Propagation of cosmic rays in intermittent magnetic fields

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Outline

1. Interstellar magnetic fields: intermittency from dynamo action and shock compression

2. Cosmic ray diffusion in intermittent magnetic fields

   2.1. Dynamo in a single-scale chaotic velocity field

   2.2. Dynamo in a multi-scale chaotic flow

3. Scattering of cosmic rays: a correlated random walk

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Interstellar magnetic fields: intermittency from dynamo action and shock wave compression

Interstellar random magnetic fields are produced by:

- Tangling of the mean magnetic field $B \downarrow 0$ by turbulence.

  Presumably Gaussian statistics, volume-filling

  $$\frac{\partial \vec{B}}{\partial t} \simeq (\vec{B}_0 \cdot \nabla)\vec{v}, \qquad \vec{B} \simeq \vec{B}_0 \,.$$

- Fluctuation ("small-scale") dynamo

  Strongly intermittent: random filaments and ribbons

- Compression in shock-wave turbulence

  Separation of primary shocks $d \simeq 4 Ma \uparrow 4.5\ pc$, Ma=Mach number,

The fluctuation dynamo:

generation of a random magnetic field by a random flow

$$\frac{\partial \vec{B}}{\partial t} + \underbrace{(\vec{v}\cdot\nabla)\vec{B}}_{\text{advection}} = \underbrace{(\vec{B}\cdot\nabla)\vec{v}}_{\text{stretching}} + \eta\nabla^2\vec{B}, \qquad \nabla\cdot\vec{B} = 0$$

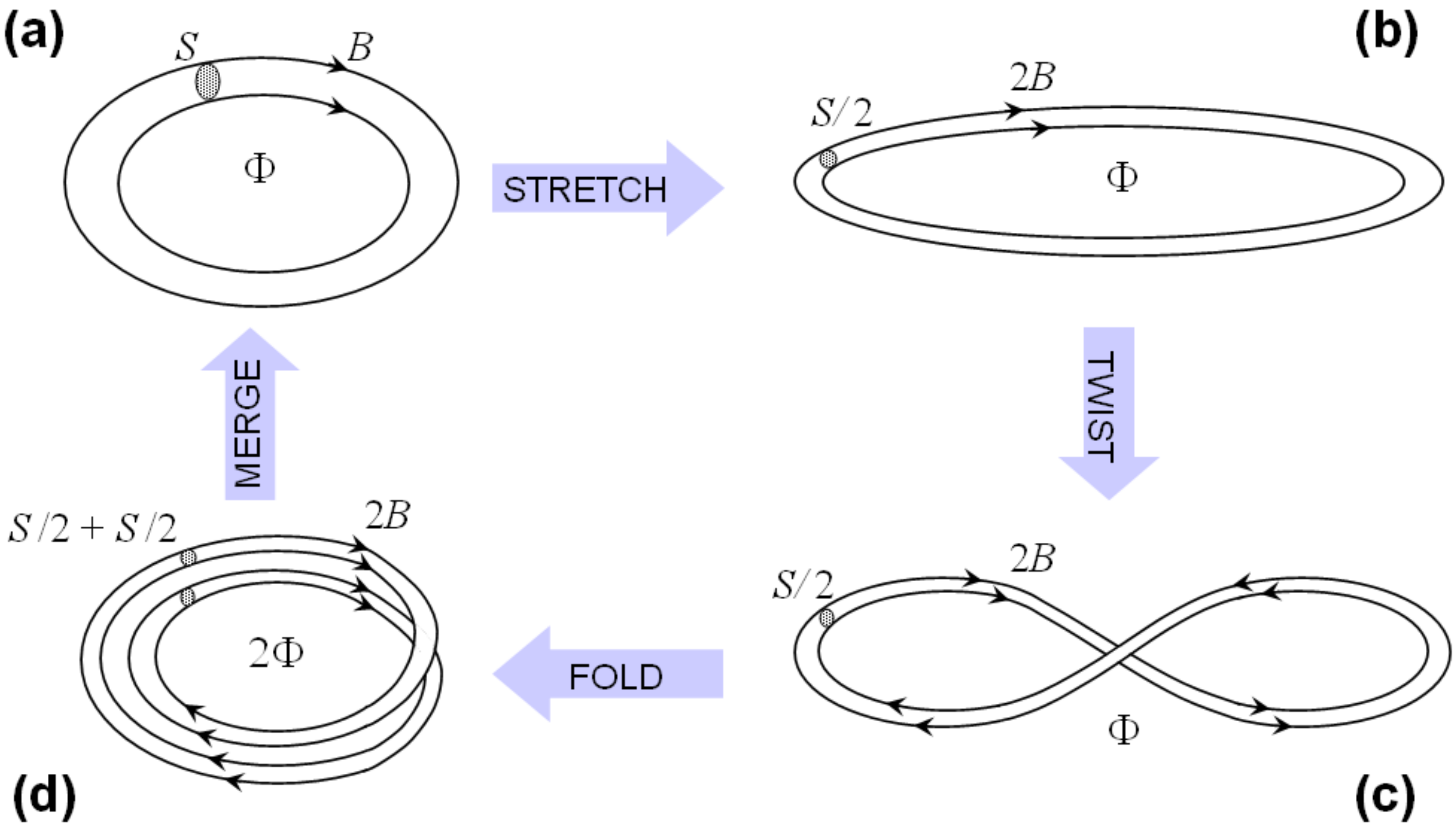
$\vec{B} \propto \exp\left[\gamma(R_{\rm m})t\right]$, $\quad \gamma \simeq \dfrac{v_0}{l_0}$ when $B \ll B_0$ (kinematic dynamo)

provided $R_{\rm m} = \dfrac{l_0 v_0}{\eta} \geq R_{\rm m,cr} = 30$–$500$

and $\vec{v}$ is a random or chaotic flow;

$B \simeq B_{\rm eq}$ in the statistically steady state, $\qquad B_{\rm eq}^2 = 4\pi\rho v^2$.

A conceptual model: Zeldovich's rope dynamo



Intermittent magnetic fields

- Rapid growth of high-order statistical moments:

  $$\langle B^{2n} \rangle \propto \exp(\gamma_{2n} t), \qquad \gamma_{2n} \propto n^2$$

- Intense, random magnetic filaments and ribbons in a sea of weaker random magnetic fields

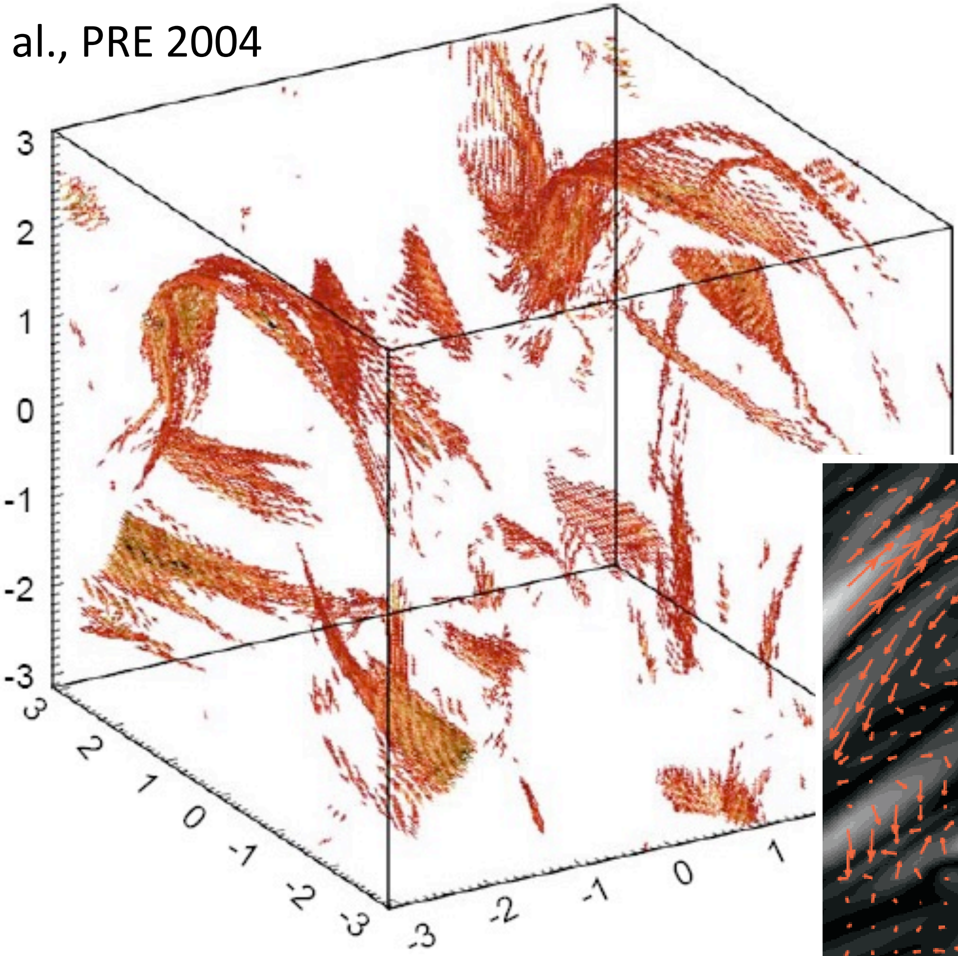
  (Zeldovich, Ruzmaikin & Sokoloff, *The Almighty Chance*, 1990)

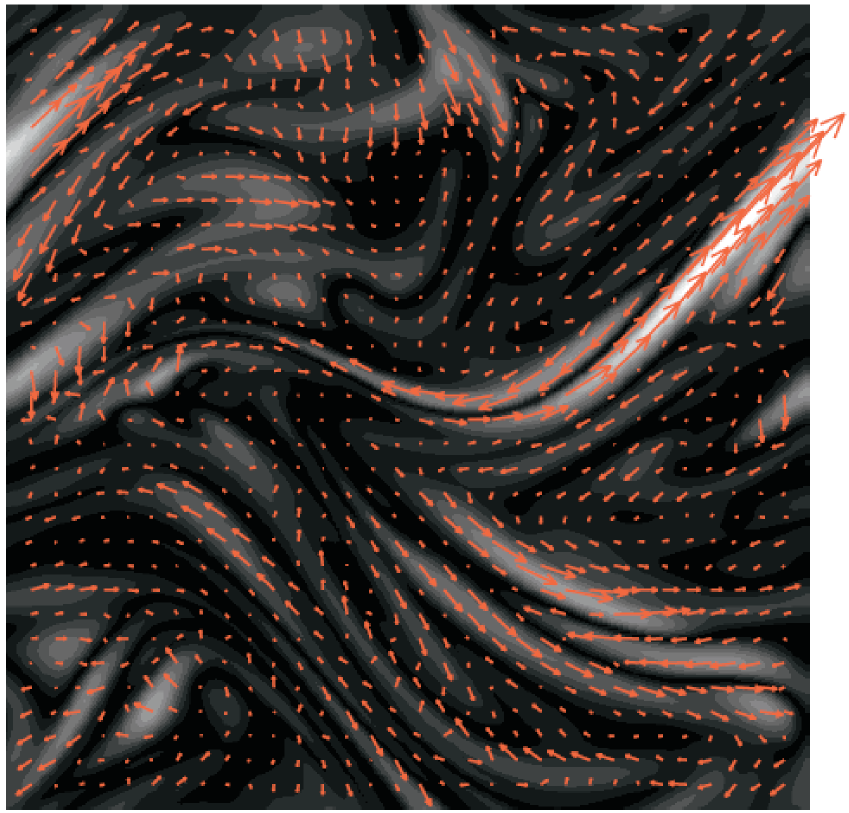
- Strongly non-Gaussian random magnetic fields

- $\langle B^2 \rangle \simeq 0.1 B_{\text{eq}}^2$

- Very different from random magnetic fields used in cosmic ray propagation models

Haugen et al., PRE 2004



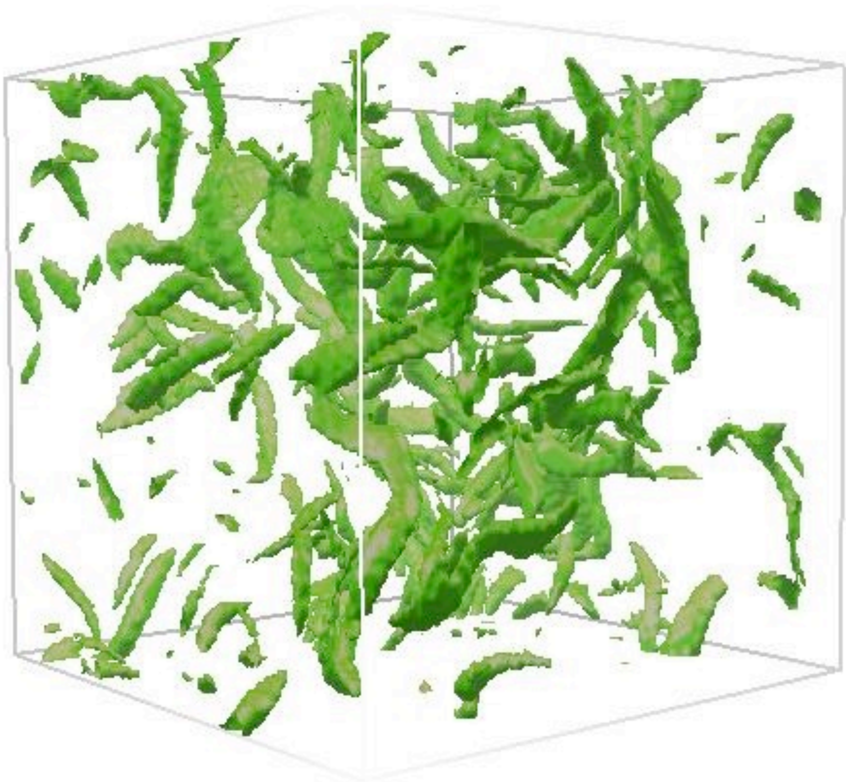


Schekochihin et al., ApJ 2004

Morphology of magnetic structures

Wilkin et al., *PRL*, 2007: dynamo in an isotropic chaotic flow:

$$\vec{v} = \sum_{n=1}^{N} \left[\vec{C}_n \times \vec{k}_n \cos(\vec{k}_n \cdot \vec{x}) + \vec{D}_n \times \vec{k}_n \sin(\vec{k}_n \cdot \vec{x}) \right], \qquad E(k_n) \propto k_n^{-s}$$



Magnetic structures:

thickness: $l_1 \propto l_0 R_{\mathrm{m}}^{-2/(1-s)}$

width: $l_2 \propto l_0 R_{\mathrm{m}}^{-0.55}$

length: $l_3 \simeq l_0$

$R_{\mathrm{m}} \gg 1$: filaments, l_1 thick, l_0 long

Subramanian, *PRL*, 1999:

steady state: $R_{\mathrm{m,eff}} = R_{\mathrm{m,cr}} \simeq 10^2$

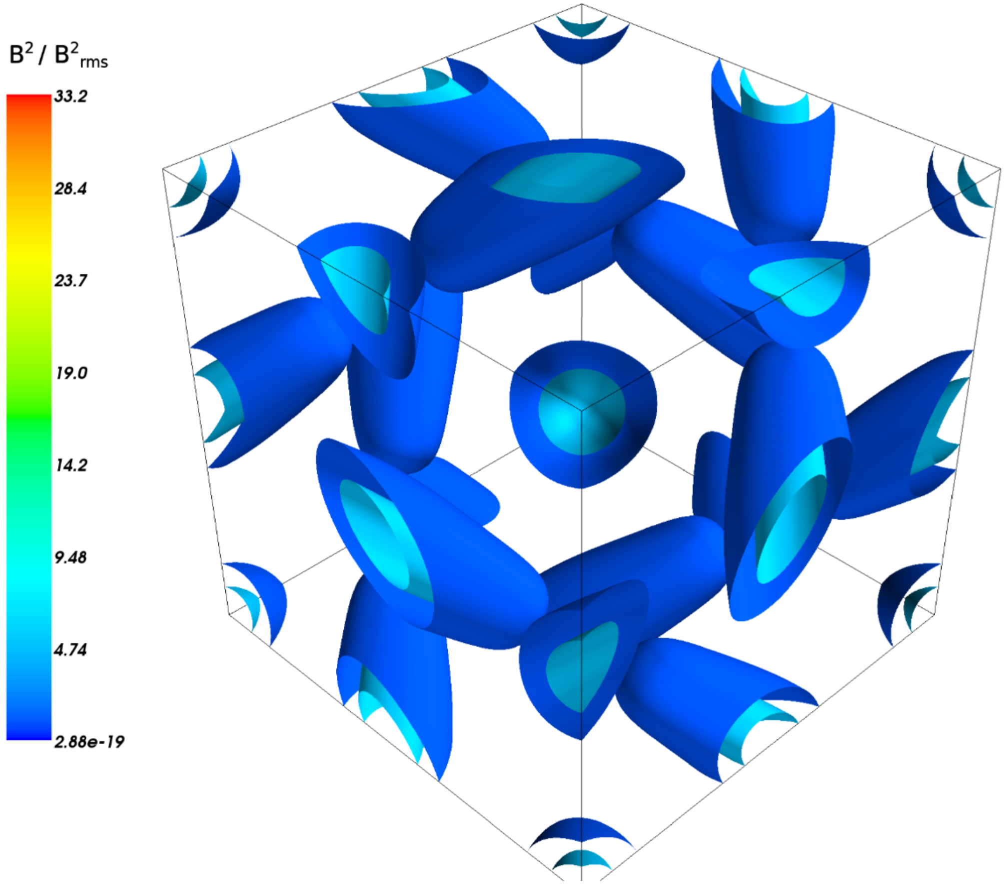
2

Cosmic ray diffusion in intermittent magnetic fields

2.1. A single-scale chaotic velocity field (W)

$$\vec{v} = \frac{2}{\sqrt{3}}(\sin y \cos z,\ \sin z \cos x,\ \sin x \cos y),$$

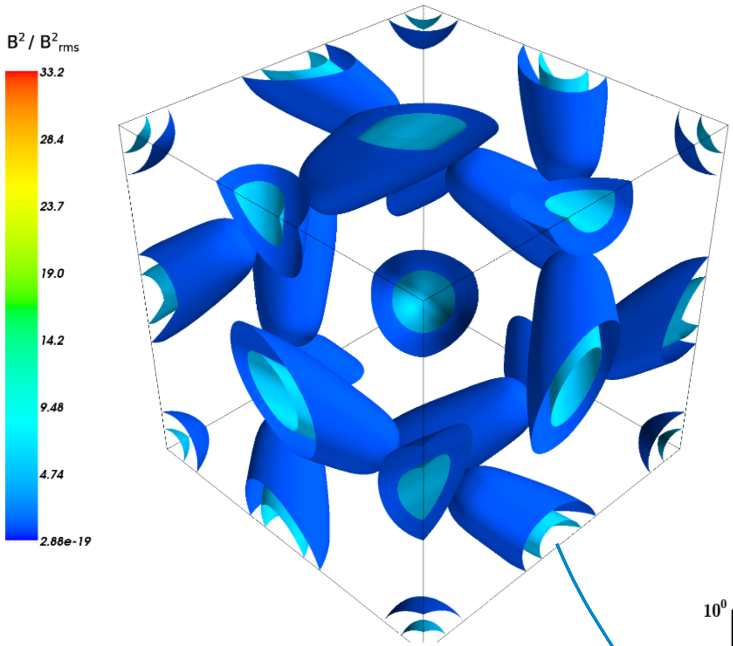
$$R_{\mathrm{m,cr}} \approx 1.8$$



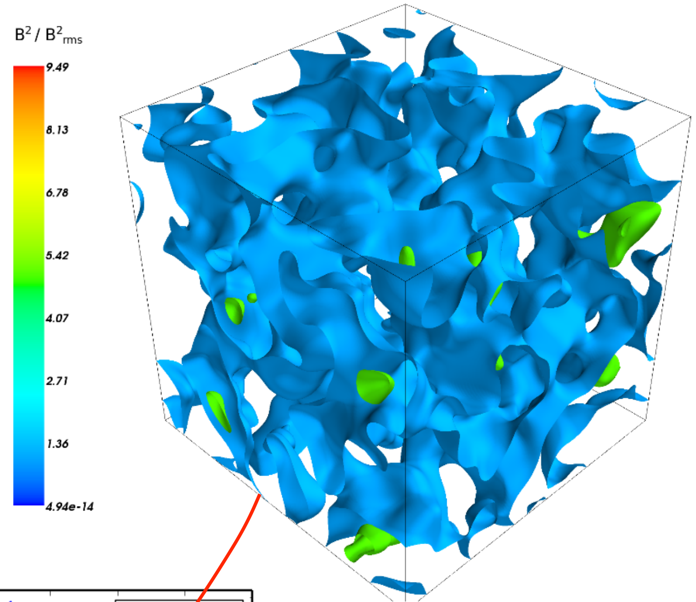
Magnetic field produced by the dynamo: isosurfaces of $|B|$ at $R_m = 50$: a periodic array of magnetic filaments

Magnetic intermittency: non-Gaussian statistics

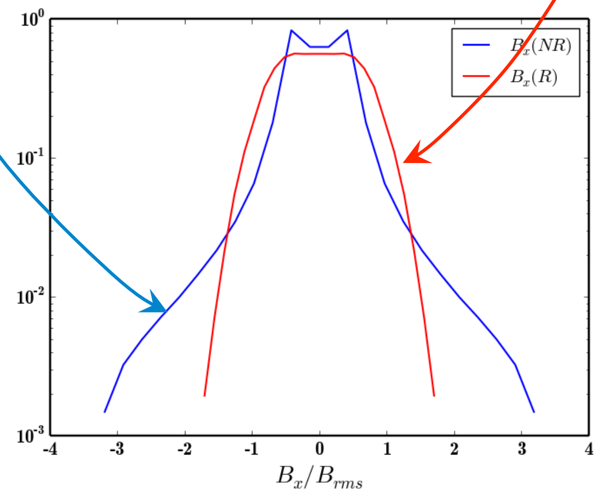
Dynamo solution



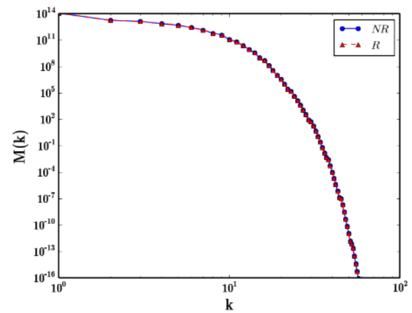
Fourier mode phases randomised



Probability density of $B_i / \langle B^2 \rangle$: heavy tails in the intermittent field, Gaussian after phase randomisation

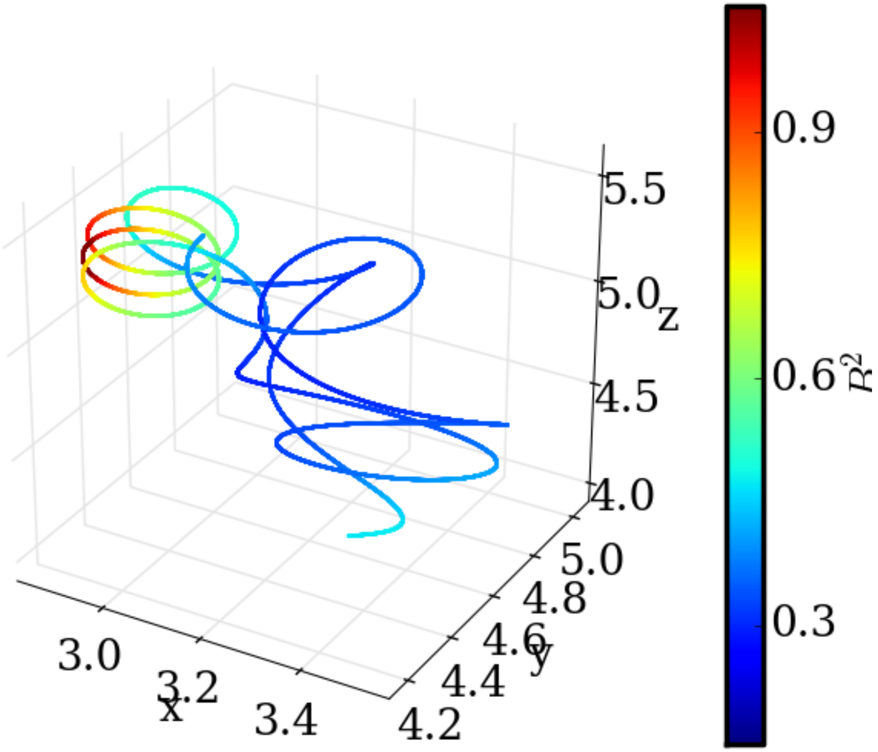


Identical power spectra

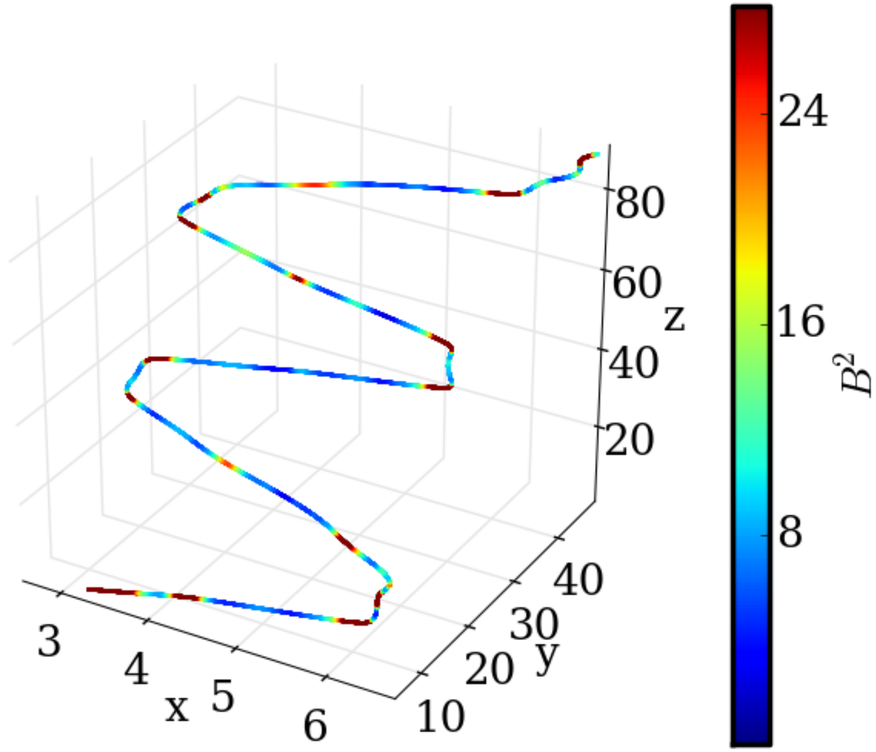


Test particle simulations of cosmic ray propagation

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{e}{c}\vec{u} \times \vec{B}, \qquad \frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = \vec{u}, \qquad \text{1024 particles.}$$



Low-energy particle trajectory

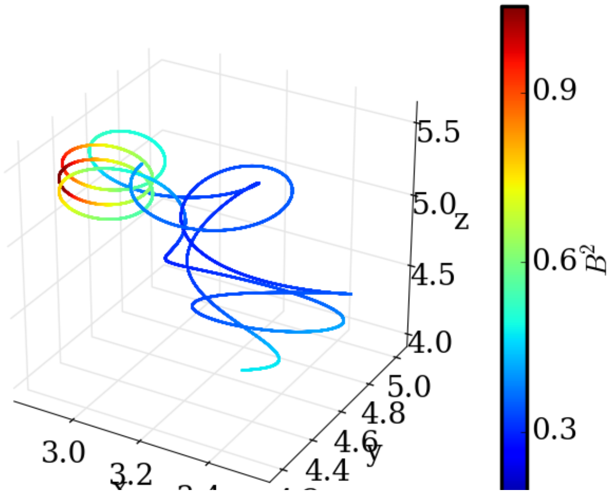


High-energy particle trajectory

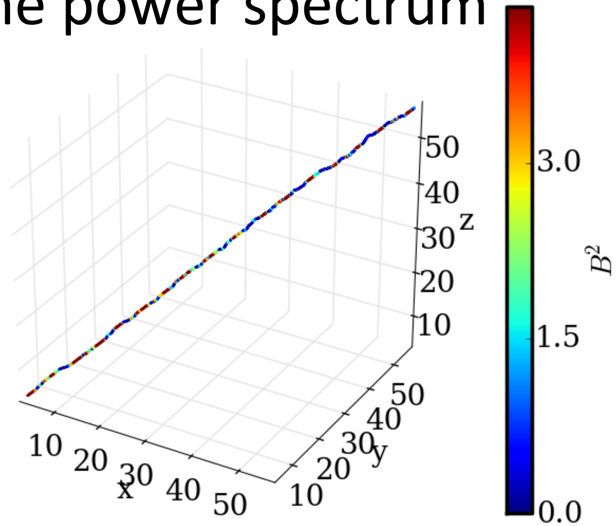
The effect of intermittency on particle trajectories

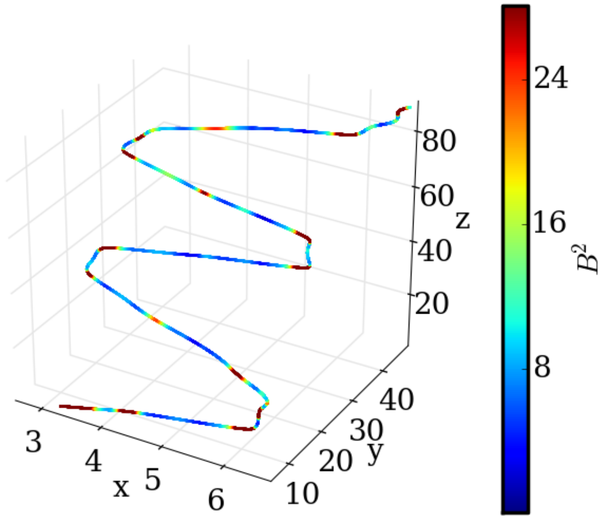
In an intermittent magnetic field

In a Gaussian random field with the same power spectrum

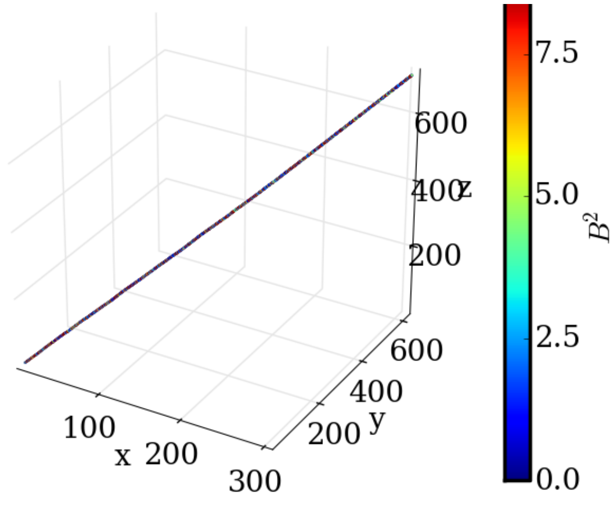


Low energy





High energy



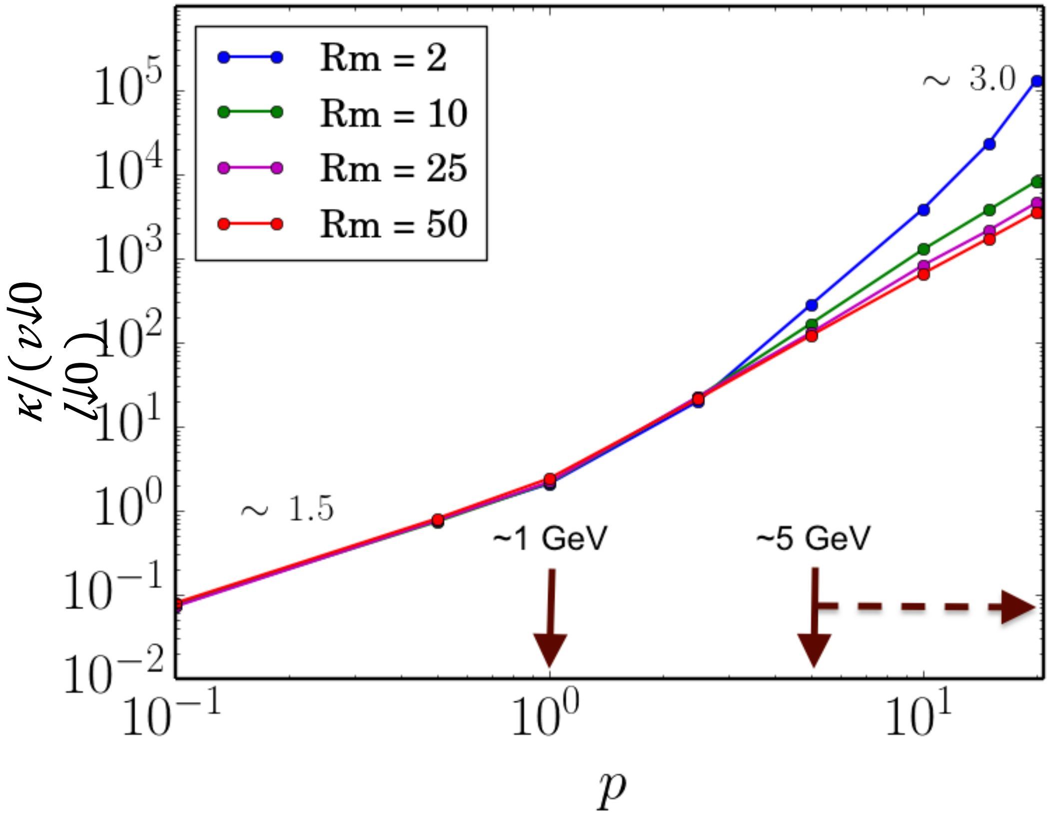
The effect of intermittency on the diffusion coefficient

- A random system of magnetic filaments $\simeq l_0$ apart, of a thickness $d \simeq l_0 R_m^{-1/2}$.

- Cosmic ray particles that are sensitive to the magnetic structures: $r_L < l_0$ or

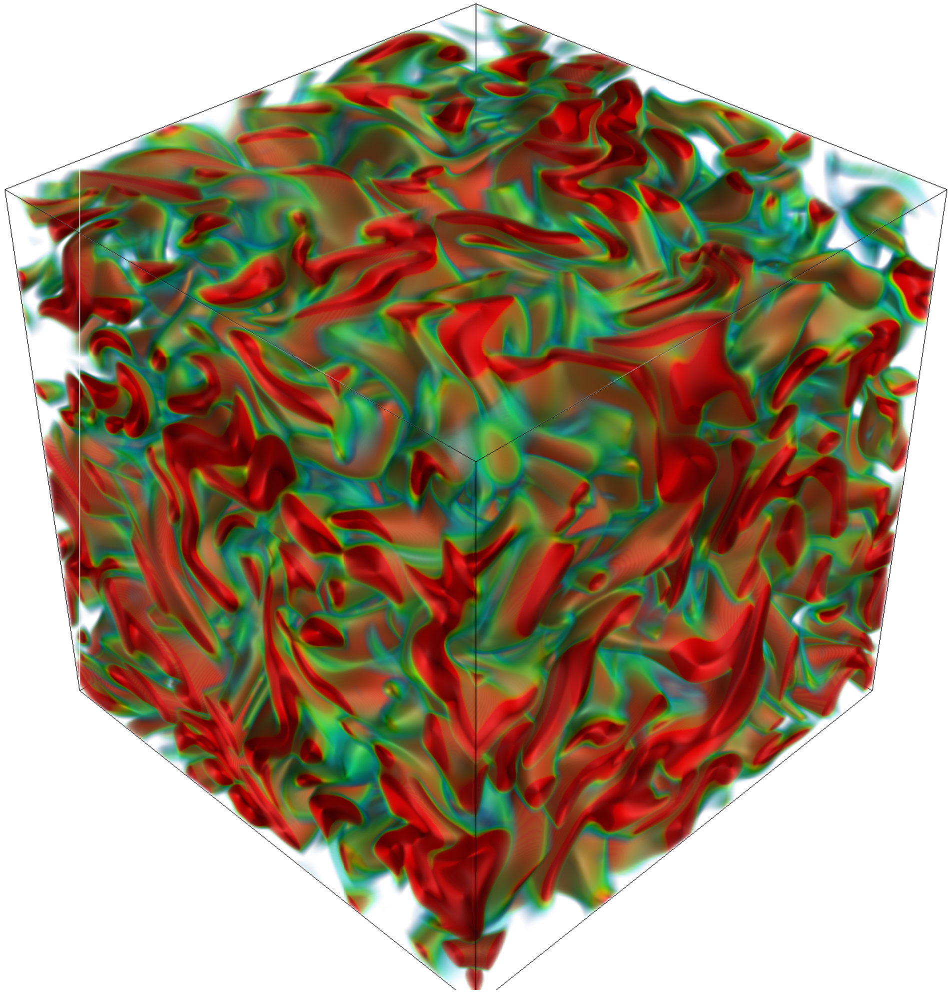
$$\frac{E}{1\,\mathrm{GeV}} \lesssim 10^9 \frac{l_0}{1\,\mathrm{kpc}} \frac{B}{1\,\mu\mathrm{G}}$$

r_L = Larmor radius



2.2. A multi-scale chaotic flow (KS)

$$\vec{v} = \sum_{n=1}^{N} \left[\vec{C}_n \times \vec{k}_n \cos(\vec{k}_n \cdot \vec{x}) + \vec{D}_n \times \vec{k}_n \sin(\vec{k}_n \cdot \vec{x}) \right], \qquad E(k_n) \propto k_n^{-5/3}$$



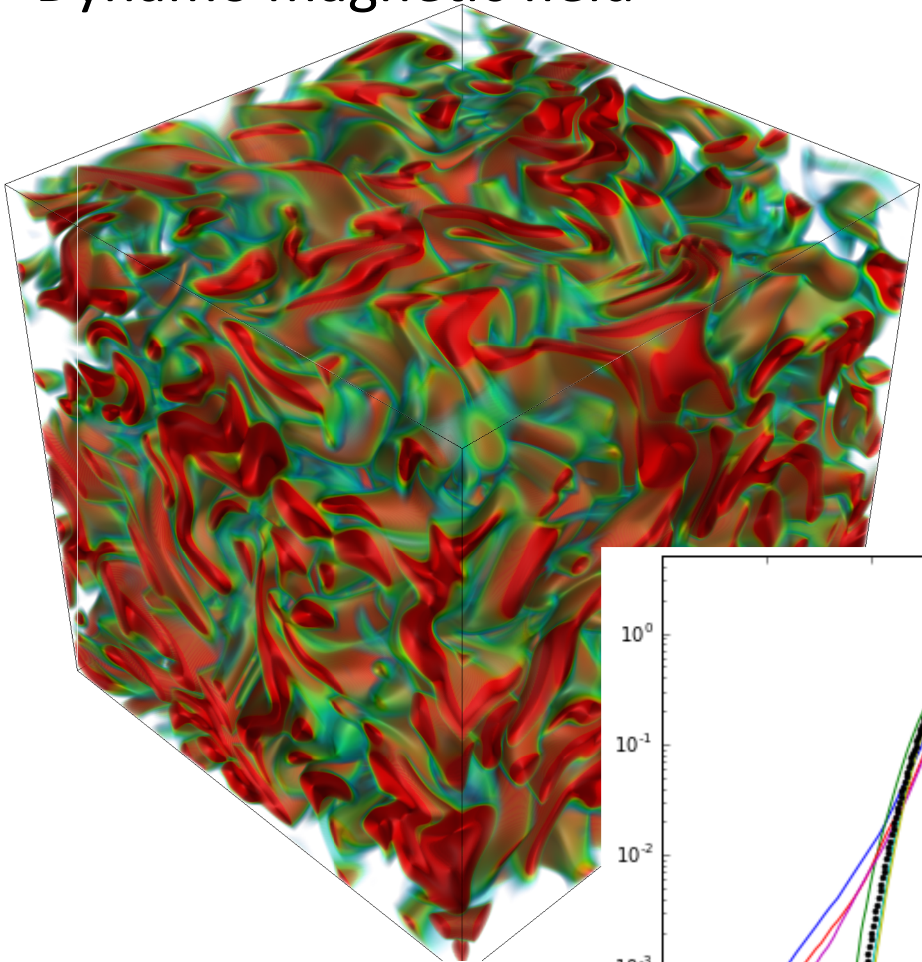
Isosurfaces of $|\vec{B}|$,

$R_{\rm m} = 850$,

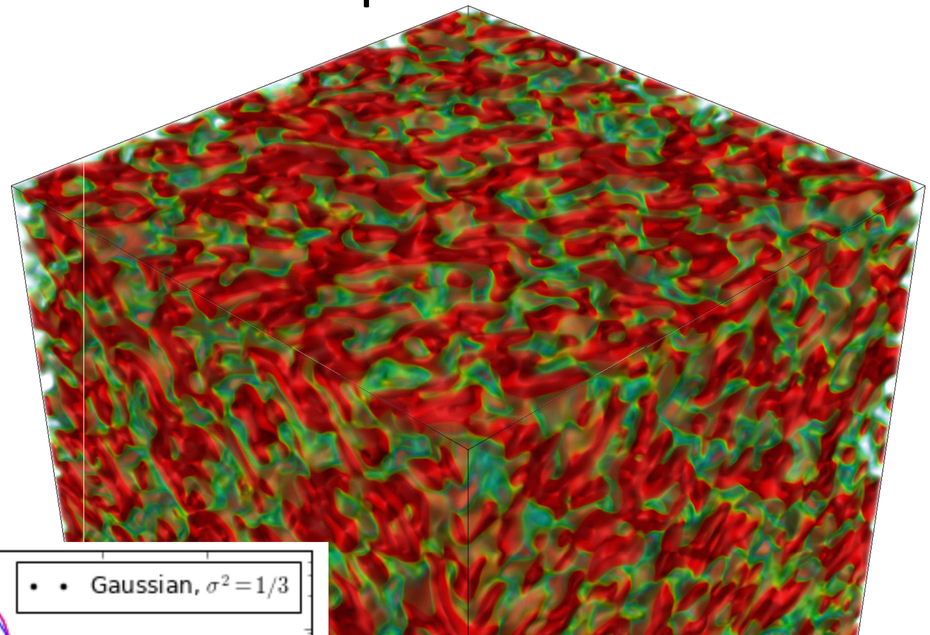
$R_{\rm m,cr} = 750$.

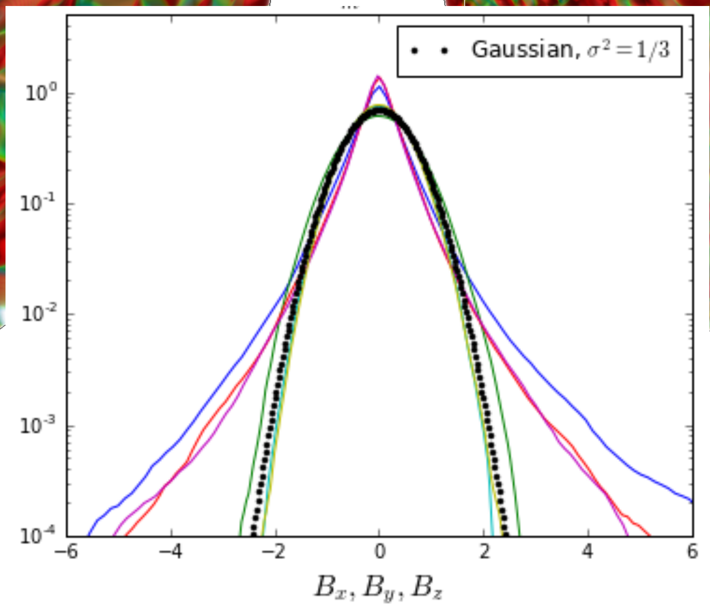
Magnetic intermittency: non-Gaussian statistics

Dynamo magnetic field

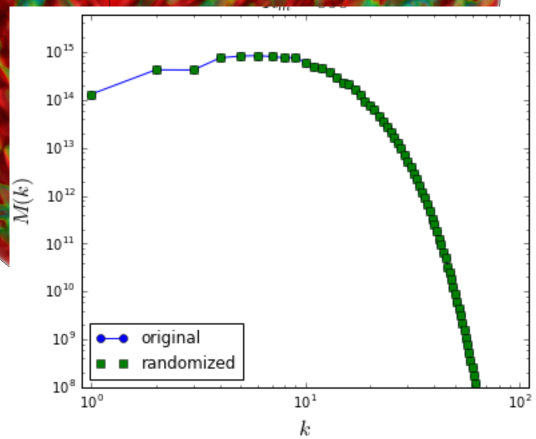


Fourier mode phases randomised

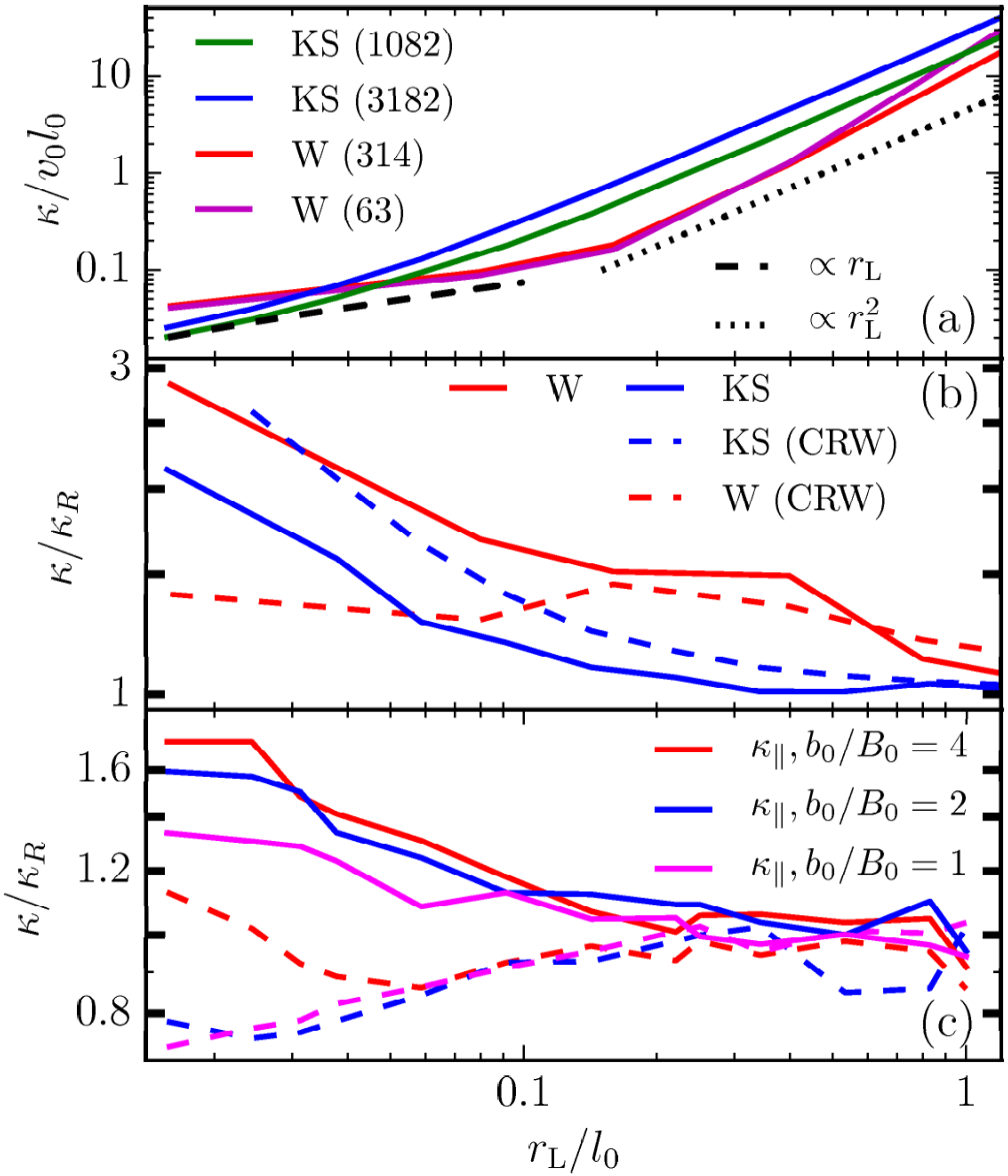




Very different PDFs



Identical power spectra



Diffusivity dependence on the Larmor radius: sensitive to the type of the flow (KS and W) and R_m

The effect of intermittency: ratio of diffusivities in intermittent and randomised fields (solid); correlated random walk model (dashed)

Imposed uniform magnetic field B_0 :

ratio of diffusivities in an intermittent and randomised fields, $\kappa_\parallel$: solid, $\kappa_\perp$: dashed

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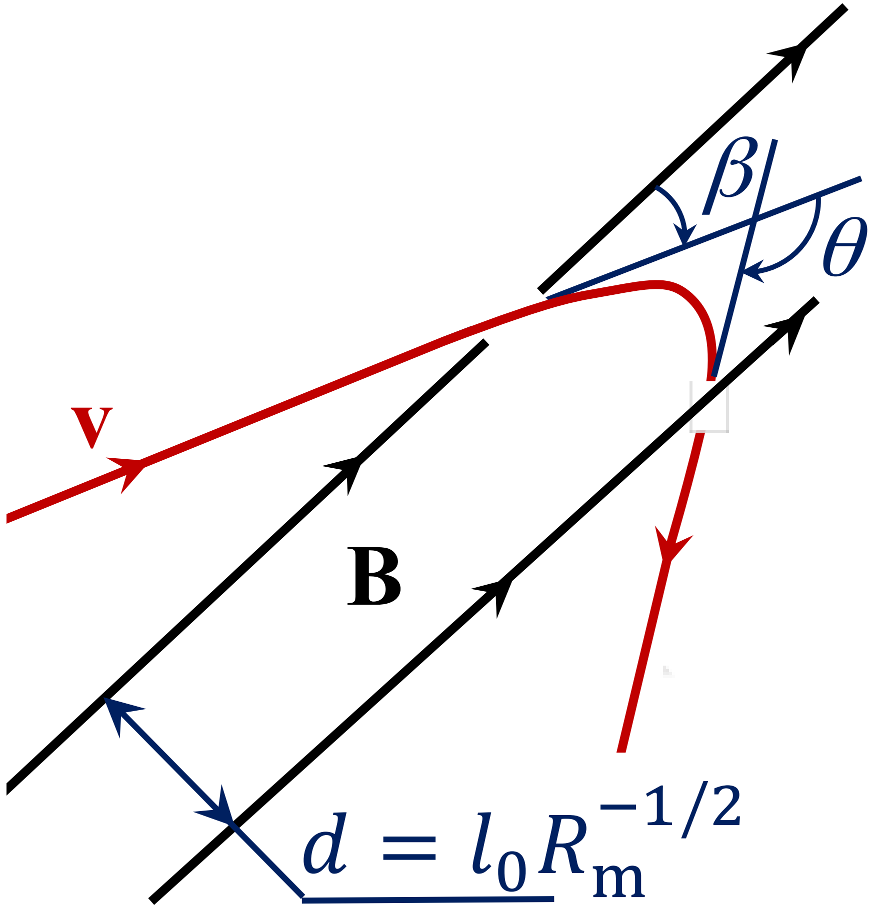
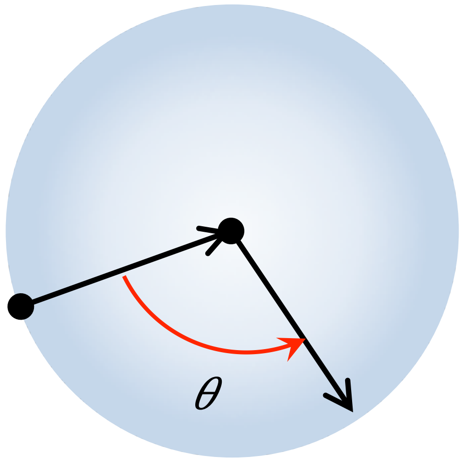
Scattering of cosmic rays:
a correlated random walk



$$\langle \cos\theta \rangle = \pi^{-1} \int_0^{\pi} \cos\left(a/\sin\beta\right) \mathrm{d}\beta$$
$$= 1 - \tfrac{1}{2}\pi a \left[J_0(a)\mathcal{H}_{-1}(a) - J_{-1}(a)\mathcal{H}_0(a)\right]$$

$$a = \frac{d}{r_\mathrm{L}}$$

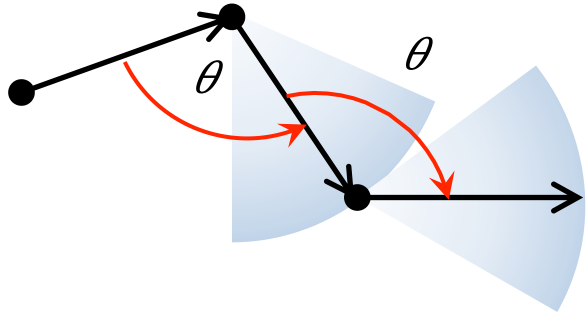
Brownian motion



$\theta = U(0, 2\pi)$

$$\kappa = \frac{\langle l^2 \rangle}{2\tau}$$

Correlated random walk

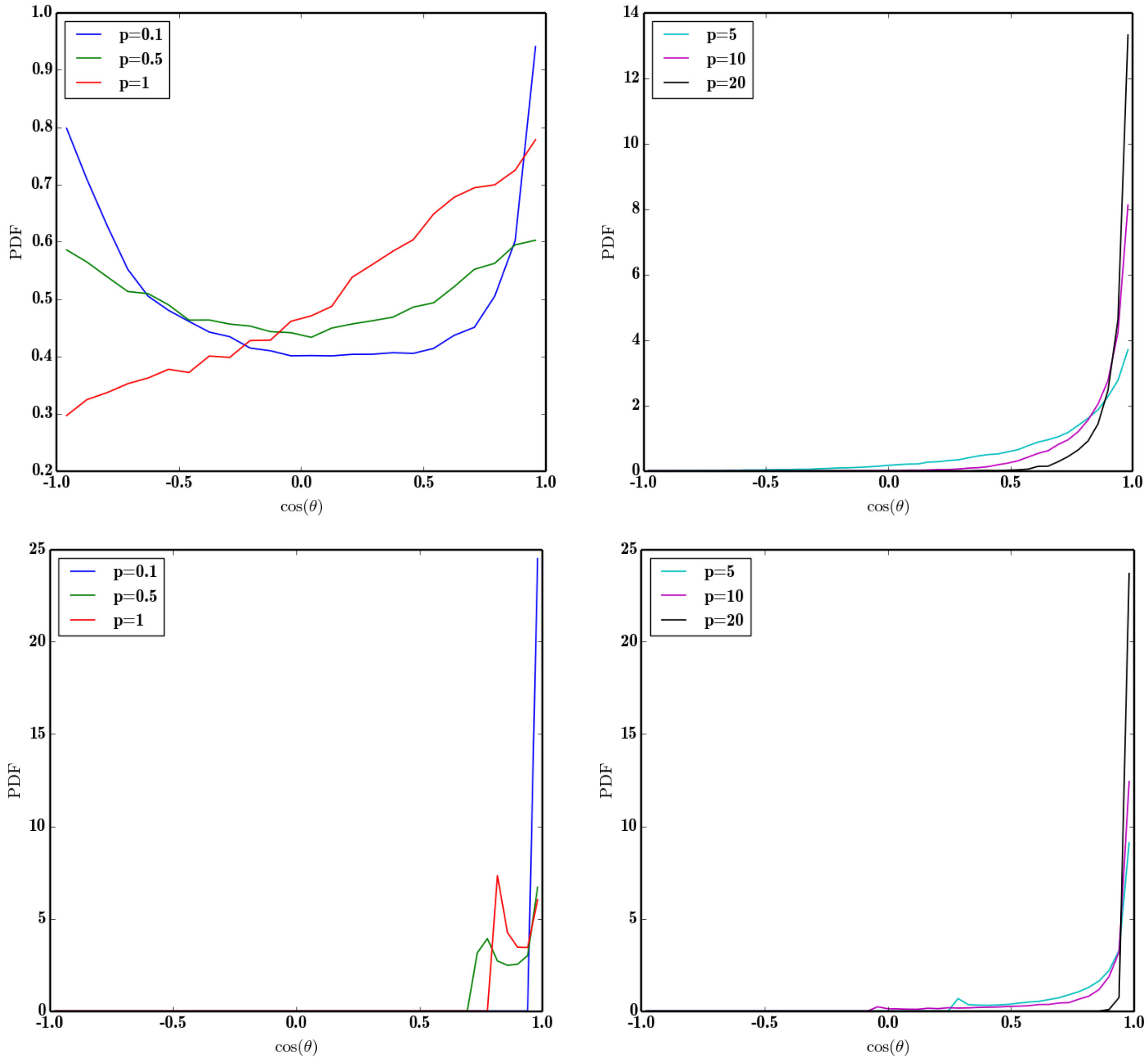


$\theta_1 \leq \theta \leq \theta_2$, $\theta_1 > 0$, $\theta_2 < 2\pi$

$$\kappa = \frac{\langle l^2 \rangle}{2\tau} + \frac{\langle l \rangle^2}{\tau} \frac{\langle \cos\theta \rangle}{1 - \langle \cos\theta \rangle} \quad \text{in 2D},$$

$$\langle \cos\theta \rangle = f(B_0, R_{\mathrm{m}}).$$

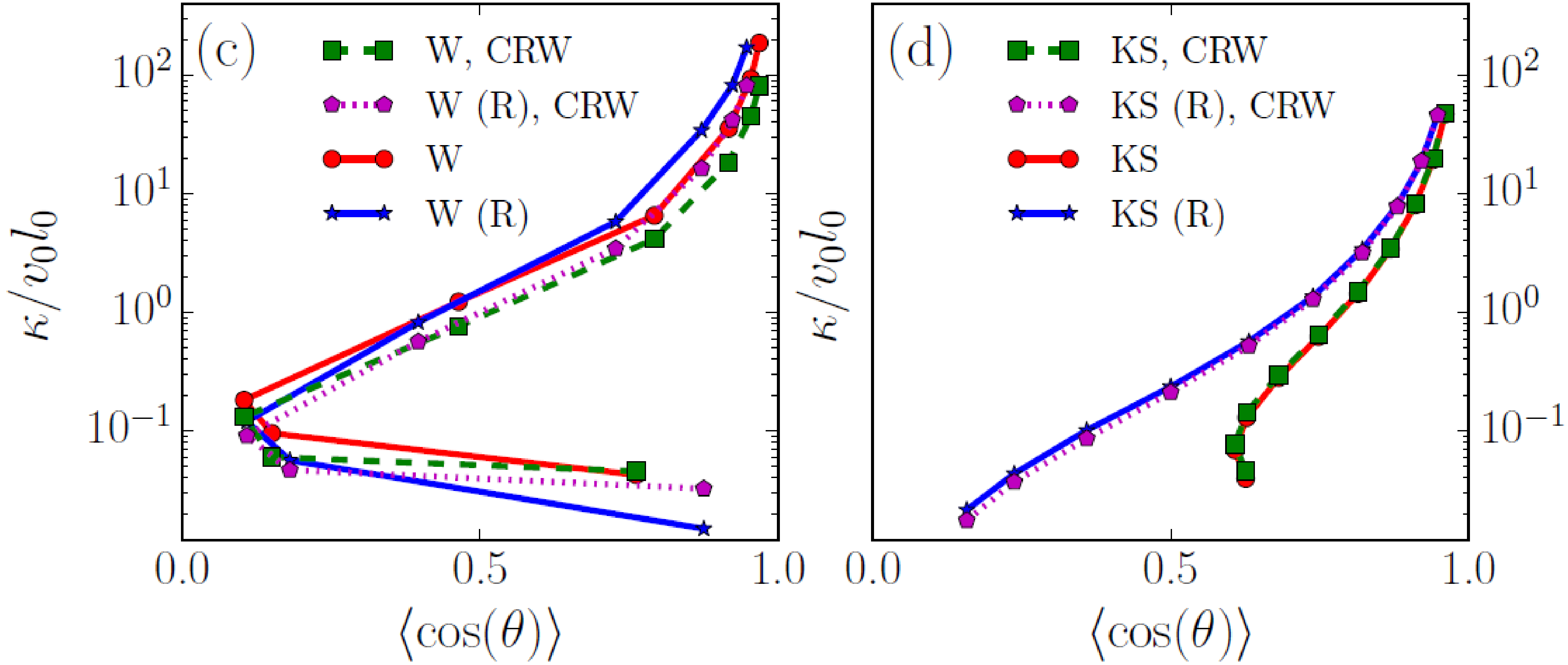
Probability density of cosθ



The single-scale chaotic flow, intermittent magnetic field, $R_{\mathrm{m}} = 50.$

The single-scale chaotic flow, magnetic field with randomized Fourier phases, $R_{\mathrm{m}} = 50.$

Diffusivity dependence on $\langle cos\theta \rangle$



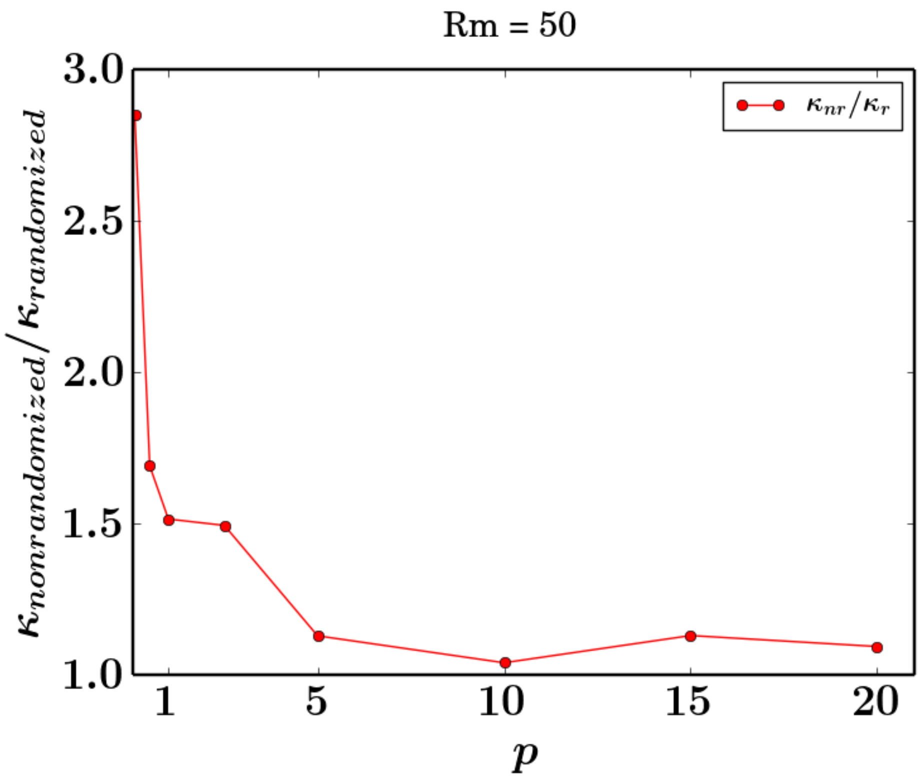
Data points: particles of various energies

Correlated random walk model is impressively successful with both intermittent and Gaussian random magnetic fields

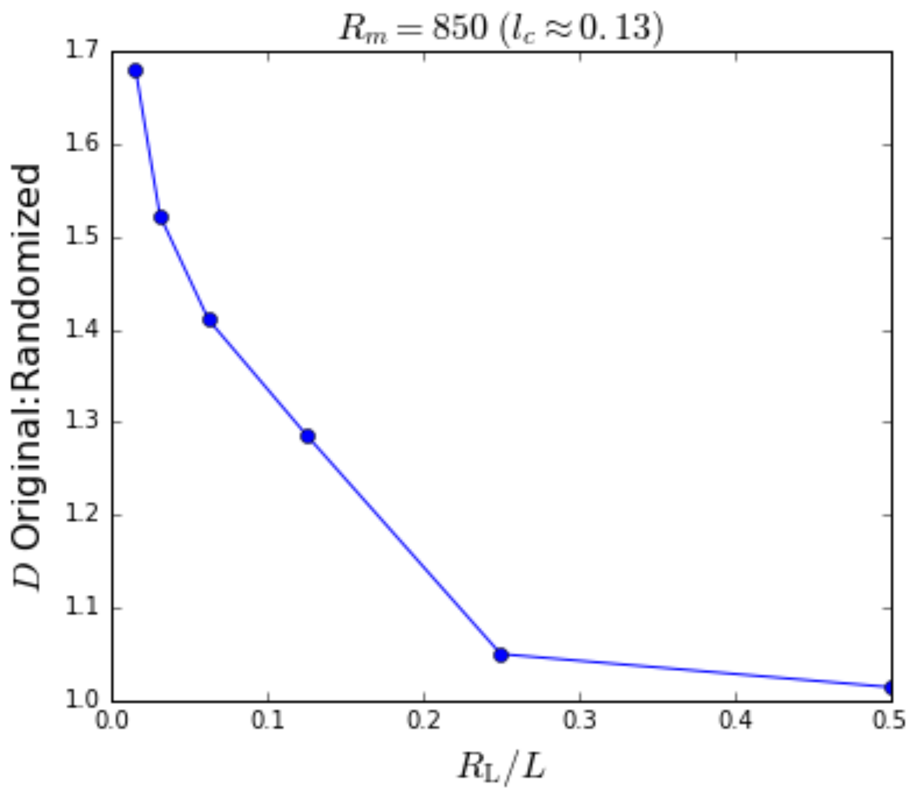
Diffusivity in intermittent vs. Gaussian magnetic fields

$$\frac{D_{\mathrm{intermittent}}}{D_{\mathrm{Gaussian}}} \quad \text{vs. particle energy}$$

The single-scale flow



The multi-scale flow



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Conclusions

- Astrophysical random magnetic fields can be (and most often, are) intermittent.
- Cosmic rays propagate differently in intermittent and Gaussian random magnetic fields of identical power spectra.
- Propagation of cosmic rays in random magnetic fields is not a Brownian motion but a correlated random walk.