CMB polarization

B. Revenu

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1 Why is the CMB polarized?

1.1 Thomson diffusion

At the epoch of last scattering, the universe was mainly filled with baryons, electrons, photons and dark matter (DM). DM is already decoupled from the others species and the coupling is very strong between baryons, electrons and photons because of the Thomson diffusion. The associated cross section is given by:

$$\frac{d\sigma}{d\Omega} \propto |\hat{\epsilon}.\hat{\epsilon}'|^2$$

where $\hat{\epsilon}$ and $\hat{\epsilon}'$ are two vectors caracterizing the state of polarization of the incoming and outgoing photons respectively. The scalar product between the two vectors of polarization shows that the diffusion favorizes an outgoing photon with a state of polarization close to that of the incoming photon. For instance, in Fig. 1, the x' component of the electric field will not be attenuated after the diffusion

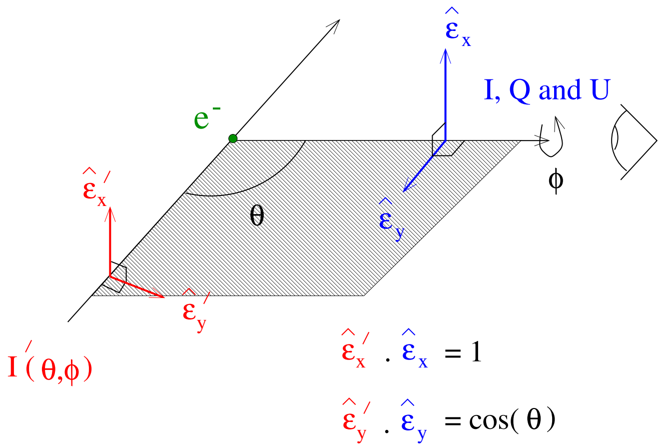

Figure 1: Thomson diffusion of a photon by an electron.

since $\hat{\epsilon}'_x.\hat{\epsilon}_x = 1$ whereas the y' component will be attenuated by a factor $\hat{\epsilon}'_y.\hat{\epsilon}_y = \cos\theta$. Schematically, the x polarization is transmitted and the y polarization is attenuated (or zeroed depending on the value of θ). No net polarization is produced if the electron is surrounded by an isotropic and homogeneous distribution of unpolarized photons.

This result is wrong if the photon distribution is not isotropic, as it is the case at last scattering epoch. To qualitatively understand this point, please refer to Fig 2. The diffusion of a photon coming from the left will transmit the vertical polarization component and that of a photon coming from the right will transmit the horizontal polarization component. If the "left" and "right" intensities are equal, no polarization will be produced. On the contrary, if the intensities are different (let's say more intense for the photons coming from the left), the vertical component of the electric field of the outgoing photon will be greater than the horizontal component, resulting in linear polarization. In this way, linear polarization can be produced if the photon distribution present a *quadrupolar anisotropy*. Such anisotropy exists at last scattering: density distribution, vorticity of the photon-baryon fluid and gravitational waves, all present a quadrupolar anisotropy. This is the reason why we expect the CMB to be polarized. The rate of polarization in the CMB does not depends strongly on the cosmological

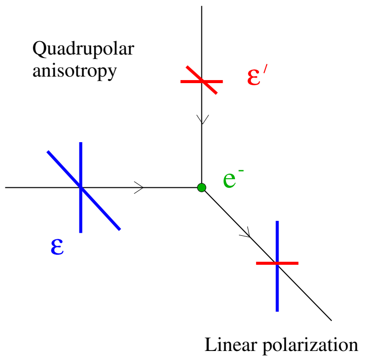

Figure 2: Diffusion of photons by an electron. A quadrupolar anisotropy in the photon distribution generates a linear polarization.

model and is expected to be at a level of approximatively 10% of the temperature fluctuations. We will explain in the following what is precisely the rate of polarization.

2 Quantitative description of polarization

2.1 The Stokes parameters

For a detailed review of the subject, see Chandrasekhar (1960). Polarized light can be described by the *Stokes parameters* I, Q, U and V. Not circular polarization can be generated through Thomson scattering so we can forget V, which is specific to circular polarization. We can write the electromagnetic vibration as:

$$\begin{aligned} E_x &= E_{0x}\sin(\omega t - \varphi_x) \\ E_y &= E_{0y}\sin(\omega t - \varphi_y) \end{aligned}$$

The light is polarized if $E_{0x}/E_{0y} = Cte$ and if $\varphi_x - \varphi_y = Cte$. The Stokes parameters are defined by:

$$\begin{aligned} I &= E_{0x}^2 + E_{0y}^2 && (1) \\ Q &= E_{0x}^2 - E_{0y}^2 = I\cos 2\xi && (2) \\ U &= 2E_{0x}E_{0y}\cos(\varphi_x - \varphi_y) = I\sin 2\xi && (3) \end{aligned}$$

where ξ is the direction of the polarization. We have the following relation between I, Q and U:

$$I^2 = Q^2 + U^2.$$

We describe the light made of a mixture of unpolarized and polarized lights with two sets of Stokes parameters:

$$\begin{aligned} &\left\{I - \left(Q^2+U^2\right)^{1/2}, 0, 0\right\} && \text{for the unpolarized component} \\ &\left\{\left(Q^2+U^2\right)^{1/2}, Q, U\right\} && \text{for the polarized component.} \end{aligned}$$

I stands for the *total* intensity. The *polarized intensity* is:

$$I_P = \sqrt{Q^2+U^2}.$$

When one says that the CMB is polarized at a level of 10% of the temperature fluctuations, one means that $I_p \sim 10\% \, (\delta T/T)$, that is to say of the order of μK.

2.2 Measurement of the Stokes parameters with a polarimeter

The Stokes parameters are not invariant by rotation. Consider a system of coordinates made of two perpendicular axes $\hat{e}_x$ et $\hat{e}_y$ (see FIG. 3). In this system of coordinates, we define the Stokes parameters with the equations 1, 2 and 3. If the axes are turned by an angle ϕ, the Stokes parameters transform as:

$$I' = I \tag{4}$$
$$Q' = Q\,\cos 2\phi + U\,\sin 2\phi \tag{5}$$
$$U' = -Q\,\sin 2\phi + U\,\cos 2\phi. \tag{6}$$

We see that I is a scalar (invariant under rotations) whereas Q and U are not invariant. They are "spin-2" quantities. It means that the 2-vector (Q, U) transforms as:

$$(Q \pm iU)' = e^{\mp 2i\phi}(Q \pm iU). \tag{7}$$

A polarimeter making an angle α with $\hat{e}_x$ measures:

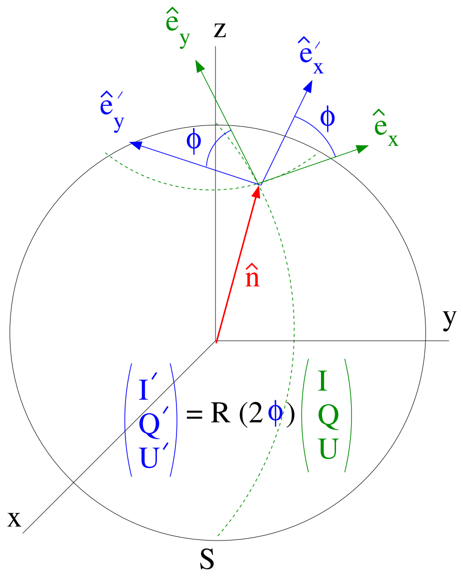

Figure 3: Transformation of the Stokes parameters under a rotation ϕ around $\hat{n}$.

$$M = \frac{1}{2}\left(I + Q\,\cos 2\alpha + U\,\sin 2\alpha\right). \tag{8}$$

So we need at least 3 polarimeters in order to measure the 3 Stokes parameters. The direction of these polarimeters is defined modulo π. There exists an optimized geometrical configuration for the polarimeters: they have to be at equal angular separation and they must cover the angle π. For instance, for 4 polarimeters, they must be placed at 0, $\pi/4$, $\pi/2$ and $3\pi/4$.

3 Analysis of a polarized sky

As in the case of the temperature, we can work with polarization on the sphere. Temperature is a scalar quantity so we use scalar functions to describe it on the sphere; these functions are the usual spherical harmonics. For polarization, we need the spin-2 spherical harmonics: ${}_{\pm 2}Y_\ell^m$. Given two sky maps of Q and U, we first compute the coefficients $a_{\pm 2\ell m}$ of these fields by:

$$Q \pm iU = \sum_{\ell m} a_{\pm 2\ell m} \ {}_{\pm 2}Y_\ell^m$$

or

$$a_{\pm 2\ell m} = \int \ (Q \pm iU)(\hat{\boldsymbol{n}}) \ {}_{\pm 2}Y_\ell^{m\,*}(\hat{\boldsymbol{n}}) \ \sin\theta \ d\theta \ d\phi.$$

From $Q \pm iU$, we can construct two scalar real fields, E and B. They are defined by:

$$\begin{aligned} E(\hat{\boldsymbol{n}}) &= \sum_{\ell m} a_{\ell m}^E \ Y_\ell^m(\hat{\boldsymbol{n}}) \\ B(\hat{\boldsymbol{n}}) &= \sum_{\ell m} a_{\ell m}^B \ Y_\ell^m(\hat{\boldsymbol{n}}) \\ \text{with } a_{\ell m}^E &= -\frac{a_{2\ell m} + a_{-2\ell m}}{2} \\ \text{and } a_{\ell m}^B &= i\ \frac{a_{2\ell m} - a_{-2\ell m}}{2} \end{aligned}$$

These new fields E and B are invariants under rotations and they describe completetly the polarization field on the 2-sphere. The associated power spectra are defined by:

$$\begin{aligned} C_\ell^E &= \frac{1}{2\ell+1} \sum_{m=0}^{+\infty} \left|a_{\ell m}^E\right|^2 = \left\langle \left|a_{\ell m}^E\right|^2 \right\rangle \\ C_\ell^B &= \frac{1}{2\ell+1} \sum_{m=0}^{+\infty} \left|a_{\ell m}^B\right|^2 = \left\langle \left|a_{\ell m}^B\right|^2 \right\rangle \end{aligned}$$

We have the following relations:

$$\begin{aligned} a_{\ell m}^{E\ *} &= (-1)^m \ a_{\ell,-m}^E \\ a_{\ell m}^{B\ *} &= (-1)^m \ a_{\ell,-m}^B \end{aligned}$$

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