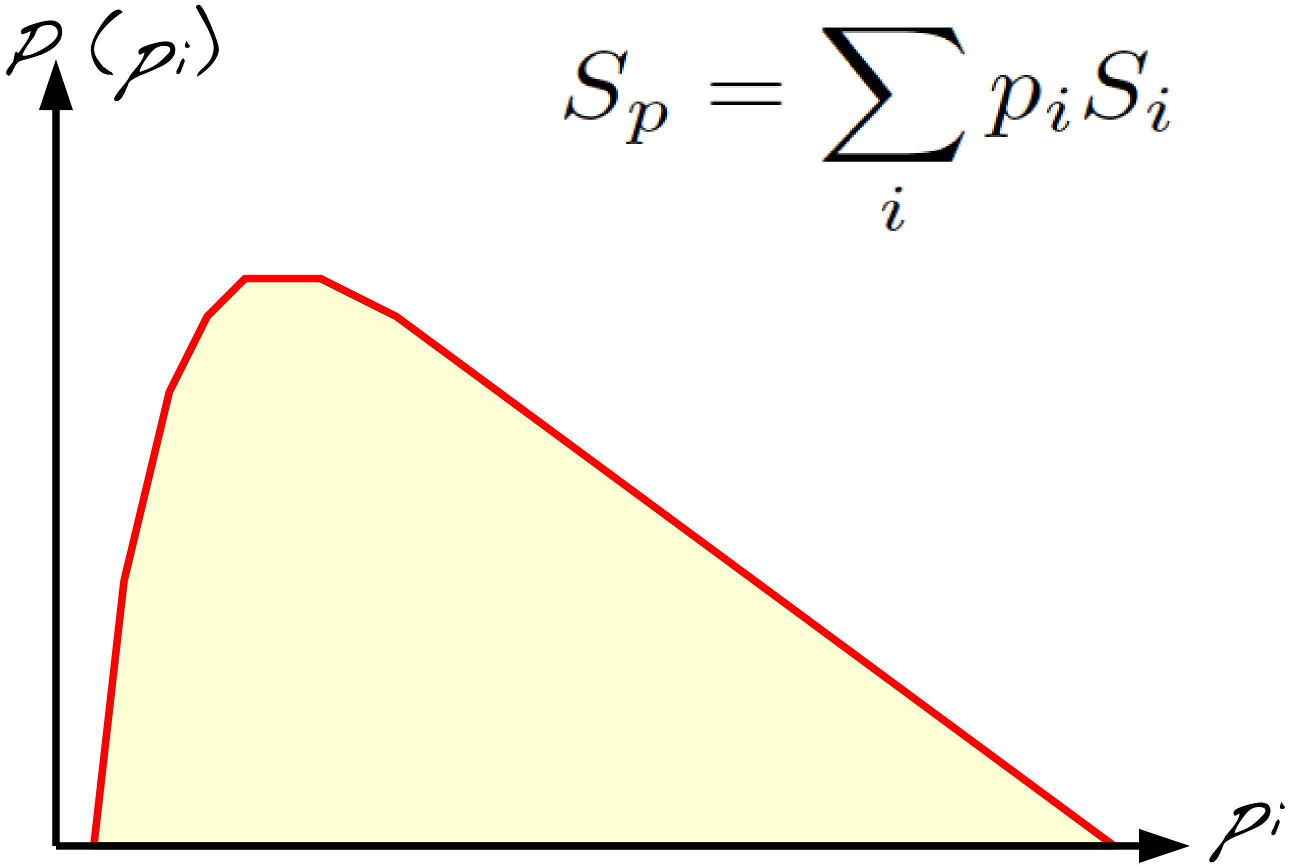
Wiener filter

$$m_p = \langle s \rangle_{(s|d,p)} = D_p \, j$$

$$D_p = \left[S_p^{-1} + M \right]^{-1} M = R^\dagger N^{-1} R$$

$$j = R^\dagger N^{-1} d$$

Spectral prior



$$S_p = \sum_i p_i S_i$$

$$P(p_i) = \frac{1}{q_i\,\Gamma(\alpha_i - 1)} \left(\frac{p_i}{q_i}\right)^{-\alpha_i} \exp\left(-\frac{q_i}{p_i}\right)$$

Generic filter formula

$$m_{p^*} = D_{p^*}\, j$$

$$p_i^* = \frac{1}{\gamma_i + \varepsilon_i}\left(q_i + \frac{1}{2}\mathrm{Tr}[(m_{p^*}\, m_{p^*}^\dagger + \delta_i\, D_{p^*})\, S_i^{-1}]\right)$$

$$D_p = \left[S_p^{-1} + M\right]^{-1}$$

$$\gamma_i = \alpha_i - 1 + \varrho_i/2$$

$$\varrho_i = \mathrm{Tr}[S_i^{-1} S_i]$$

Jeffreys prior

$$
\begin{aligned}
m_{p^*} &= D_{p^*}\, j \\
p_i^* &= \frac{1}{\varrho_i + 2\,\varepsilon_i} \left(\mathrm{Tr}[(m_{p^*} m_{p^*}^\dagger + \delta_i\, D_{p^*})\, S_i^{-1}] \right)
\end{aligned}
$$

$$D_p = \left[S_p^{-1} + M\right]^{-1}$$

$$\gamma_i = \varrho_i / 2 \qquad \alpha_i = 1$$

$$\varrho_i = \mathrm{Tr}[S_i^{-1} S_i]$$

Classical map

$$m_{p^*} = D_{p^*}\, j$$

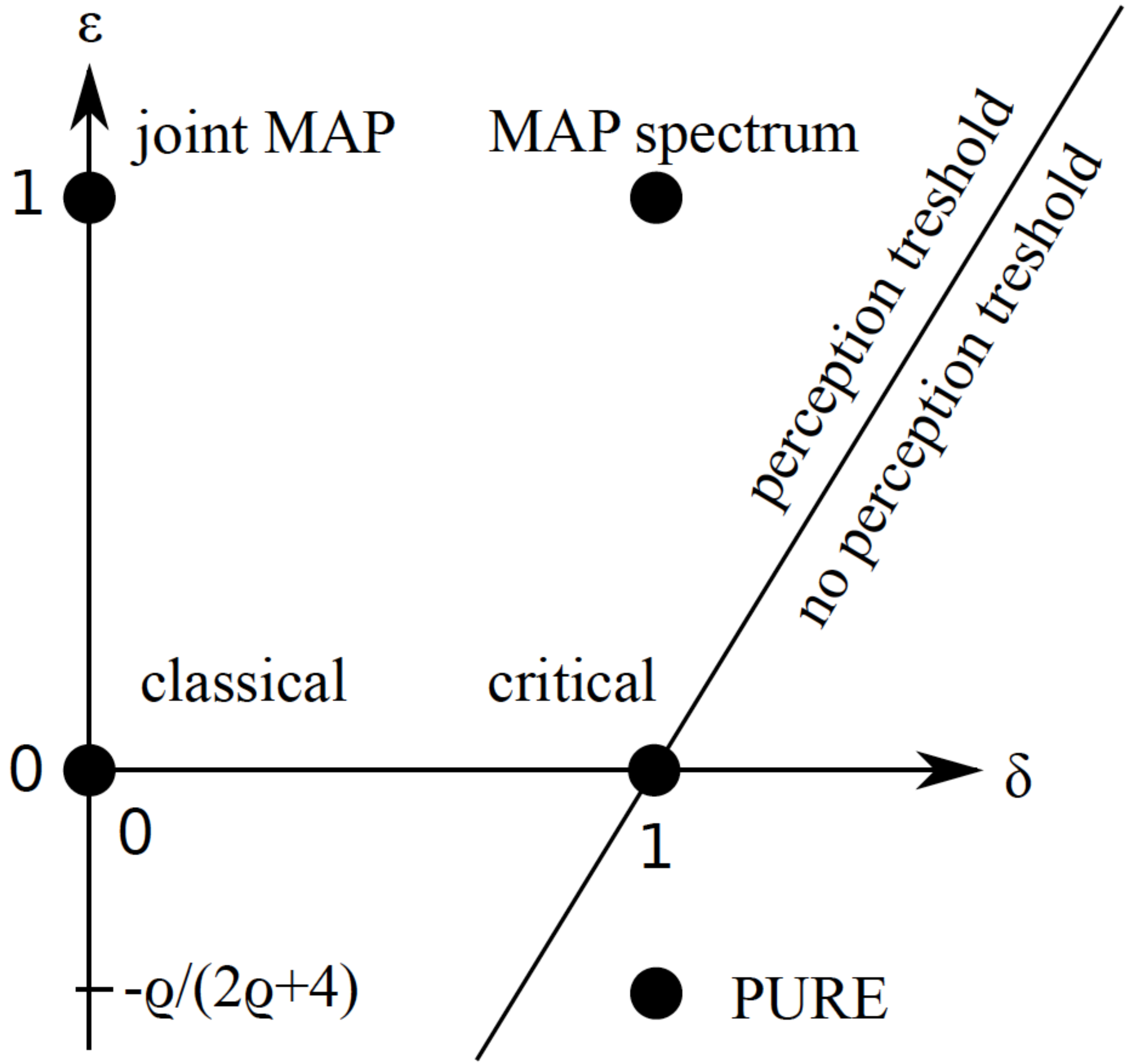
$$p_i^* = \frac{1}{\varrho_i} m_{p^*}^\dagger S_i^{-1} m_{p^*}$$

$$D_p = \left[S_p^{-1} + M\right]^{-1}$$

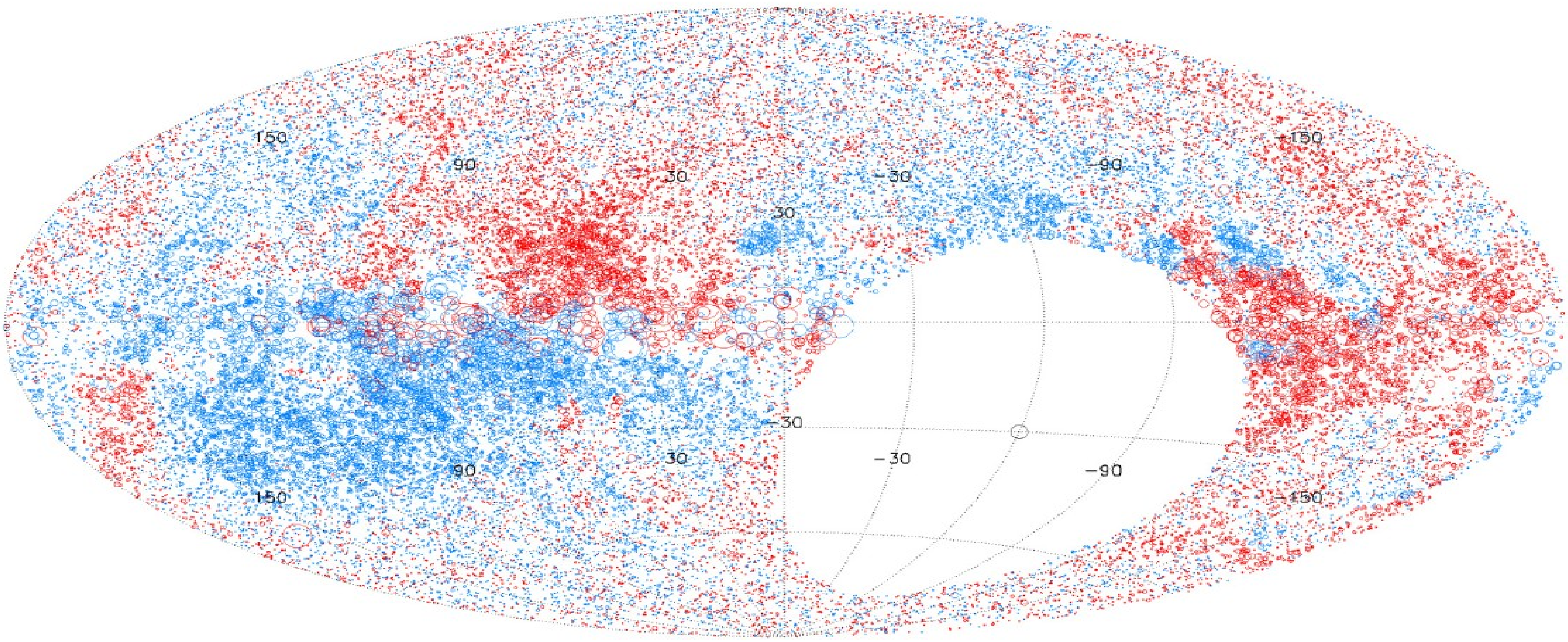
$$\gamma_i = \varrho_i / 2 \qquad \alpha_i = 1$$

$$\varrho_i = \mathrm{Tr}[S_i^{-1} S_i]$$

Filter parameters

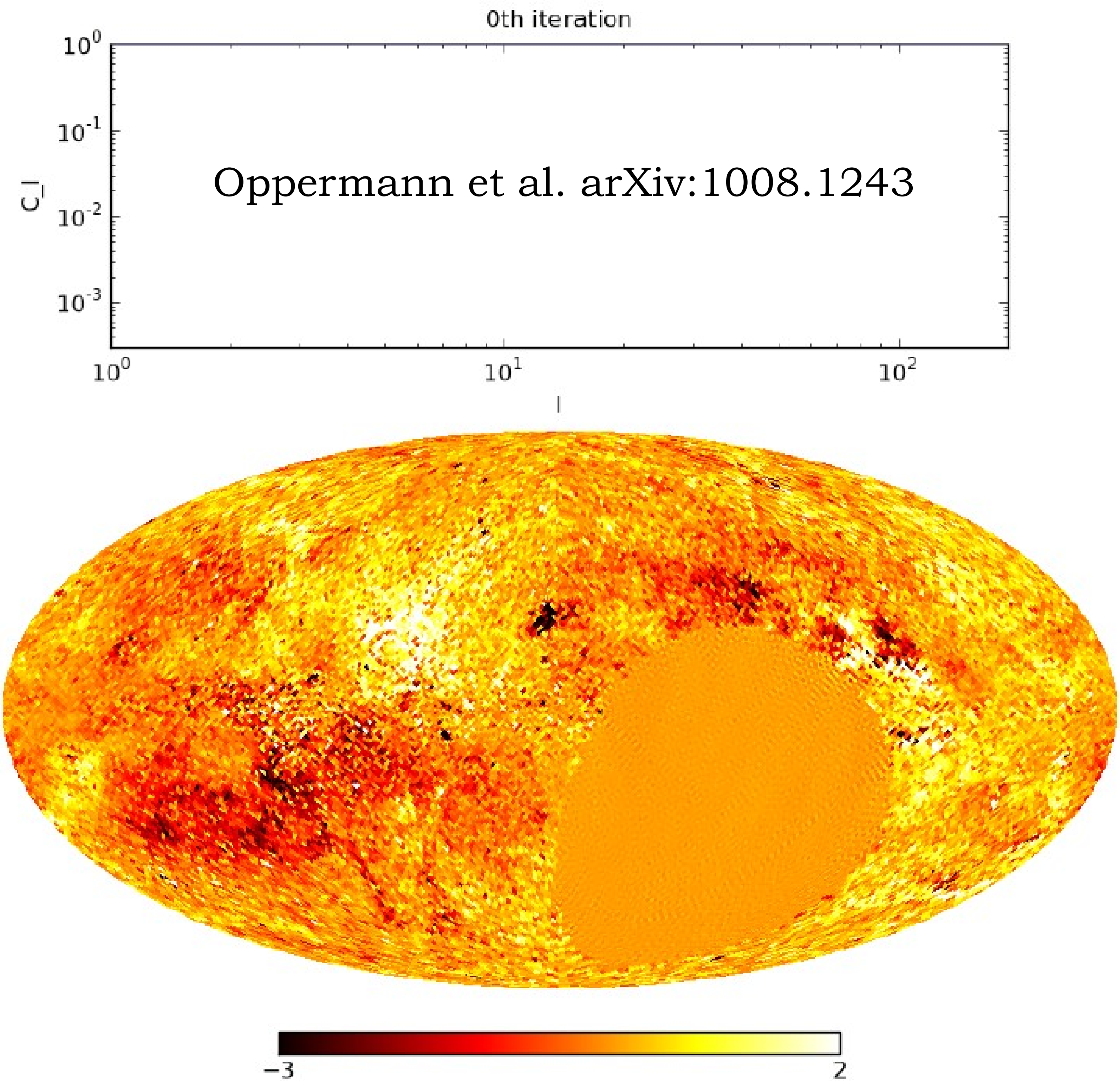


The Faraday rotation sky

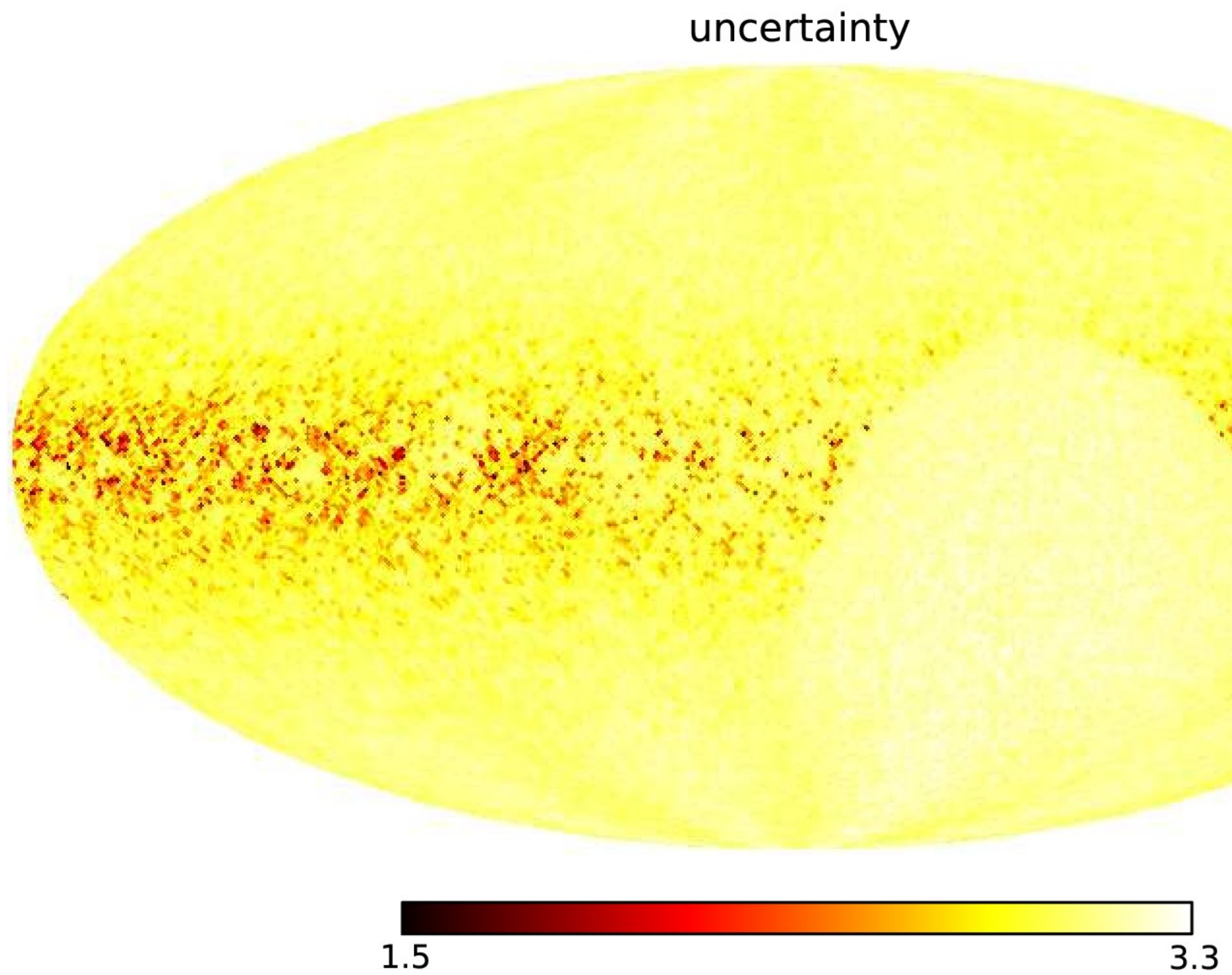


Taylor,Stil,Sunstrum 2009

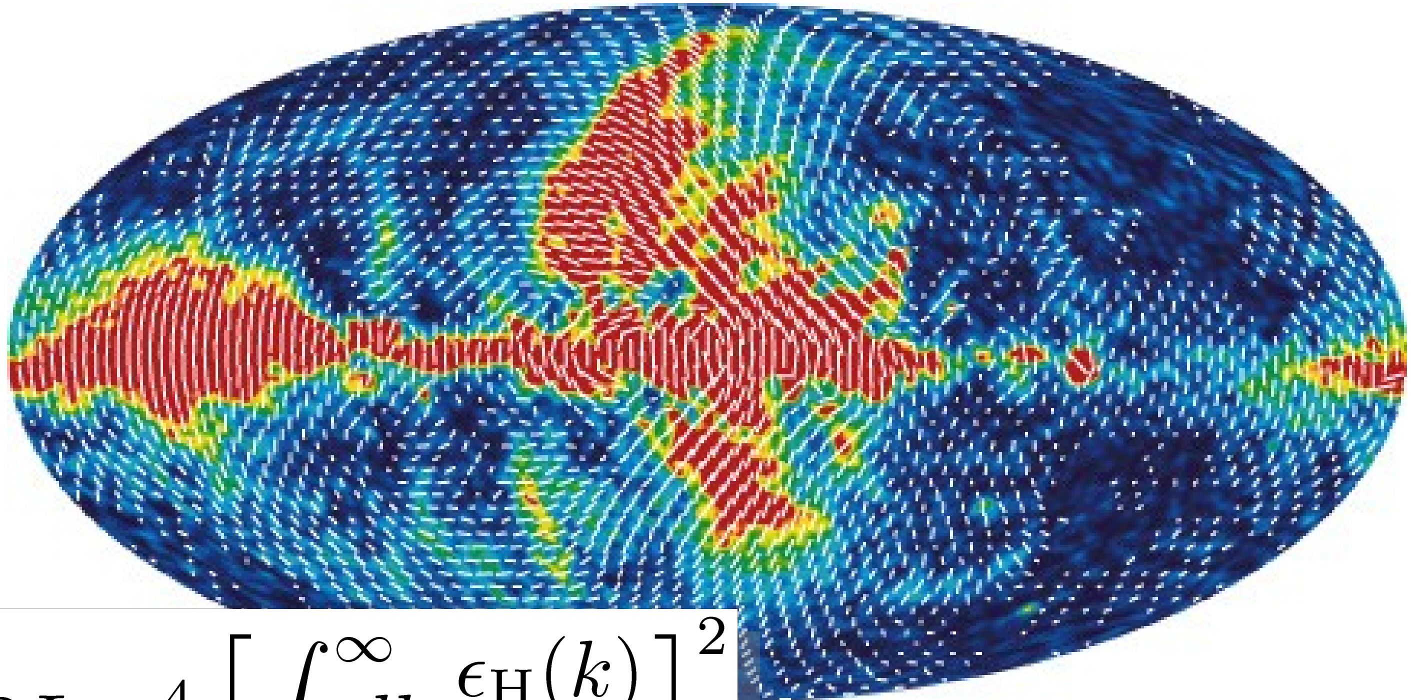
37,534 RM extragalactic RM-sources in the northern sky


0th iteration
Oppermann et al. arXiv:1008.1243
C_l
l
10^0
10^{-1}
10^{-2}
10^{-3}
10^1
10^2
−3
2

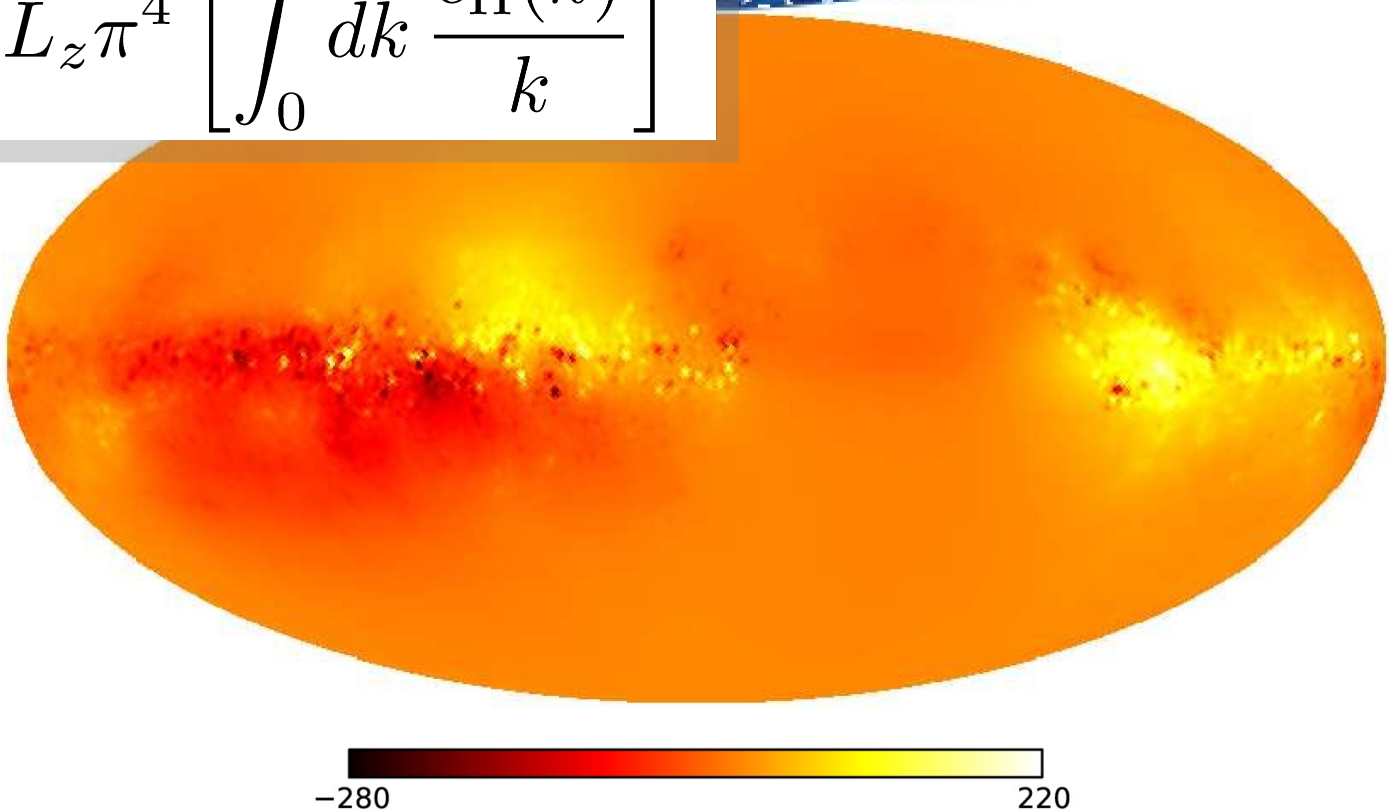
The Faraday rotation sky



Information Field Theory



$$\langle G\, P^* \rangle = 2 L_z \pi^4 \left[\int_0^\infty dk \, \frac{\epsilon_{\rm H}(k)}{k} \right]^2$$



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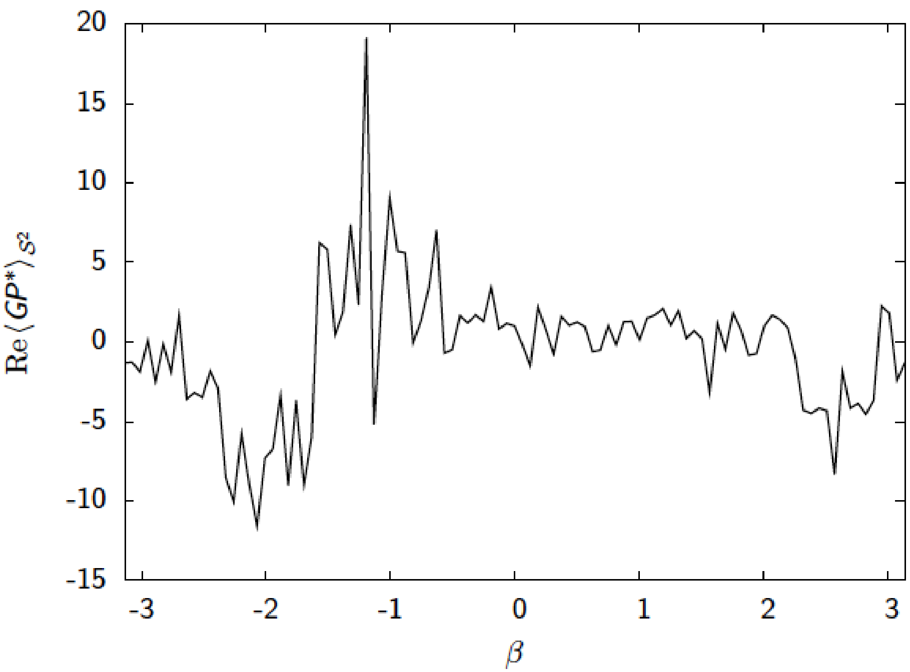
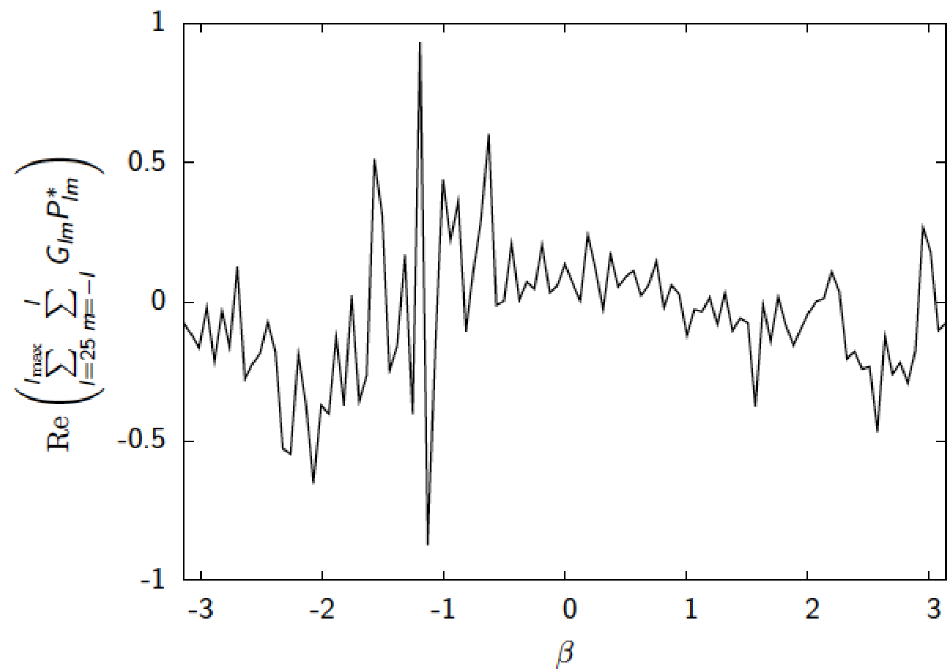
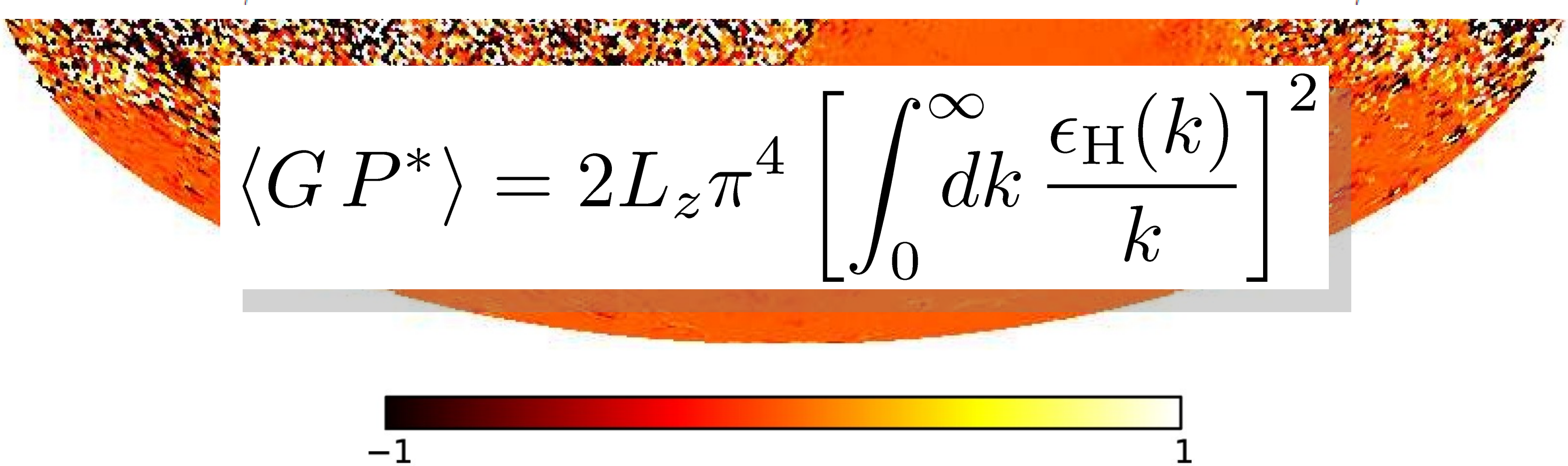
Local Inference Test for Magnetic fields Uncovering HeliceS

contributions of all scales



small-scale contributions





$$\langle G\,P^*\rangle = 2L_z\pi^4\left[\int_0^\infty dk\,\frac{\epsilon_{\mathrm{H}}(k)}{k}\right]^2$$

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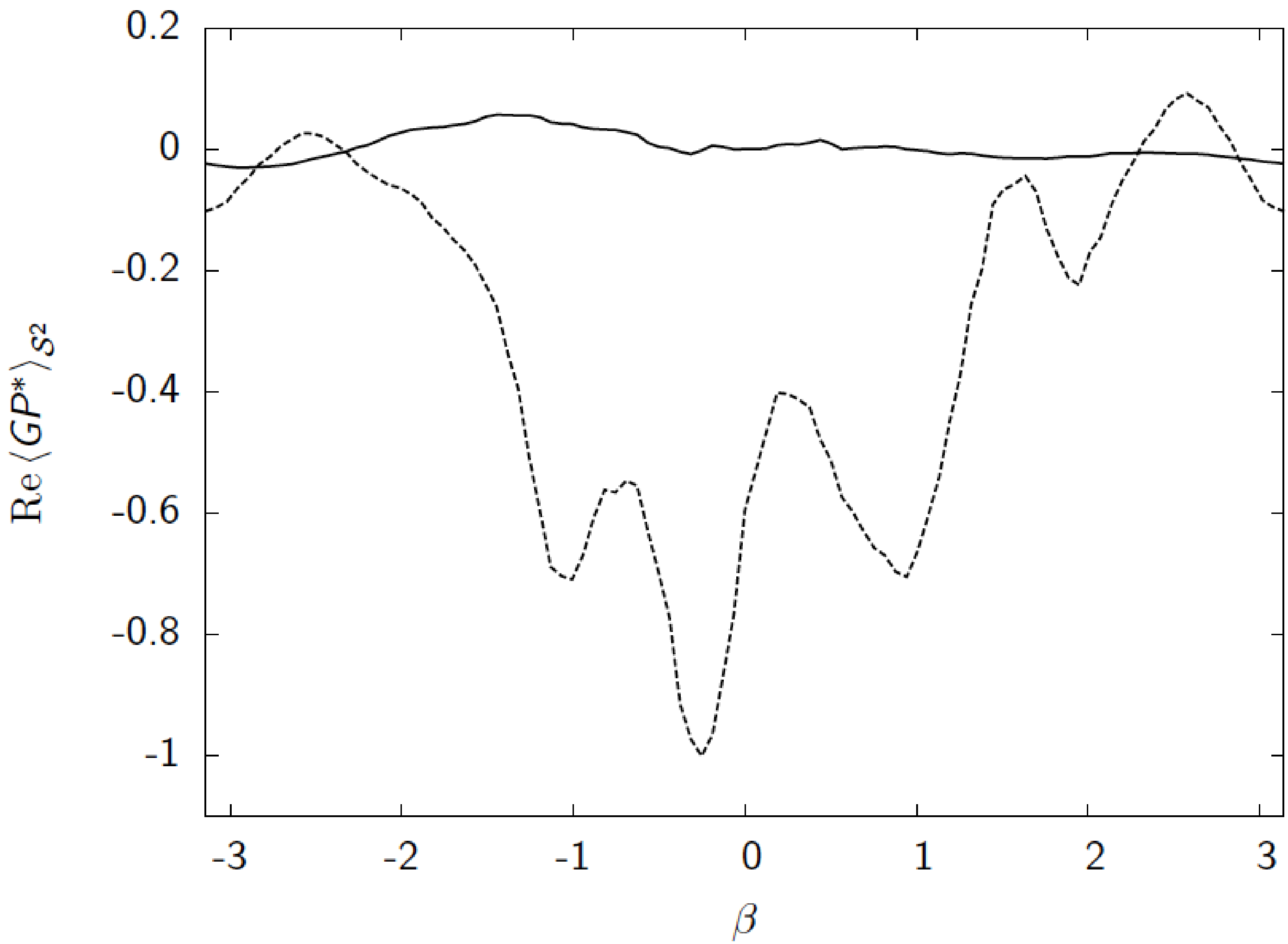
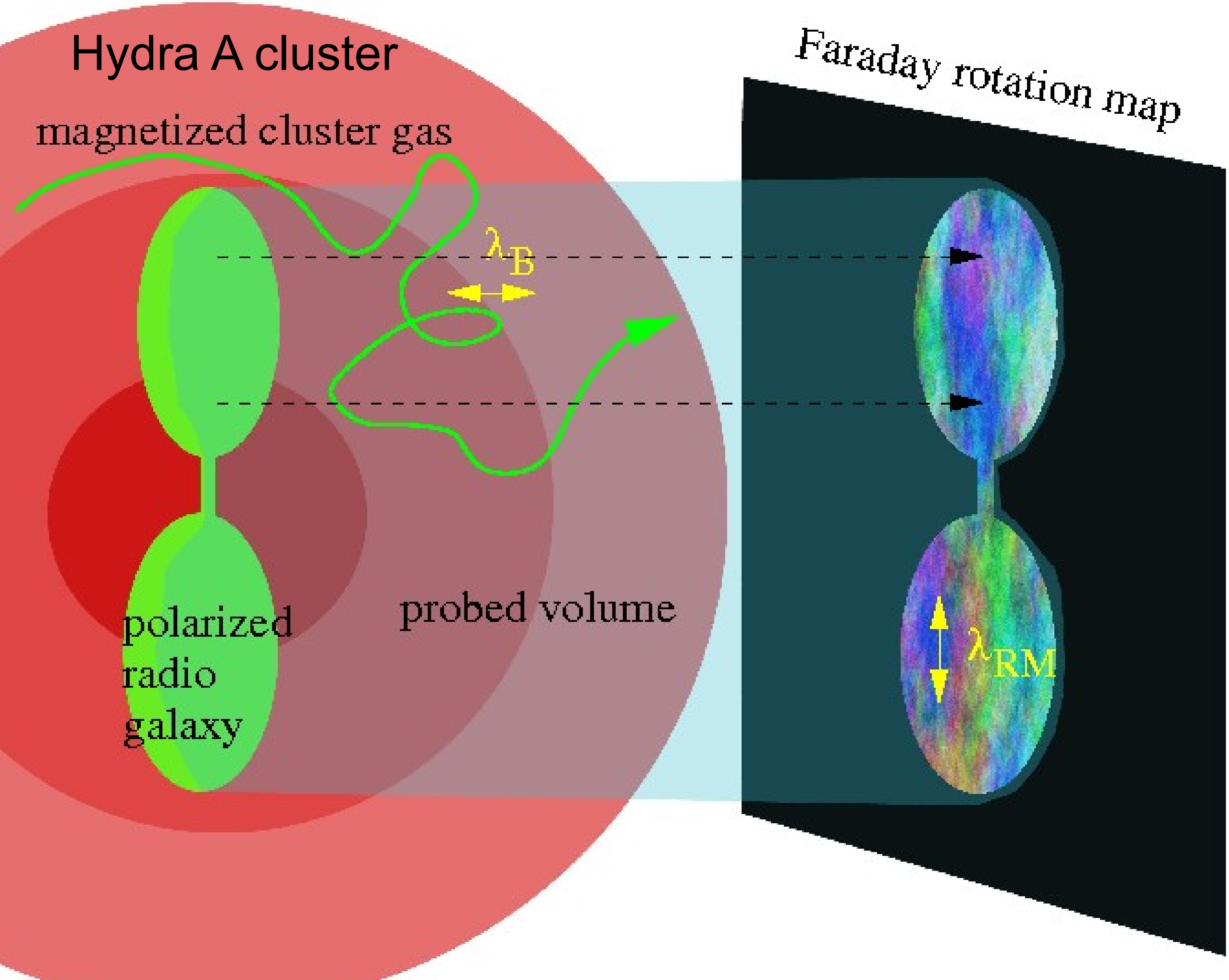
Local Inference Test for Magnetic fields Uncovering HeliceS

Test case with non-trivial electron densities:

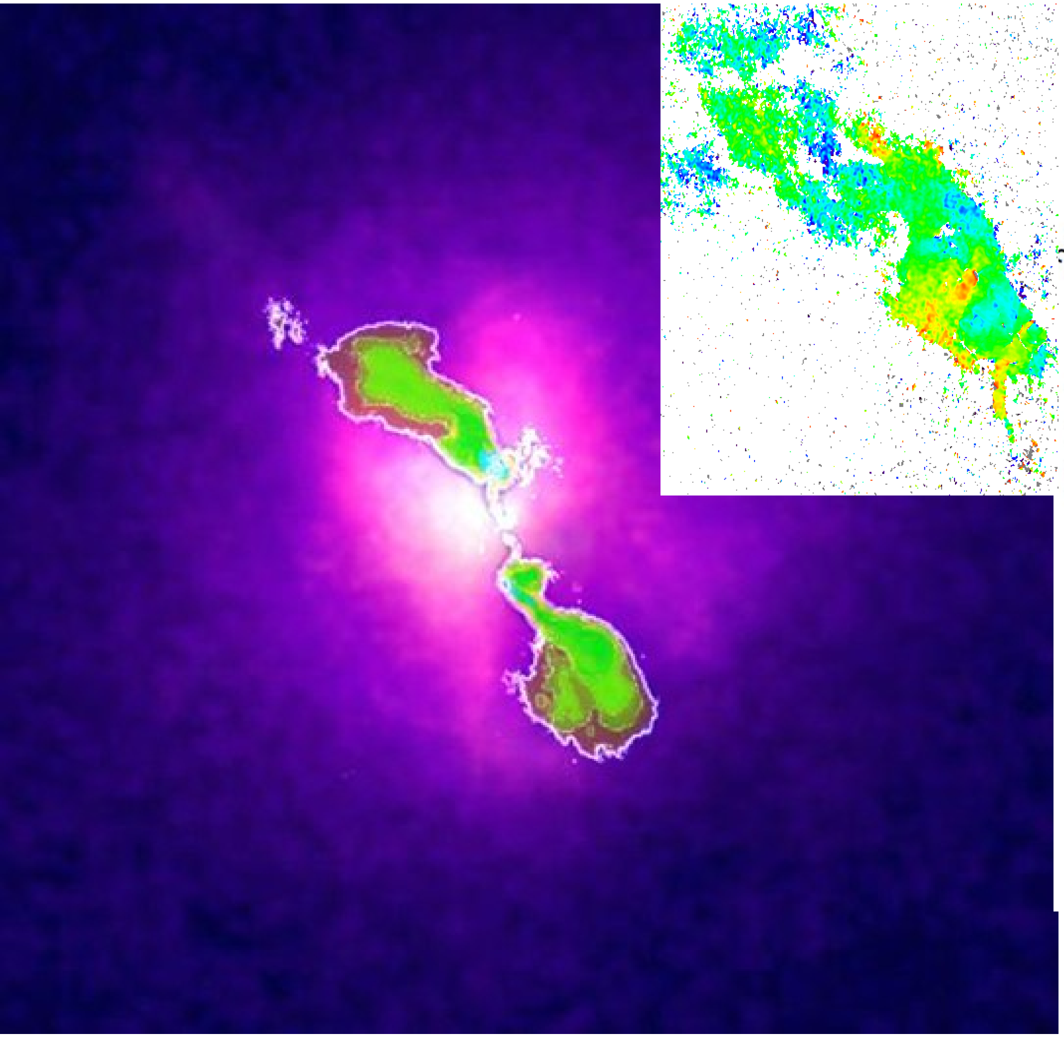


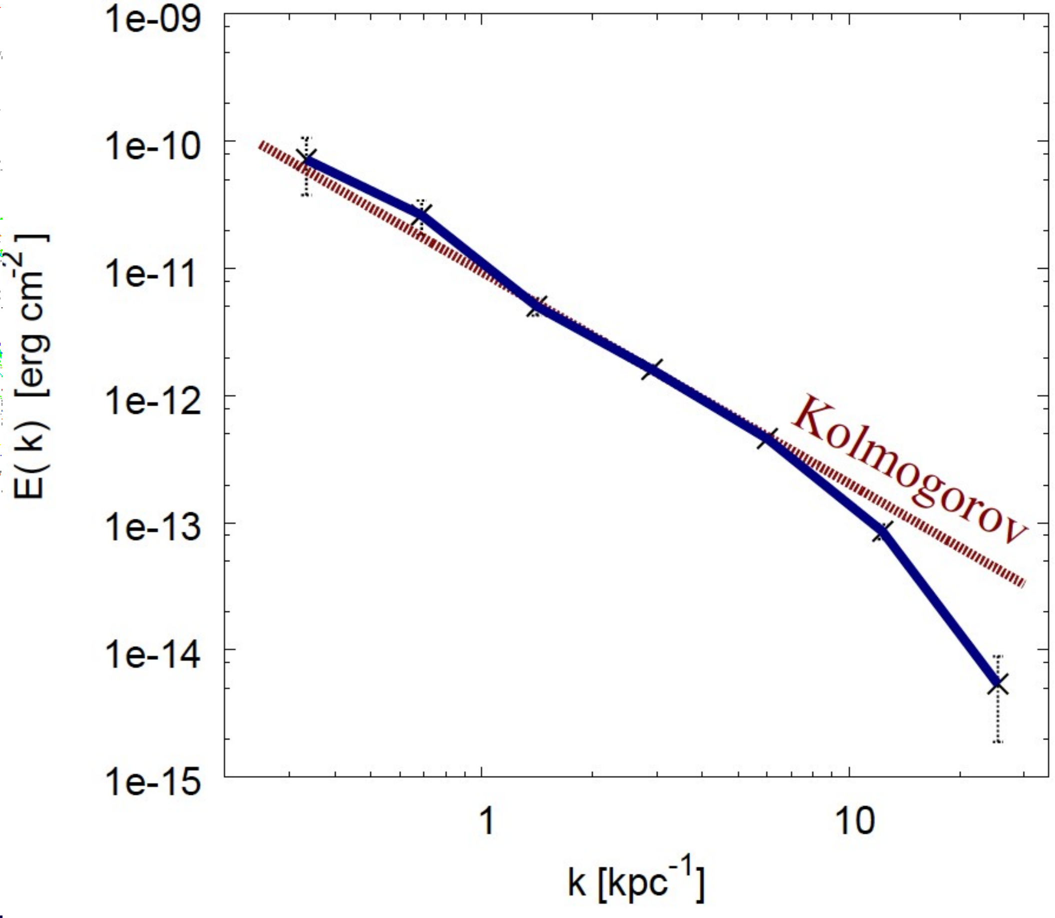
Observational Setup



Magnetic power spectra in the cool core of the Hydra A cluster

Kuchar & Enßlin (2009)

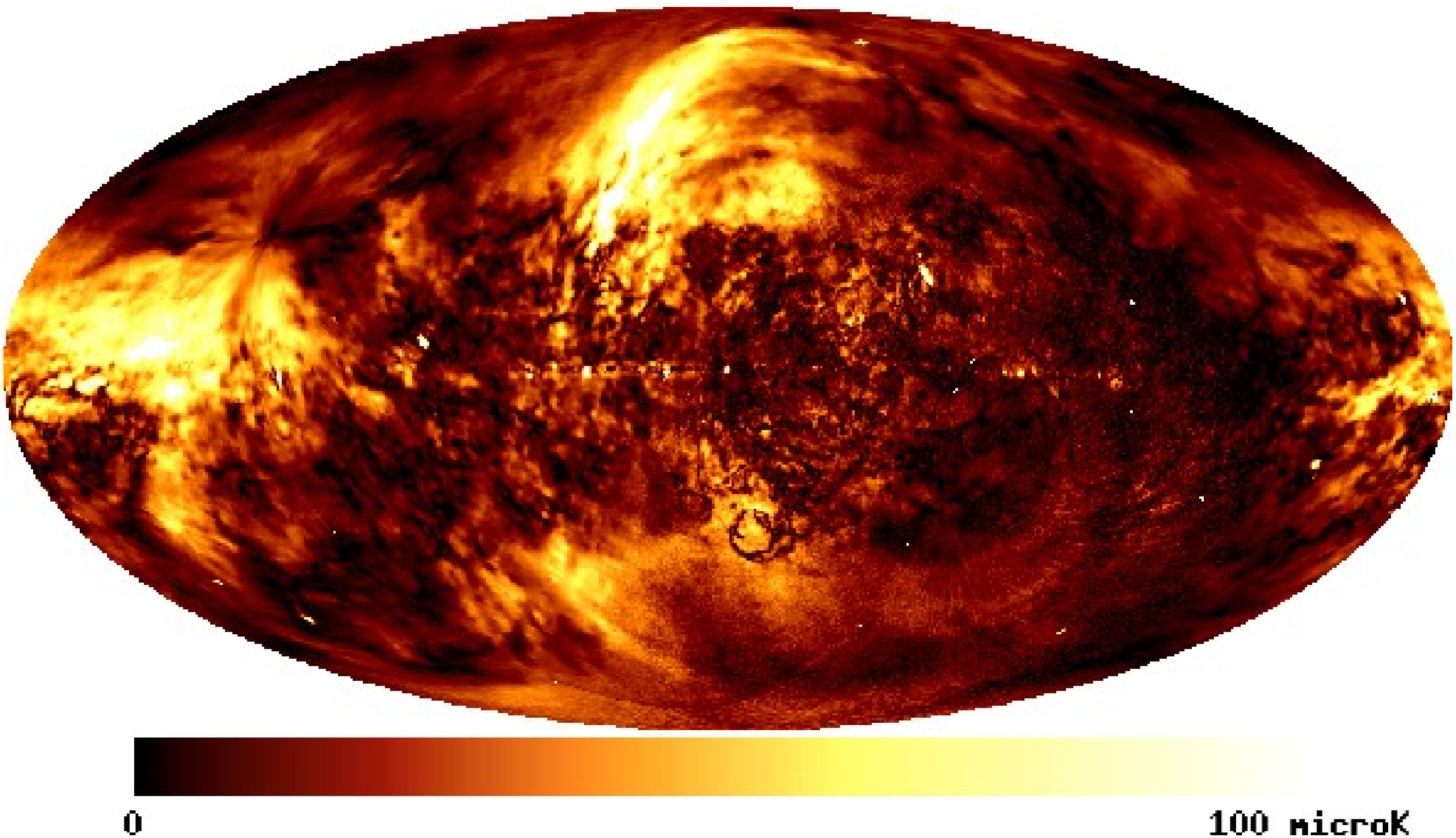




Conclusions

- magnetic field observables allow deduction of helicity and energy spectrum
- inference problems require information theory
- information field theory is a statistical field theory
- information Hamiltonian $= - \mathit{log}\, P(\mathrm{data, signal})$
- information propagator = Wiener variance, ...
- perturbative, renormalization, thermodynamical, & sampling methods can be used
- providing us with the mean map, signal uncertainty, ...

Outlook



1.4 GHz: Reich & Wolleben

22 GHz: WMAP team


Thank you !