Notions of geodesy and the UTM coordinate system. Proposition of a local cartesian coordinate system for the southern Auger site

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1 The shape of the Earth

1.1 The ideal case

In most applications, it is sufficient to approximate the shape of the Earth as a spherical **rigid** body with a spherical symetric distribution of masses and a radius of $\sim 6\,370$ km. In this case, the geometrical center of the Earth coincides with its center of mass. We can use various coordinates system such as a cartesian one $OXYZ$, fixed with respect to the Earth, whose center is the center of the Earth, (OZ) is the rotation axis of the Earth and (OX) is the intersection between the equator plane and the plane defined by the Greenwich meridian. In this reference system, we can use, instead of X, Y and Z the usual spherical coordinates defined by the latitude λ and the longitude ϕ (see Fig. 1).

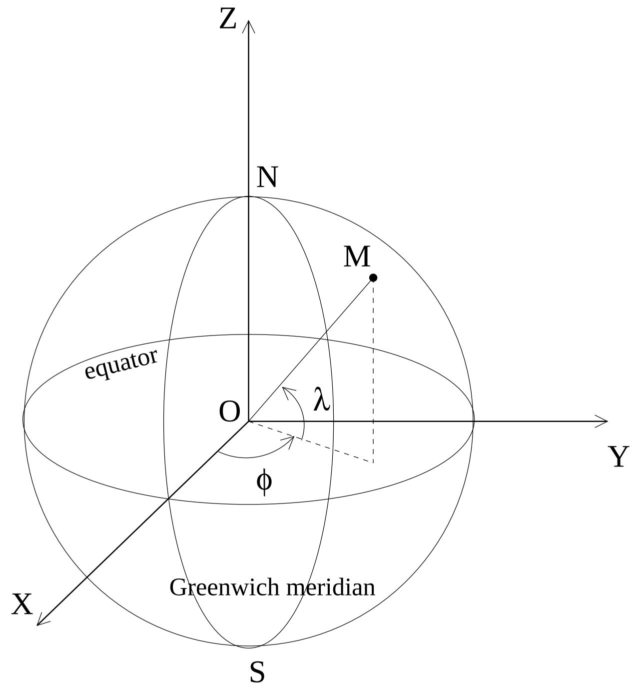
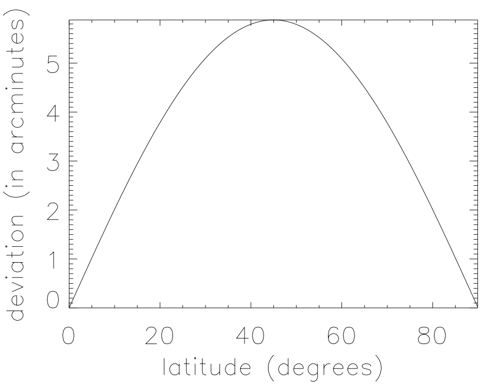


Figure 1: The spherical representation of the Earth with its canonical geocentric reference system.

We define the vertical at a location such as being the direction indicated by a plumb line at this location. The plumb line direction is the result of the superposition of two forces: the pure gravitationnal force (directed towards the center of mass of the Earth in this case) and the centrifugal force due to the rotation of the Earth. This inertial force leads to a small deviation α between the radius OM and the vertical. This deviation is easy to calculate in this case and is given by

$$\cos\alpha = \left(1 + \frac{\Omega^4 (R+z)^2 \cos^2\phi \sin^2\phi}{\left(\dfrac{GM}{(R+z)^2} - \Omega^2 (R+z) \cos^2\phi \right)^2} \right)^{-1/2}$$

where M is the mass of the Earth ($M \sim 6 \times 10^{24}$ kg), G is the universal constant of gravitation ($G \sim 6.67 \times 10^{-11}$ m^3.kg^{-1}.s^{-2}), R is the radius of the Earth ($R \sim 6\,370$ km), Ω is the rotationnal speed of the Earth ($\Omega \sim 7.27 \times 10^{-5}$ rad.s^{-1}), z is the altitude (we took $z = 0$ km here) and ϕ the latitude. The value of this deviation is at most of ~ 6 arcmin (see Fig 2(a)). The deviation is maximum at a latitude of $\pm 45°$ and is equal to zero at the poles and on the equator. We have defined here the altitude as being the distance $OM - R$ above the surface of the sphere.



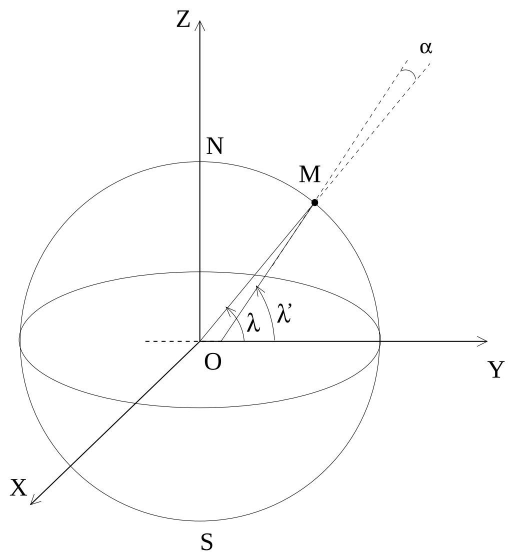


Figure 2: (a): Angular separation between the pure gravitationnal force (along (OM)) and the vertical (which includes the effect of the rotation of the Earth). (b): Definition of the astronomic latitude.

Because of this angular separation, we can define the **astronomic** latitude λ' (see Fig 2(b)). It is defined as being the angle between the vertical and the plane of the equator (the geocentric latitude is the angle between (OM) and the plane of the equator). Without the rotation of the Earth, these two latitudes would be equal.

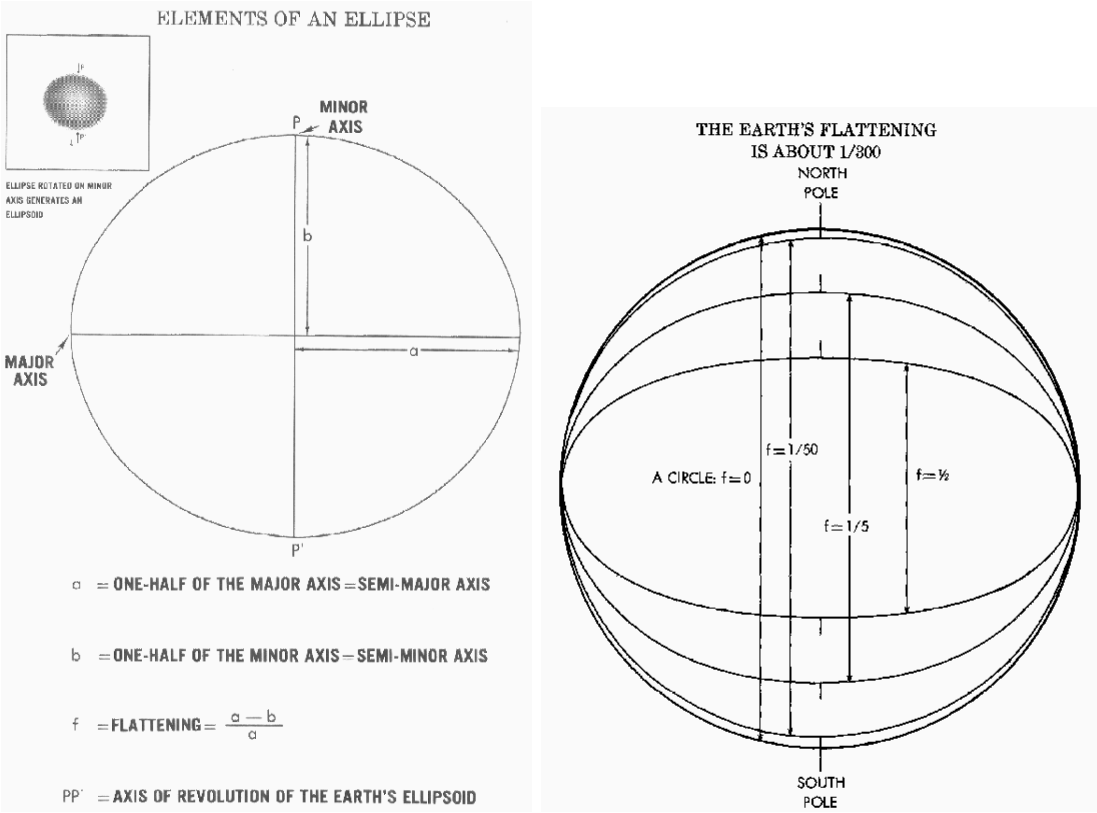
1.2 The ellipsoid approximation

Due to the rotation of the Earth, which is not rigid, the surface is closer to an ellipsoid of revolution than to a sphere. **Locally** (until the firsts GPS satellites), we have been able to compute the elements of the best ellipsoid fitting the local surface. These elements (see Fig. 3) can vary from one point to the other. Since the possibility to have a global description of the Earth with the satellites, it has been necessary to determine a **global** ellipsoid of revolution. With time, increasing precision lead to new determination of the elements of the ellipsoid.

The GPS system uses the WGS84 ellispoid whose elements are:

- semi-major axis $a = 6\,378\,137.0$ m
- flattening $f = 1/298.257\,223\,563$

The definition of latitude becomes more complicated. It is possible to define three different latitudes for a point M located at the surface of the ellipsoid:



(a) (b)

Figure 3: (a): Elements of an ellipse. The ellipsoid of revolution is obtained by rotating the ellipse along one of its axis. (b): Flattening of the Earth. A circle has a flattening of 0 and a straight line of 1. The Earth has a flattening of $\sim 1/300$.

1. the astronomic latitude, is the same that in the spherical case. It is the angle between the vertical (plumb line) at M and the plane of the equator;

2. the geocentric latitude, is the angle between the line defined by the center of the ellispoid and M and the plane of the equator;

3. the geodetic latitude, is the angle between the perpendicular to the ellipsoid at M and the plane of the equator.

The difference between the astronomic and geodetic latitudes and between astronomic and geocentric latitudes are generally less than 3" and 12' respectively. The longitude and latitude given by a GPS are geodetic longitudes and latitudes. The altitude given by a GPS is the height above the reference ellipsoid. Note that in a case of an ellipsoid, the pure gravitational force is not always oriented towards the center of mass of the Earth (it's only true on the equator plane and on the poles plane).

1.3 The geoid (or spheroid)

A definition can be:
The geoid coincides with the surface to which the oceans would conform over the entire Earth if free to adjust to the combined effect of the Earth's mass attraction and the centrifugal force of the Earth's rotation.

Another one, equivalent of course, is:
The geoid is a surface along which the gravity potential is everywhere equal and to which the direction of gravity (that of a plumb line) is always perpendicular. (gravity = gravitation + rotation of the Earth).

Since the Earth is not homogeneous, the ellipsoid surface and the geoid do not coincide. The separations are referred to as geoid undulations, geoid heights or geoid separations. Fig. 4 illustrates the differences between geoid and ellipsoid. The distance between both can reach few hundreds meters.

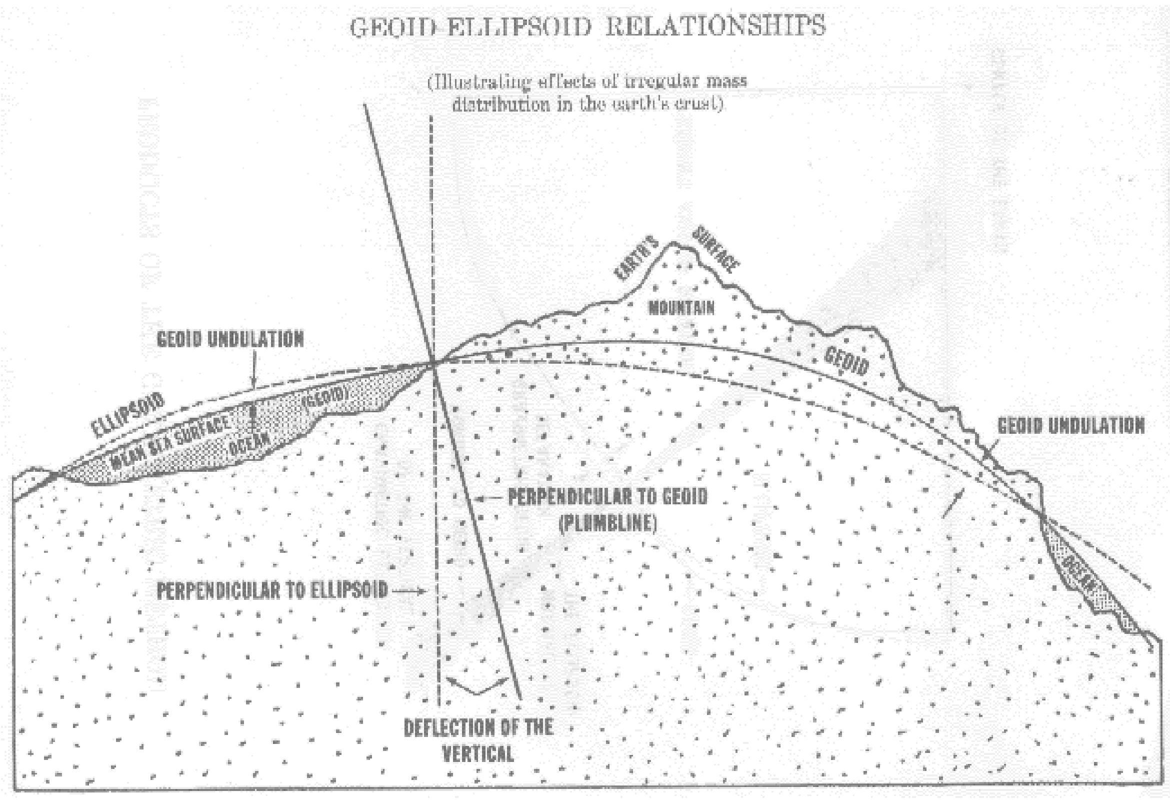


Figure 4: Differences between ellipsoid and geoid

As we have seen before, the canonical latitude associated with the geoid is the astronomic latitude (since given by the plumb line). The altitude is defined as being the height above the geoid.

2 Planimetry and the UTM projection

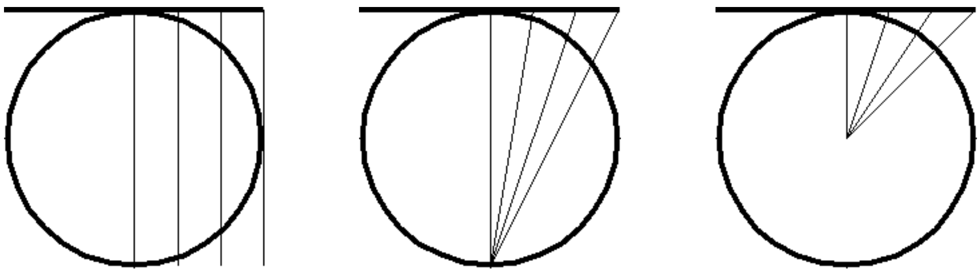
In order to deal with distances in an easy way, it is useful to have a representation of the Earth on a flat surface. Since the transformation from 3D-surface to 2D-surface is not possible without loss of information, we have to choose between what to neglect and what to preserve.

2.1 Planimetry

The different types of projection can be sorted like this:

- construction (see Fig. 5)

  1. direct: these are constructed by straight lines going from a point through the surface of the planet and to the projection plane. The starting point can be located infinitely far away (orthographic projection), on the surface itself, opposite the projected point (stereographic projection) or in the center of the planet (gnomic projection).



(a) (b) (c)

Figure 5: **Azimuthal projections**. (a): **orthographic projection**. Gives a "natural" picture of the planet but it's neither length-, area- nor angle preserving. A hemisphere is the largest area that can be mapped. (b): **stereographic projection**. This projection is angle preserving, but it's neither length- nor area preserving. Theoretically larger areas than a hemisphere can be mapped, but one gets servere distortion on the circumference. (c): **gnomic projection**. This projection is neither length-, area- nor angle preserving, but it has the property that all greatcircles (orthodromes) are mapped as straight lines. It is therefore used when one navigates with greatcircles (airplanes, ships, etc...)

  2. indirect: these are constructed by applying geometrical laws or some mathematical principle.

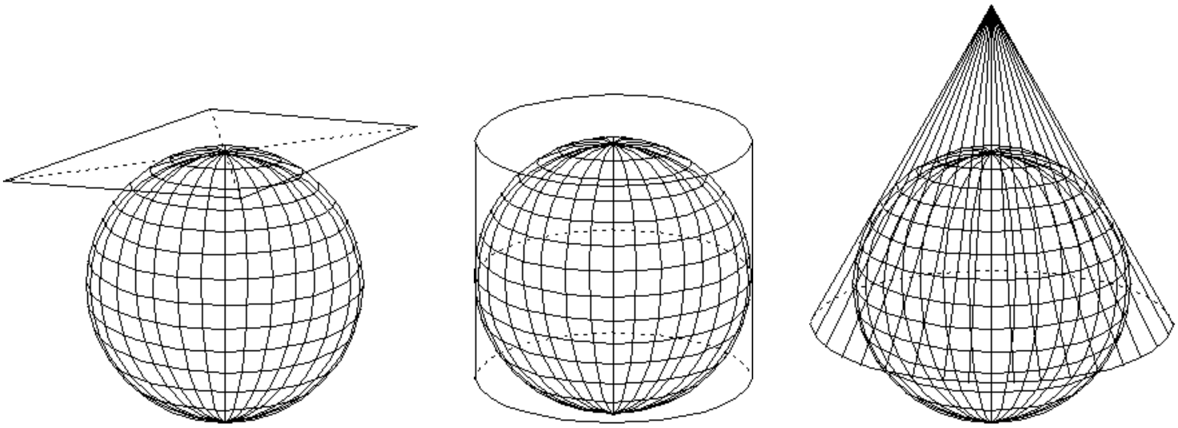
- projection plane (see Fig. 6)

  1. azimuthal: the projection plane is a plane surface tangenting the surface to be projected (see also Fig. 5);
  2. cylindrical: the projection plane is a cylinder circumfering the planet, either tangenting or cutting (see also Fig. 7);
  3. conical: the projection plane is a cone circumfering the planet, either tangenting or cutting.

- orientation of the projection plane

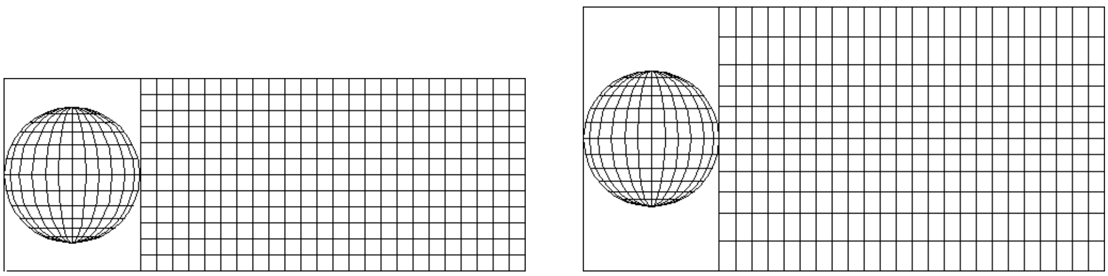
  1. normal: the main axis is parallel to the axis of the planet;
  2. transversal: the main axis is orthogonal to the axis of the planet.

- the properties of the projection



(a) (b) (c)

Figure 6: (a): Plane (azimuthal), (b): Normal cylindrical and (c): Normal conic projections



(a) (b)

Figure 7: **Cylindrical projections**. (a): **square projection**. This projection is length preserving on the equator and the meridians, but distortion is increased as you come closer to the poles. (b): **Mercators projections**. This conformal projection have very strange properties. It is angle preserving for small areas near the equator and these have also the correct shape. Distorsions increase as you come closer to the poles. This mapping is the only where all compass directions (loxodromes) are mapped as straight lines. Loxodromes are a compass direction that cut all meridians at the same angle. In reality, these lines are curved but on a Mercator projection there are straight lines. This property has made this projection widely used for charts since it's very easy to take out a compass direction on such a map.

1. length preserving (equidistant): the projection preserves lengths in certain directions, e.g. on the equator for a tangenting cylindrical projection, or on several directions starting in a certain point. NO projection can preserve lengths on the whole surface though!
2. area preserving: usually the projection is area preserving on the whole surface, but the shape will most certainly change! This means that area preserving projections can't be angle preserving.
3. angle preserving (conformal): usually the projection is conformal on the whole

surface. This means that an angle on the surface of the planet is the same as on the projection.

2.2 The UTM (Universal Transverse Mercator) coordinate system

2.2.1 Construction

As we saw before, only *direct* constructions are easy to visualize (and even to physically realize with optical projection systems). The more general "map projection" actually means "mathematical transformation of the globe onto some other surface".

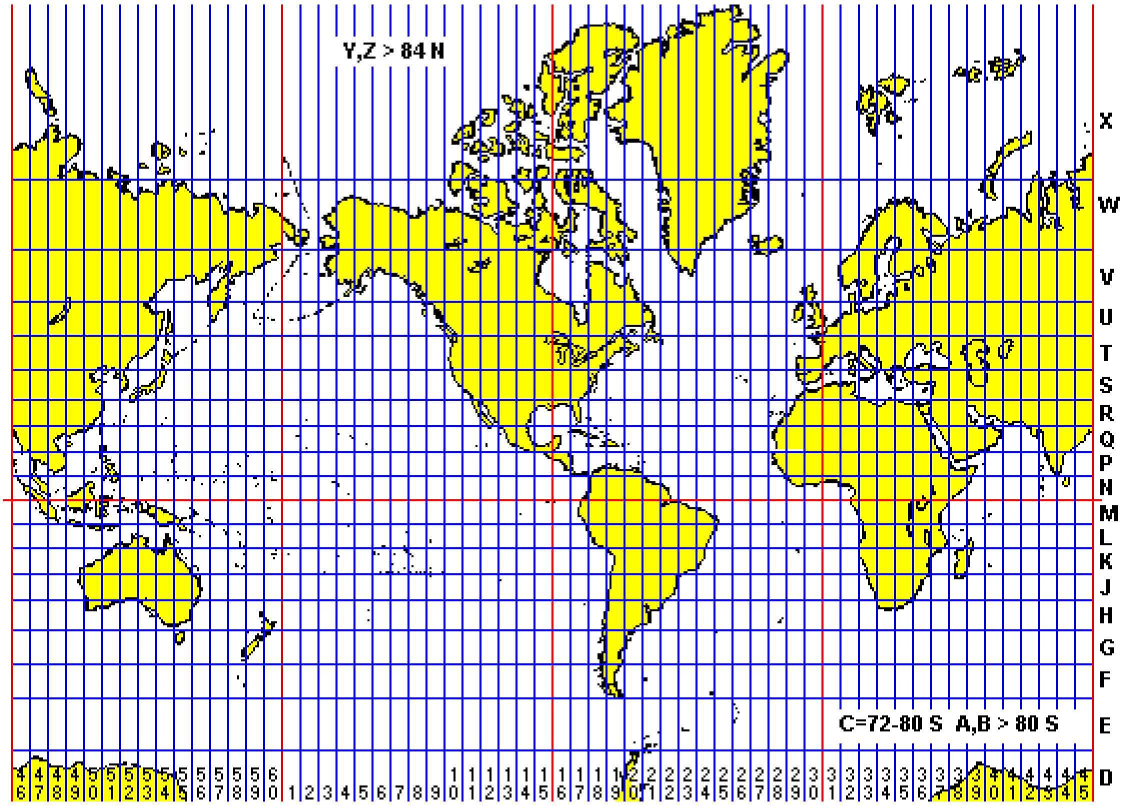


Figure 8: **UTM projection of the Earth**. The Earth is divided into 20 latitude zones lettered from 'C' (not shown here) to 'X' and 60 longitude zones (from 1 at 180° West to 60 at 180° Est). In this projection, the zones appear as straigth lines.

The Mercator projection is a conformal projection, meaning that angles and small shapes on the globe project as the same angles or shapes on the map. The price paid by conformal projections is great variation in scale away from the central portions of the map. For instance, Greenland on a Mercator map looks as big as South America, though it has actually only 1/8 the area (see Fig. 8). However, a small portion of the Greenland has the same shape on the map as it does on the ground.

The cylinder encircling the Earth can have any orientation: if its axis is parallel (resp. perpendicular, oblique) to the Earth's axis of rotation, it is a simple (resp. transverse, oblique) Mercator projection. Because the Transerver Mercator projection is very accurate in narrow zones, it has become the basis for a global coordinate system called the *Universal* Transverse Mercator. In this Universal system, the globe is first subdivided into narrow longitude zones, which are projected onto a Transverse Mercator projection. A grid is constructed on the projection and used to locate points. The upside of the grid system is that, since the grid is rectangular and decimal, it is far easier to use than latitude and longitude. The downside is that, unlike latitude and longitude, there is no way to determine grid locations independently.

In order to get a UTM map (see Fig. 8), the steps to follow are:

- subdivide the Earth into 60 longitude zones (from 1 to 60), each 6° wide. The numbering begins at zone 1 at 180° longitude West (with respect to Greenwich) and proceeds eastward.
- subdivide the Earth into 19 latitude zones (from letter 'C' to letter 'W' omitting 'I' and 'O' to avoid confusion) of 8° wide each and 1 latitude zone of 12° (letter 'X'). The first zone 'C' begins at 80° South and the last zone 'X' begins at 72° North. Because of the distortions inherent to the Transerver Mercator projection, the polar regions (from 90° South to 80° South and from 84° North to 90° North) are omitted (and should have been lettered 'A,B' for the South polar cap and 'Y,Z' for the North polar cap).
- for each zone, place the cylinder — used for the projection — so that the central meridian of the zone (3° apart from the West and East sides of the zone) belongs to the section of the cylinder (see Fig. 9).

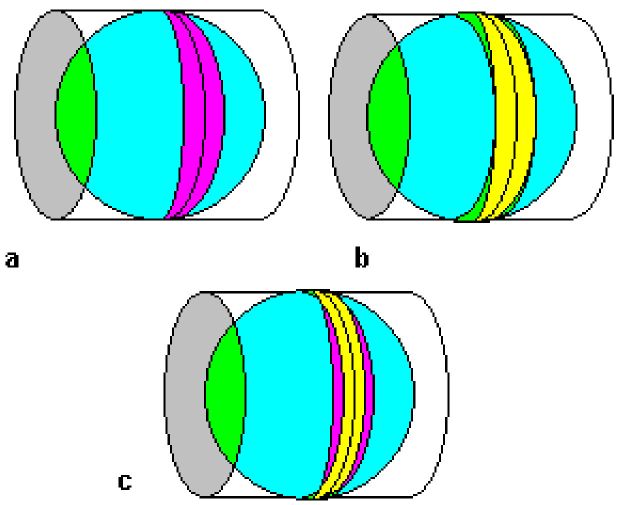


Figure 9: **Position of the cylinder used for the projection**. For each zone, the cylinder is turned by 6° around the Earth's rotation axis. (a): a cylinder touching the globe at the central meridian of a longitude zone lies entirely outside the Earth, and areas away from the central meridian project larger than on the globe. (b): a cylinder that touches the outer edges of the zone lies entirely inside the earth within the zone, and areas within the zone project smaller than their true size on the globe. (c): the scale is actually set to be the best overall compromise. We not only want the grid to be useful for specifying location, but we want distances measured on the grid to be as close as possible to distances on the ground.

- make the projection for each zone (complicated mathematical formulas, impossible to visualize by pure geometry) and see the result in Fig. 8.

2.2.2 Northings and Eastings

A square grid is superimposed on each zone. It's aligned so that vertical grid lines are parallel to the central meridian. UTM grid coordinates are expressed as a distance in meters to the east, referred to as the *easting*, and a distance in meters to the north, referred to as the *northing*.

Eastings. UTM easting coordinates are referenced to the central meridian of the considered zone. The central meridian is assigned an easting value of 500 000 meters East. Since this 500 000 m value is arbitrarily assigned, eastings are sometimes referred to as "false eastings". An easting of zero will never occur, since a 6 wide zone is never more than 674 000 meters wide. Minimum and maximum easting values are: 160 000 mE and 834 000 mE at the equator and 465 000 mE and 515 000 mE at 84° N.

Northings. UTM northing coordinates are measured relative to the equator. For locations north of the equator the equator is assigned the northing value of 0 meters North. To avoid negative numbers, locations south of the equator are made with the equator assigned a value of 10 000 000 meters North. Some UTM northing values are valid both north and south of the equator. In order to avoid confusion the full coordinate needs to specify if the location is north or south of the equator. Usually this is done by including the letter for the latitude band.

3 From UTM coordinates to regional cartesian coordinates

By construction, the Mercator representation of a sphere (or a quasi-spherical ellipsoid) is *conformal*: on any "region" (small portion of the sphere), it is approximately the combination of a scaling and a rotation of the axes. With the usual (equatorial) projection, the meridians and parallels transform into vertical and horizontal axes, and there is no rotation with respect to the usual geographic frame; with the Transverse Mercator system, the placeholder of the "equator" is now the central meridian of the zone, and the new "transverse meridians" and "transverse parallels" are no more lines of constant latitude or longitude: the transformation from (Easting, Northing) to regional cartesian coordinates implies a rotation (except on the central meridian of the UTM zone). The principle of the transformation is illustrated on Fig. 10. Of course, the vertical cartesian coordinate is the altitude corrected for the curvature of the Earth.

In a small region around a given central point, the transformation from UTM (Easting E, Northing N, altitude A) to regional cartesian coordinates (x towards East, y towards North, z vertical) with origin at (E_0, N_0, A_0) may be expressed in a very simple form, neglecting second order terms:

$$x = (1+\beta)(E-E_0) + \alpha(N-N_0) \tag{1}$$

$$y = (1+\beta)(N-N_0) - \alpha(E-E_0) \tag{2}$$

$$z = A - A_0 - \frac{(E-E_0)^2 + (N-N_0)^2}{2R} \tag{3}$$

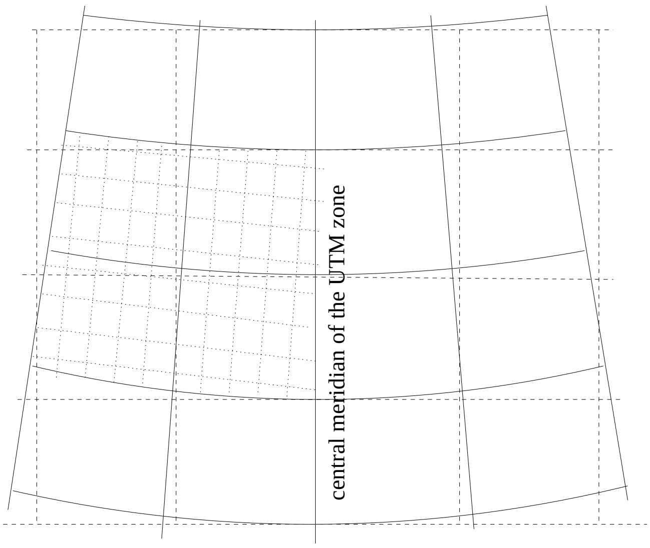


Figure 10: Top view of a small region of the Earth (exaggerating the distortions). Solid lines: meridians and parallels; dashed: UTM coordinates; dotted: local cartesian coordinates.

where R is the radius of the Earth[1], and conversely:

$$E = E_0 + (1-\beta)x - \alpha y \tag{4}$$
$$N = N_0 + (1-\beta)y + \alpha x \tag{5}$$
$$A = A_0 + z + \frac{x^2+y^2}{2R} \tag{6}$$

The scaling factor β is optimized to be 1 in average in the UTM zone; the rotation angle α varies between $-\delta \sin\lambda$ and $\delta \sin\lambda$, where λ is the latitude, and $2\delta = 6$ deg is the width of the zone;. Let us also note that β increases with the altitude by a term A/R.

Let us define the "center" of the site of Pampa Amarilla (by convention) as the point of **latitude 35.25 S, longitude 69.25 W** (or equivalently: zone 19H, Northing 6099203.68, Easting 477256.66) and **altitude 1400 m a.s.l**. At this point one finds $\beta \simeq 6\times10^{-4}$ (in other terms 1 "UTM meter" represents 1.0006 "true" meters), and $\alpha \simeq 2.6\,\mathrm{mrad}$.

3.1 Parametrization for the AUGER Site

To define the values of parameters that minimize the distortions of the parametrization defined by the formulae (1,2,3) over the AUGER region, we have computed exactly the transformation UTM → (latitude, longitude) → local cartesian for the points of a grid around the AUGER center (± 35 km in Easting, ± 30 km in Northing, with 5 km spacing) and we have determined α, β and $\gamma = 1/2R$ from a least squares fit over the full grid. The results are shown of Fig. 11 and 12: within the area covered by the Ground Array and the Telescopes, the distortions of the parametrization are kept within ±40 cm in x, y, z; moreover they are smooth functions of the position, then the differential distortions over a few kilometers (typical extension of a shower) are absolutely negligible.

[1] the fact that the Earth is an ellipsoid is negligible in the quadratic term; anyway at medium latitudes the two radii of curvature (along NS or WE directions) are almost equal.

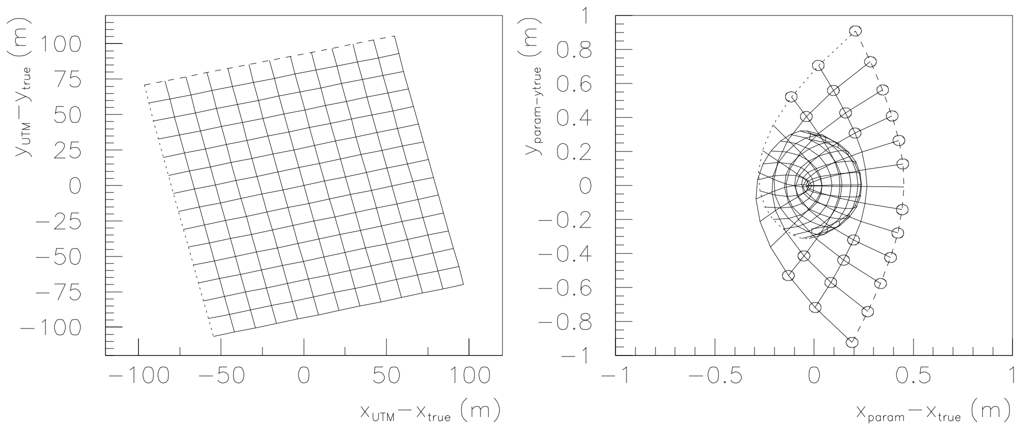


Figure 11: Distortions on horizontal coordinates on 15 × 13 points of a grid (70 × 60 km^2) around AUGER center (69.25 W, 35.25 S). Left: differences between "raw" UTM (without rescaling and rotation) and true cartesian coordinates with axes along WE and SN directions at the center. The dashed line is the southern bound, the dotted line is the western bound. Right: differences between parametrized coordinates and true ones. By chance, the points with the largest distortions (circles) are outside the Ground Array area (NW and SW corners of the grid)

The fitted parameters are:
$\alpha = 2.52 \times 10^{-3}$; $\beta = 6.03 \times 10^{-4}$; $\gamma = 7.853 \times 10^{-8}\,\mathrm{m}^{-1}$ (i.e. $R = 6368\,\mathrm{km}$)

3.2 Regional and Local Coordinates

Let us call (x, y, z) the coordinates in the regional cartesian frame, defined at the center C. At a point (x, y), the direction of the local vertical differs by $\sqrt{x^2 + y^2}/R$ from the regional z axis and the local North direction at fixed latitude λ differs by $x \tan\lambda/R$ from the regional y axis: these differences are not negligible with respect to the angular precision expected on the shower reconstruction.

To simplify the transformation, we can define a *local* system at any point P of the region, through the following transformation of coordinates and unit vectors:

$$x' = x \quad ; \quad y' = y \quad ; \quad z' = z + \frac{x^2 + y^2}{2R}$$
$$\vec{u}'_x = \vec{u}_x - \frac{x}{R}\,\vec{u}_z \quad ; \quad \vec{u}'_y = \vec{u}_y - \frac{y}{R}\,\vec{u}_z \quad ; \quad \vec{u}'_z = \frac{x}{R}\,\vec{u}_x + \frac{y}{R}\,\vec{u}_y + \vec{u}_z$$

(with a normalization factor on the unit vectors if needed for mathematical consistency).

At first order, this is equivalent to a rotation around an *horizontal* axis perpendicular to CP, followed by a translation such that the horizontal coordinates of C are still (0,0). The local vertical is along $\vec{u}'_z$, and the local origin is such that z' represents the vertical height above the *local* altitude A_0 (see Fig. 13). In other terms, we use a "flexible" cartesian layer, applied onto a small region of the earth with minimal deformation (no shear).

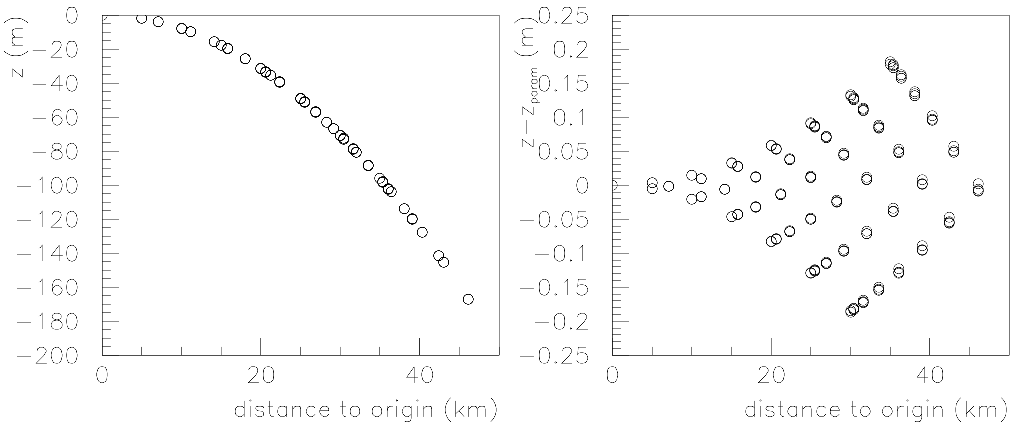


Figure 12: Distortion on the vertical coordinate, on the points of the grid. Left: difference between the altitude and the true cartesian z. Right: differences after quadratic correction.

This new system is orthonormal within second order corrections $(x^2 + y^2)/2R^2$ on the angles (in radians), and on the coordinates (in relative value), that is less than 1.2×10^{-5} everywhere in the region.

The advantages of choosing local coordinates (x, y, z') are:

- the coordinates of the stations and the telescopes may be computed once for all; changes of coordinates and angles needed when combining two telescopes, or in the hybrid reconstruction, are very simple.
- z' is related to the "true" altitude and the zenith angle θ is physically the angle with the z' axis. Over a few kilometers (the extension of a shower at ground), no corrections need to be done.

The price to pay is that the local North is not exactly along $\vec{u}'_y$. This may bias the description of the magnetic field, but a rotation of 5 mrad has probably quite negligible consequences on the visible magnetic effects.

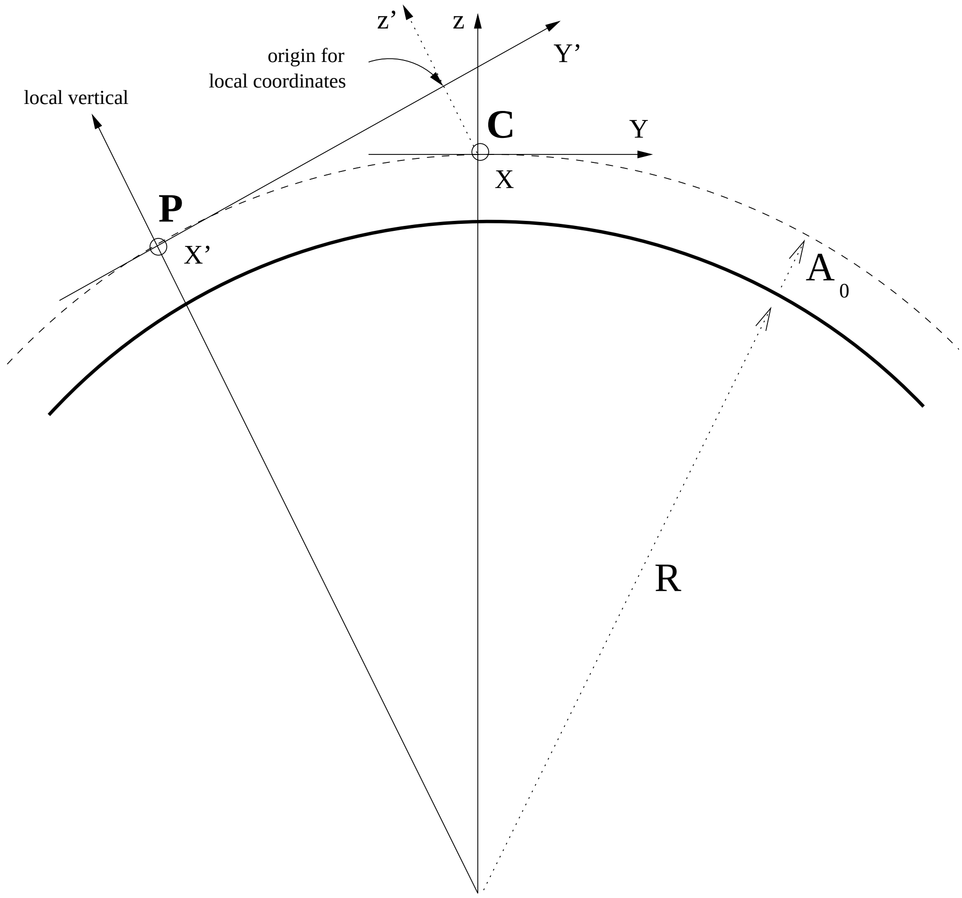


Figure 13: Transformation from regional frame at C to the local frame at P, as seen in a meridian plane containing C and P (with exaggerated Earth curvature). For clarity, the axes x and y are rotated into X and Y in such a way that Y is along CP and X perpendicular to it. In the region of interest, the origin for local coordinates is on the same vertical as C within a negligible error.

Appendix

A Conversion UTM $\longleftrightarrow$ geodetic longitude, latitude

In all these (horrible) formulas, a is the semi-major axis and ϵ is the excenticity of the ellipsoid ($\epsilon^2 = 1 - (b/a)^2$), which can be WGS84, ED50, NTF... The conversion programs (in C++) can be downloaded at:
`http://cdfinfo.in2p3.fr/~revenu/`

A.1 geodetic longitude, latitude $\longrightarrow$ UTM

Given the geodetic latitude λ and the longitude ϕ, we can compute the Easting and the Northing in the corresponding UTM zone (which is trivial to determinate).

The Eastings and Northings are given by:

$$E = FE + k_0 N \left(A + (1 - T + C)\frac{A^3}{6} + (5 - 18T + T^2 + 72C - 58\epsilon'^2)\frac{A^5}{120} \right)$$

$$N = FN + k_0 \left(M - M_0 + N \tan\phi \left(\frac{A^2}{2} + (5 - T + 9C + 4C^2)\frac{A^4}{24} + (61 - 58T + T^2 + 600C - 330\epsilon'^2)\frac{A^6}{720} \right) \right)$$

where FE and FN are false eastings and northings, can be taken as 0, and

$$T = \tan^2\phi, \quad C = \frac{e^2}{1-e^2}\cos^2\phi = \epsilon'^2\cos^2\phi, \quad A = (\lambda - \lambda_0)\cos\phi$$

$$M = a\left(\left(1 - \frac{1}{4}\epsilon^2 - \frac{3}{64}\epsilon^4 - \frac{5}{256}\epsilon^6 - \frac{175}{16384}\epsilon^8 + ...\right)\phi - \left(\frac{3}{8}\epsilon^2 + \frac{3}{32}\epsilon^4 + \frac{45}{1024}\epsilon^6 + \frac{105}{4096}\epsilon^8 + ...\right)\sin 2\phi \right.$$
$$+ \left(\frac{15}{256}\epsilon^4 + \frac{45}{1024}\epsilon^6 + \frac{525}{16384}\epsilon^8 + ...\right)\sin 4\phi - \left(\frac{35}{3072}\epsilon^6 + \frac{175}{12288}\epsilon^8 + ...\right)\sin 6\phi$$
$$\left. + \left(\frac{315}{131072}\epsilon^8 + ...\right)\sin 8\phi + ... \right).$$

M_0 is the value of M for $\phi = \phi_0$, origin of the latitude (equal to 0 if on the equator). λ_0 is the longitude of the origin (0 if on the Greenwich meridian).

A.2 UTM $\longrightarrow$ geodetic longitude, latitude

$$\phi = \phi_1 - \frac{N_1\tan\phi_1}{\rho_1}\left(\frac{D^2}{2} - \left(5 + 3T_1 + 10C_1 - 4C_1^2 - 9\epsilon'^2\right)\frac{D^4}{24} \right.$$
$$\left. + \left(61 + 90T_1 + 298C_1 + 45T_1^2 - 252\epsilon'^2 - 3C_1^2\right)\frac{D^6}{720} \right)$$

$$\lambda = \lambda_0 + \frac{1}{\cos\phi_1}\left(D - (1 + 2T_1 + C_1)\frac{D^3}{6} + \left(5 - 2C_1 + 28T_1 - 3C_1^2 + 8\epsilon'^2 + 24T_1^2\right)\frac{D^5}{120} \right)$$

where:

$$N_1 = \frac{a}{\sqrt{1 - \epsilon^2\phi_1}}, \quad \rho_1 = \frac{a(1-\epsilon^2)}{(1 - \epsilon^2\sin^2\phi_1)^{3/2}}, \quad \epsilon_1 = \frac{1 - \sqrt{1-\epsilon^2}}{1 + \sqrt{1-\epsilon^2}}$$

$$\mu_1 = \frac{M_1}{a\left(1 - \frac{1}{4}\epsilon^2 - \frac{3}{64}\epsilon^4 - \frac{5}{256}\epsilon^6 - \frac{175}{16384}\epsilon^8\right)}$$

$$M_1 = M_0 + \frac{N - FN}{k_0}, \quad D = \frac{E - FE}{N_1 k_0}$$

$$T_1 = \tan^2\phi_1, \quad C_1 = \epsilon'^2\cos^2\phi_1, \quad \epsilon'^2 = \frac{\epsilon^2}{1-\epsilon^2}$$

$$\phi_1 = \mu_1 + \left(\frac{3}{2}\epsilon_1 - \frac{27}{32}{\epsilon_1}^3 + ...\right)\sin 2\mu_1 + \left(\frac{21}{16}{\epsilon_1}^2 - \frac{55}{32}{\epsilon_1}^4 + ...\right)\sin 4\mu_1$$
$$+ \left(\frac{151}{96}{\epsilon_1}^3 + ...\right)\sin 6\mu_1 + \left(\frac{1097}{512}{\epsilon_1}^4 + ...\right)\sin 8\mu_1$$

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