Cosmological models of seed fields

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Outline

- short review of primordial generation mechanisms
- model and evolution of the field
- model independent constraints from Nucleosynthesis and gravitational waves

PRIMORDIAL MAGNETIC SEEDS
amplified by structure formation

$B \sim 10^{-9}$ Gauss or 10^{-21} Gauss

(collapse) (galactic dynamo)

at about 100 kpc

Primordial generation mechanisms

CAUSAL : in the radiation or matter dominated universe

phase transitions, charge separation...

- standard physics, quite easy to get
- correlation scale too small : maximum the causal horizon

NON CAUSAL : inflation, pre big bang...

- generated at every scale
- not very predictive

amplitude depends on average method, evolution, parameters of the model...

more than 100 primordial mechanisms, but are some of these preferred?

PRIMORDIAL PHASE TRANSITIONS

- **first order :** charge separation at bubble walls + amplification by MHD turbulence (both EW and QCD)

  Hogan 1983, Quashnock et al 1989, Cheng and Olinto 1994, Baym et al 1996, Sigl et al 1996, Ahonen and Enqvist 1997, Stevens and Johnson 2010...

- **second order EW :** generated by the gradients in the Higgs field

  Vachaspati 1991, Davidson 1996, Grasso and Riotto 1997, Hindmarsh and Everett 1997, Tornkvist 1998, Diaz-Gil et al 2008...

- **helicity generation :** $$H = \frac{1}{V}\int_V d^3x\, \mathbf{A}\cdot\mathbf{B}$$

  EW baryogenesis: decay of EW strings $h \sim \frac{n_b}{\alpha}$

  Cornwall 1997, Vachaspati 2001, Copi et al 2008...

  coupling with a pseudoscalar $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$

  Field and Carroll 1998, Campanelli and Giannotti 2005...

GENERATION BY CHARGE SEPARATION AND VORTICITY

$$\partial_t B + \nabla \times E = 0$$
$$\nabla \times B - \partial_t E = 4\pi J$$

$$J = en(v_p - v_e) \qquad \Box B = 4\pi en(\Omega_p - \Omega_e)$$

electrons do Thomson scattering (Harrison 1973)

- vorticity by wiggly strings, superconducting strings or string loops

  Vachaspati and Vilenkin 1991, Davis and Dimopoulos 2005, Battefeld et al 2007...

- vorticity by second order perturbations

  Berezhiani and Dolgov 2003, Gopal and Sethi 2004, Matarrese et al 2004, Takahashi et al 2005...

INFLATION - breaking conformal invariance of electromagnetism

- coupling of em field with the metric $-\frac{1}{4}\left(\frac{R}{m^2}\right)^n F_{\mu\nu}F^{\mu\nu}$

  Turner and Widrow 1988, Bamba and Sasaki 2007, Campanelli et al 2008...

  $$-\frac{1}{4m^2}(RF_{\mu\nu}F^{\mu\nu} + R_{\mu\nu}F^{\mu\alpha}F^{\nu}_{\alpha} + R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta})$$

- coupling of gauge field with the metric $R\,A_\mu A^\mu$

  Turner and Widrow 1988, Demozzi et al 2009...

- time dependent coupling or coupling directly to the inflaton

  Ratra 1992, Bamba and Yokoyama 2004, Campanelli et al 2008, Demozzi et al 2009...

  $e^{\alpha\phi}F_{\mu\nu}F^{\mu\nu}$ $\quad -\frac{1}{4}I^2(t)F_{\mu\nu}F^{\mu\nu}$

- introducing a charged scalar field $(D_\mu\phi)^* D^\mu\phi - m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

  Turner and Widrow 1988, Calzetta et al 1998, Giovannini and Shaposhnikov 2000, Finelli and Gruppuso 2001, Dimopoulos et al 2002, Prokopec et al 2004...

- inflation in string theory context Martin and Yokoyama 2007 $f(\phi)F_{\mu\nu}F^{\mu\nu}$

- helicity generation : axial coupling to the inflaton $\frac{1}{4}f(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}$

  Anber and Sorbo 2006, Durrer et al 2010

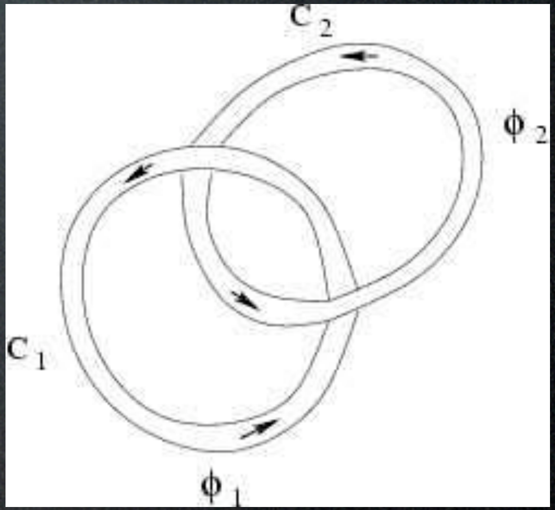
Primordial magnetic fields: model

Stochastic field, statistically homogeneous, isotropic and gaussian

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle = \delta(\mathbf{k} - \mathbf{q}) [(\delta_{ij} - \hat{k}_i \hat{k}_j) S(k) + \mathrm{i} \epsilon_{ijm} \hat{k}^m A(k)]$$

energy density $\rho_B = \int_0^\infty dk\, k^2 S(k)$

helicity density $H = \int_0^\infty dk\, k A(k)$



$$H = \frac{1}{V} \int_V d^3x\, \mathbf{A} \cdot \mathbf{B}$$

Primordial magnetic fields: model

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divergence free

at large scales $k \longrightarrow 0$:

$$S(k) \propto k^n \qquad A(k) \propto k^m$$

$$S(k) \geq |A(k)| \longrightarrow m \geq n$$

finite energy density :

$$n\,,\ m > -3$$

upper cutoff : scale of dissipation
determined by kinetic viscosity $\quad k_D(\eta)$

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Variance of the MF amplitude on a given scale λ

$$B_\lambda^2 = \int dk\, k^2\, S(k)\, e^{-k^2\lambda^2}$$

volume average

$$B_\lambda = B_L \left(\frac{L}{\lambda}\right)^{\frac{n+3}{2}}$$

$$S(k) = \frac{\lambda^{n+3}\, B_\lambda^2}{\Gamma(\frac{n+3}{2})}\, k^n$$

Primordial magnetic fields: model

Stochastic field, statistically homogeneous, isotropic and gaussian

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle = \delta(\mathbf{k}-\mathbf{q}) [\, (\delta_{ij} - \hat{k}_i \hat{k}_j) S(k) + \mathrm{i}\epsilon_{ijm} \hat{k}^m A(k)]$$

Variance of the MF amplitude on a given scale λ

$$B_\lambda^2 = \int dk\, k^2\, S(k)\, e^{-k^2 \lambda^2}$$

or total energy density

$$\rho_B = \langle B^2 \rangle = \int_0^{k_D} dk\, k^2 S(k) = \frac{\lambda^{n+3}\, B_\lambda^2}{\Gamma(\frac{n+3}{2})} \frac{k_D^{n+3}}{n+3}$$

Primordial magnetic fields: model

if the field is generated by a CAUSAL process (EWPT, QCDPT...)

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle = \delta(\mathbf{k}-\mathbf{q})[\,(\delta_{ij} - \hat{k}_i \hat{k}_j) S(k) + \mathrm{i}\epsilon_{ijm} \hat{k}^m A(k)]$$

$$\langle B_i(\mathbf{x}) B_j(\mathbf{x}+\mathbf{r}) \rangle = 0 \quad \text{for} \quad r > L \qquad\qquad L \leq \text{horizon}$$

correlation function compact support $\longrightarrow$ power spectrum analytic

Durrer et al 2000, CC and Durrer 2001...

Primordial magnetic fields: model

if the field is generated by a CAUSAL process (EWPT, QCDPT...)

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle = \delta(\mathbf{k}-\mathbf{q})[\,(\delta_{ij} - \hat{k}_i \hat{k}_j) S(k) + \mathrm{i}\epsilon_{ijm} \hat{k}^m A(k)]$$

$$k \to 0 \;:\; S(k) \propto k^2 \,, k^4 \ldots \qquad A(k) \propto k \,, k^3 \ldots$$

$n \geq 2$ even integer $\qquad$ $m \geq 3$ odd integer

$$(m \geq n)$$

Primordial magnetic fields: model

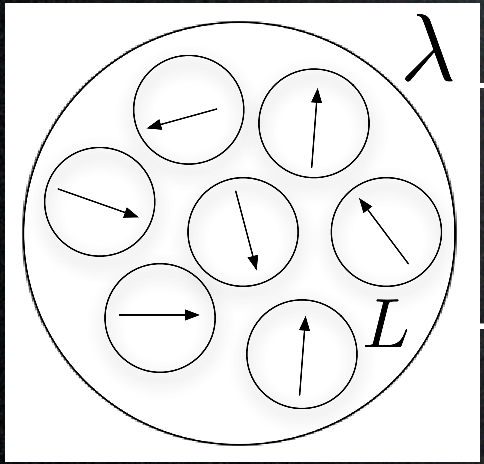
if the field is generated by a CAUSAL process (EWPT, QCDPT...)

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle = \delta(\mathbf{k}-\mathbf{q}) \left[(\delta_{ij} - \hat{k}_i \hat{k}_j) S(k) + \mathrm{i}\epsilon_{ijm} \hat{k}^m A(k) \right]$$

$$k \to 0 \;:\quad S(k) \propto k^2 \,, k^4 \ldots \qquad\qquad A(k) \propto k \,, k^3 \ldots$$

DIVERGENCE FREE IMPLIES
NO RANDOM WALK: $n \neq 0$

$$B_\lambda = B_L \left(\frac{L}{\lambda} \right)^{\frac{n+3}{2}}$$



→ cluster scale today
$0.1\,\mathrm{Mpc}$

→ horizon scale at generation
$10^{-4}\,\mathrm{pc}$

extra suppression at large scales, disfavour causal generation

Durrer and CC 2003

Power spectrum at all scales

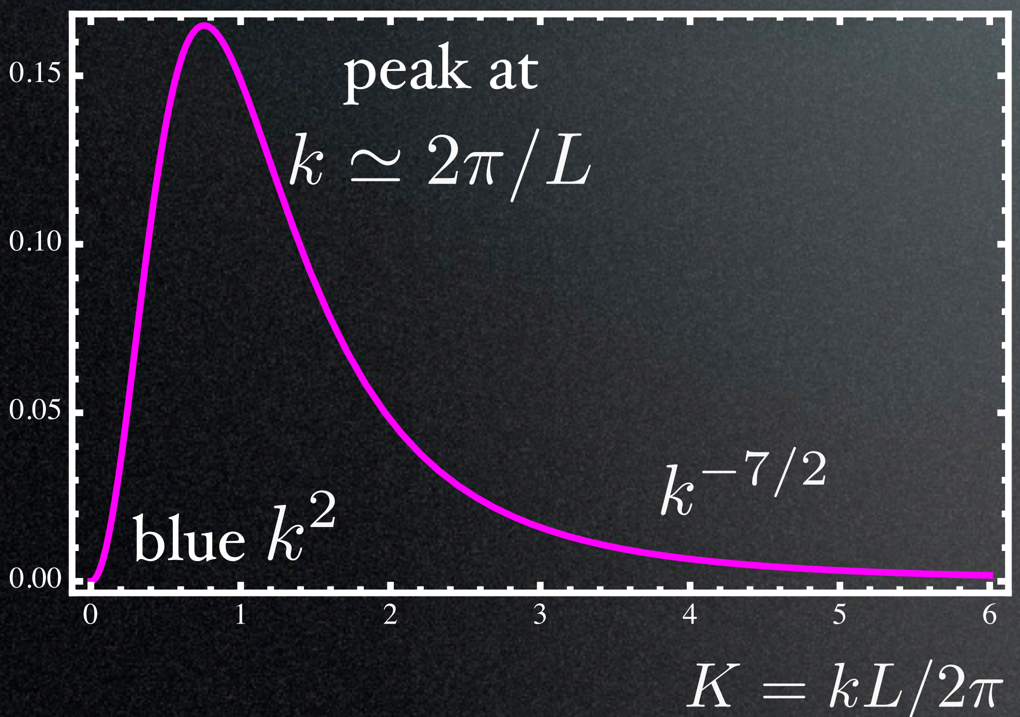
Small scales: MHD spectrum: Kolmogorov, Iroshnikov Kraichnan....

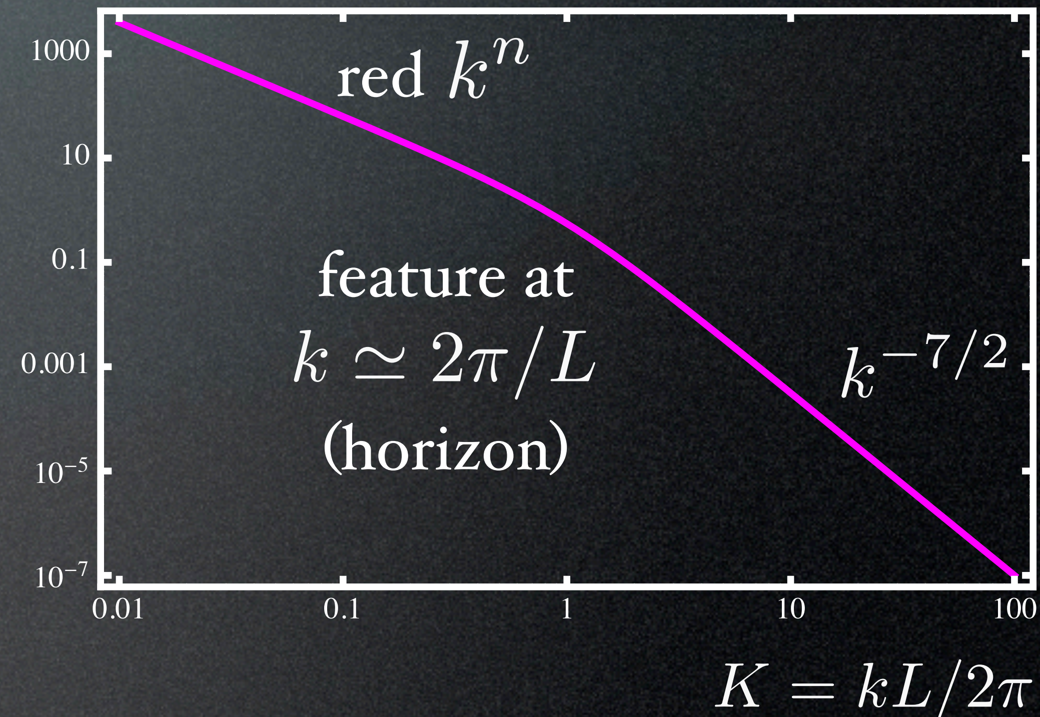
Interpolating formula:
(turbulence, Von Karman 48)

$$S(k) = \rho_B \, L^3 \, \frac{K^n}{(1 + K^2)^{(2n+7)/4}}$$

causal spectrum n = 2

a-causal spectrum n > -3





CC, Durrer and Fenu 2009

Primordial magnetic field : time evolution

- conformal transformation to flat spacetime
- ideal MHD limit $\sigma \rightarrow \infty$: flux and helicity are conserved

$$B \propto a^{-2}(\eta)$$

neutrino decoupling,
electrons non relativistic

100 GeV — 1 MeV — 0.32 eV

TURBULENT PHASE

$\nu \simeq \ell_{\nu e}$ $\quad \mathrm{Re} \gg 1$ $\quad \sigma \propto T$

Turbulent cascade

Jedamzik et al 1996, Ahonen and Enqvist 1996, Banerjee and Jedamzik 2004, Kahniashvili et al 2010...

VISCOUS PHASE

$\nu \simeq \ell_{\gamma e}$ $\quad \mathrm{Re} \simeq 1$ $\quad \sigma \propto T^{3/2}$

MHD waves in viscous fluid:
damping of magnetic energy

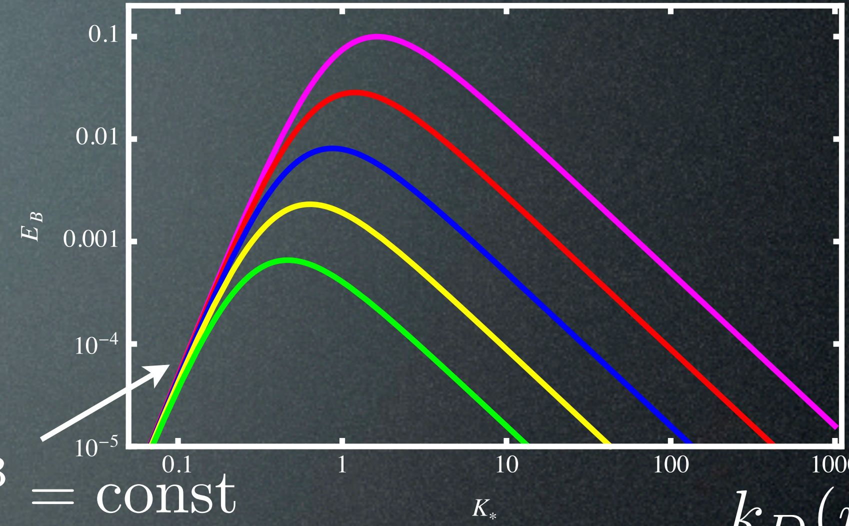
Jedamzik et al 1996, Subramanian and Barrow 1997, Banerjee and Jedamzik 2004...

Turbulent phase :

- non-helical field: DIRECT CASCADE

magnetic energy is dissipated
correlation scale grows

$2\pi/L(\eta)$

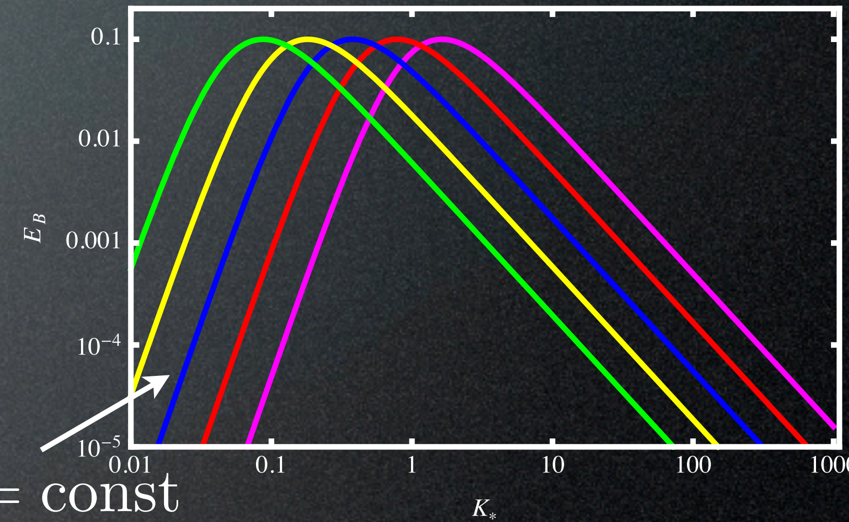


$\rho_B L^{n+3} = \text{const}$

$k_D(\eta)$

- helical field: INVERSE CASCADE

magnetic energy is transferred to larger scales
correlation scale grows



$\rho_B L = \text{const}$

Christensson et al 2002, Banerjee and Jedamzik 2004, Campanelli 2007, CC et al 2009, Kahniashvili et al 2010...

ends when entire turbulent range dissipated

$$L(\eta_{\rm fin}) = 1/k_D(\eta_{\rm fin})$$

EW non-helical: $T_{\rm fin} \simeq 180\,\mathrm{MeV}$

EW helical: $T_{\rm fin} \simeq 22\,\mathrm{MeV}$

Parameters of a primordial magnetic field

- amplitude on a given scale B_λ $\lambda = 0.1 - 1\,\mathrm{Mpc}$
- spectral index $n > -3$ (causal generation $n = 2$)
- generation time $\eta_{\rm in}$: inflation, EWPT, QCDPT, recombination....

  for causal mechanisms is related to the initial correlation length $L \leq \eta_{\rm in}$

- damping scale, upper cutoff of the spectrum due to viscosity $k_D(\eta)$
- presence of an helical component

Constraints from gravitational waves and Nucleosynthesis

$$\Omega_{\rm rel} \leq 0.1\,\Omega_{\rm rad}$$

- magnetic field generates GWs from its anisotropic stresses $$\Omega_{\rm GW} = \mathcal{E}\frac{(\Omega_B)^2}{\Omega_{\rm rad}} \leq 0.1\,\Omega_{\rm rad}$$
- once generated, GWs propagate freely without interaction
- apply Nucleosynthesis bound on GWs and induce bound on B_λ
- GW production takes place before dissipation: magnetic energy "stored" in GWs
- accounting for GWs, the bound is stronger by a factor

$$\left(\frac{k_D(\eta_{\rm nuc})}{k_D(\eta_{\rm in})}\right)^{\frac{n+3}{2}}$$

depending of MF generation time and spectral index

CC and Durrer 2001, CC et al 2009

Constraints from gravitational waves and Nucleosynthesis

$$\delta G_{\mu\nu} = 8\pi G T^B_{\mu\nu} \longrightarrow \mathcal{H}^2 h \sim G T^B \longrightarrow \dot{h} \sim G T^B / \mathcal{H}$$

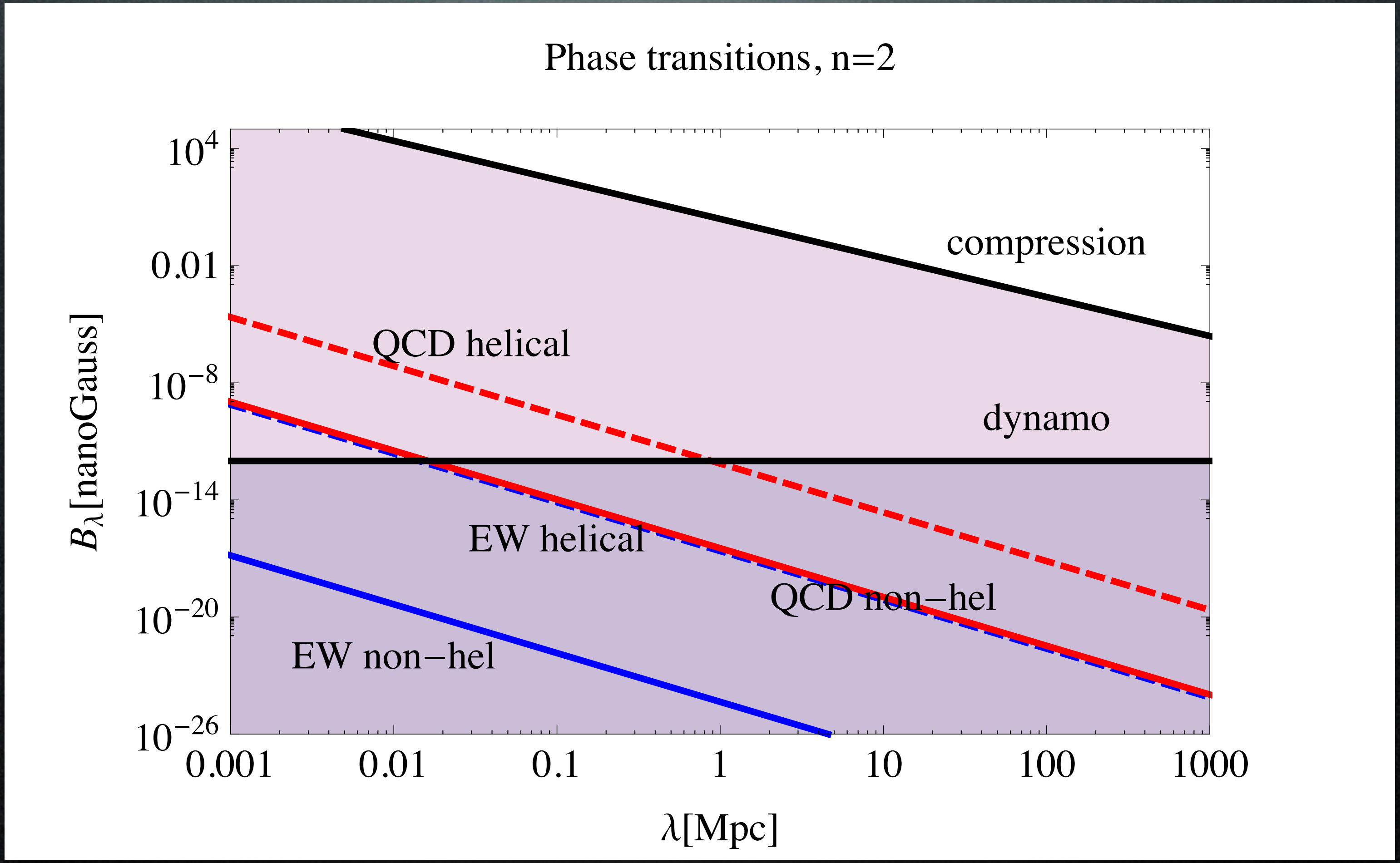
$$\rho_{GW} \sim \frac{\dot{h}^2}{8\pi G} \longrightarrow \Omega_{\rm GW} = \mathcal{E}\,\frac{(\Omega_B)^2}{\Omega_{\rm rad}} \leq 0.1\,\Omega_{\rm rad}$$

$$\Omega_B(\eta) \propto \frac{B^2_\lambda(\eta)}{\rho_c}[\lambda k_D(\eta)]^{n+3}$$

$B^2_\lambda(\eta)\,,\;\; k_D(\eta)$ depend on time due to the MHD cascade and the dissipation

- the bound on helical magnetic fields is generically less stringent because of the inverse cascade (maximally helical field)
- the dependence on n is such that magnetic fields with blue spectra are more constrained

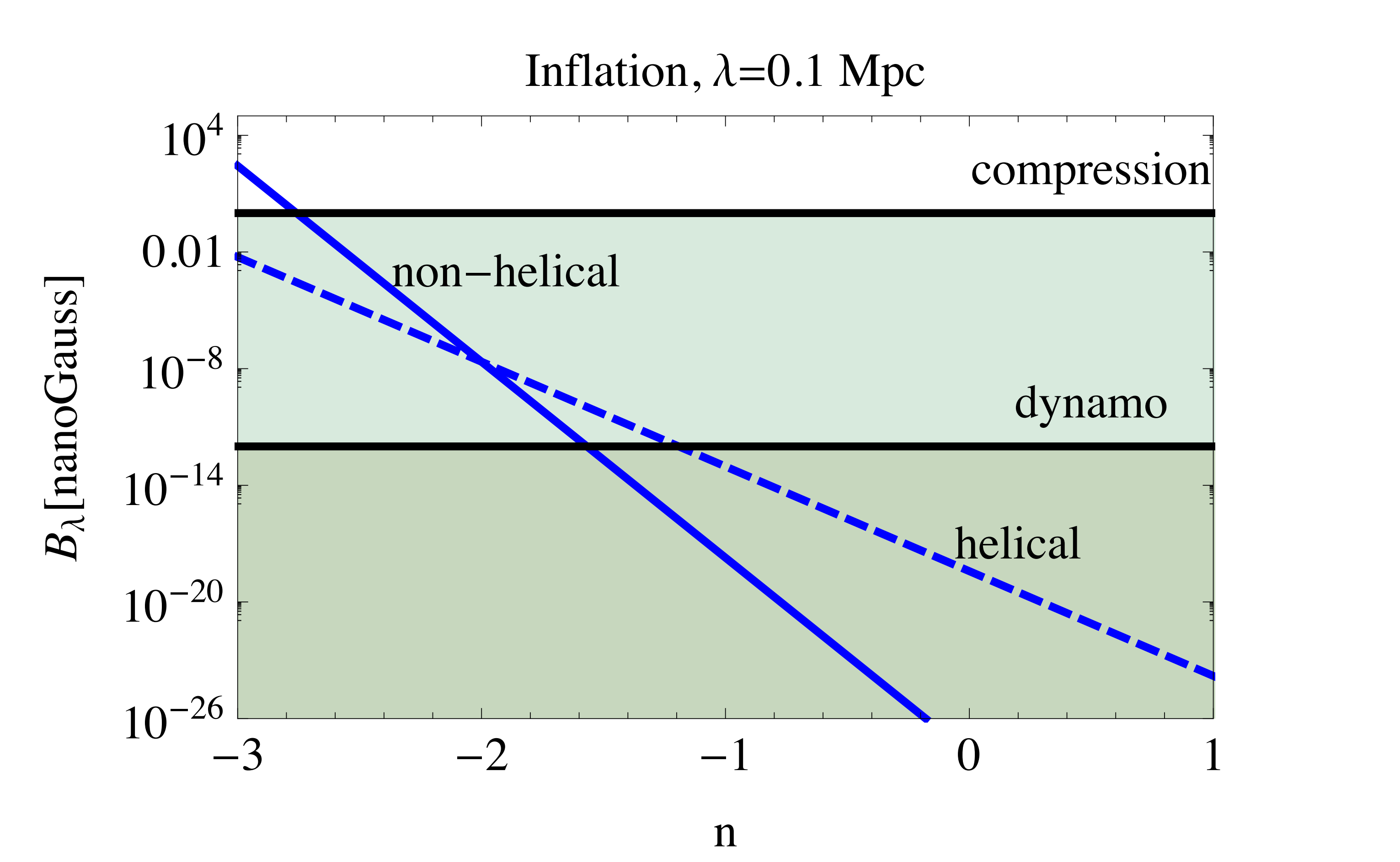
Constraints for generation at a phase transition



QCDPT (100 MeV) helical : OK to seed dynamo if $\lambda < 1\,\mathrm{Mpc}$

QCDPT (100 MeV) non-helical and EWPT helical : OK to seed dynamo if $\lambda < 15\,\mathrm{kpc}$

Constraints for generation at inflation



inflation (10^14 GeV) non-helical : OK to seed by compression if $n < -2.7$

OK to seed dynamo if $n < -1.6$

inflation (10^14 GeV) helical : OK to seed dynamo if $-1.8 < n < -1.2$

Conclusions

- the main generation mechanisms for a magnetic field in the early universe are inflation, phase transitions and charge separation at recombination
- the final amplitude depends on average method, evolution and parameters in the model
- causal generation mechanisms give rise to blue spectra, which suppresses the magnetic field amplitude at large scales : they are disfavoured
- helical fields are preferred because of inverse cascade transferring power at large scales
- Nucleosynthesis and GWs strongly constrain the generation mechanisms occurring before Nucleosynthesis
- inflation with red spectrum could seed a magnetic field simply by compression
- EWPT could seed a magnetic field by dynamo only if helical and if a smoothing scale of a few kpc is enough