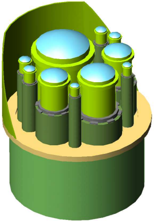
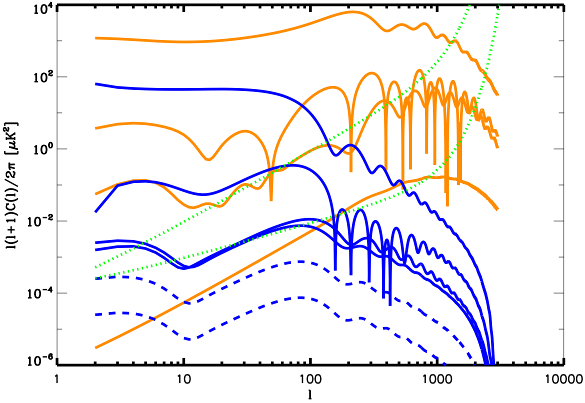


The search for B-modes: Underlying theory

Martin BUCHER, Laboratoire de Physique Théorique, Université Paris-Sud

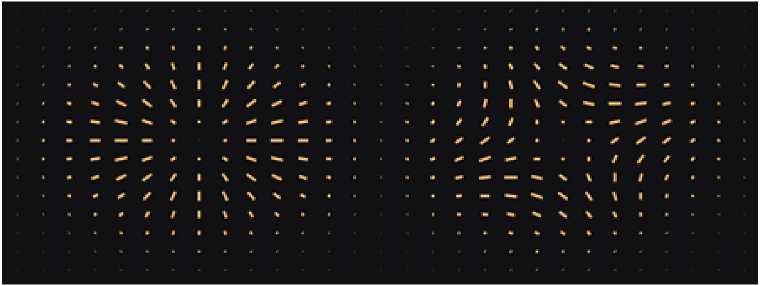
16 June 2008 / Bolometric Interferometry Workshop

Scalar and Tensor CMB Anisotropies with PLANCK



Red=scalar (top to bottom) TT, TE, EE, BB (lensing)
Blue=tensor (T/S=0.1) = TT, TE, EE, BB, dotted BB (T/S=0.01, 0.01)
Green PLANCK capabilities = top single alm, bottom agressive binning

E and B Mode Polarization



E mode B mode

$$\mathbf{Y}^{(E)}_{\ell m,ab} = \sqrt{\frac{2}{(\ell-1)\ell(\ell+1)(\ell+2)}}\left[\nabla_a\nabla_b - \frac{1}{2}\delta_{ab}\right] Y_{\ell m}(\hat{\Omega})$$

$$\mathbf{Y}^{(B)}_{\ell m,ab} = \sqrt{\frac{2}{(\ell-1)\ell(\ell+1)(\ell+2)}}\frac{1}{2}\left[\epsilon_{ac}\nabla_c\nabla_b + \nabla_a\epsilon_{bc}\nabla_c\right] Y_{\ell m}(\hat{\Omega})$$

Projection of « scalars, »« vectors »and « tensors »onto the celestial sphere

Under projection onto the celestial sphere :

$$(scalar)_3 \to (scalar)_2,$$

$$(vector)_3 \to (scalar)_2 + (vector)_2,$$

$$(tensor)_3 \to (scalar)_2 + (vector)_2.$$

There is no $(tensor)_2$ component. The E mode polarization is scalar ; the B mode is vector.

It follows that at linear order the scalar modes cannot generate any B mode polarization.

$\boxed{\text{Note crucial role of linearity assumption.}}$

Perturbations generated during inflation

$$\boxed{\hbar = c = 1, M_{pl}^{-2}} \qquad \delta\phi \approx H \qquad \frac{\delta\rho}{\bar\rho} \approx H\cdot\delta t, \quad \delta t \approx \frac{\delta\phi}{\dot\phi}$$

$$H\dot\phi \approx V_{,\phi}, \quad \dot\phi \approx V_{,\phi}/H, \quad H^2 \approx \frac{1}{M_{pl}^2}V, \frac{\delta\rho}{\bar\rho} \approx \frac{V^{3/2}[\phi(k)]}{M_{pl}^3\ V_{,\phi}}$$

Scalar perturbations :

$$\boxed{\mathcal{P}_S^{1/2}(k) \approx O(1)\cdot\frac{V^{3/2}[\phi(k)]}{M_{pl}^3\ V_{,\phi}[\phi(k)]}.}$$

Tensor perturbations :

$$\boxed{\mathcal{P}_T^{1/2}(k) \approx O(1)\cdot\frac{H}{M_{pl}} \approx O(1)\cdot\frac{V^{1/2}}{M_{pl}^2}}$$

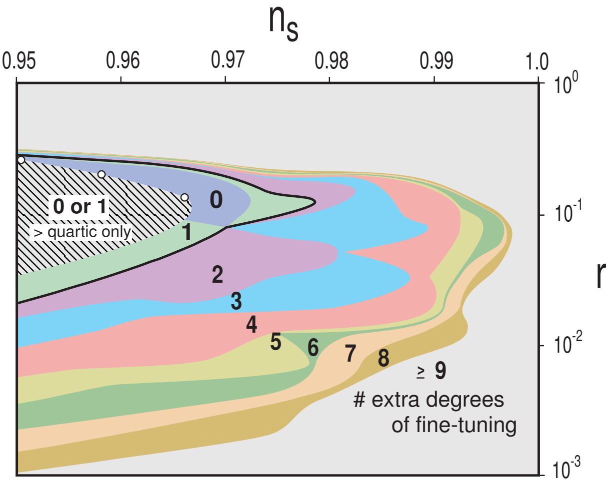
$\phi(k) \equiv$ value of ϕ at horizon crossing of the mode k

Reconstruction of the inflationary potential : the tensors measure the height of the potential, the scalars the slope.

Tests of inflation

- Order zero tests
  - Flatness, homogeneity, isotropy, no monopoles, entropy of observable universe
- Scalar perturbations
  - Scale invariance (approximate) (Harrison, Zeldovich, Peebles)
  - Gaussianity
  - Primordial character of comsological perturbations. No decaying modes observed.
- Tensor perturbations
  - Direct measure of the Hubble constant in the **very** early universe when a given mode left the horizon
  - New unique prediction of inflation

Expected (T/S) From Inflation ? (I)



From Boyle, Steinhardt and Turok.

Expected (T/S) From Inflation ? (II)

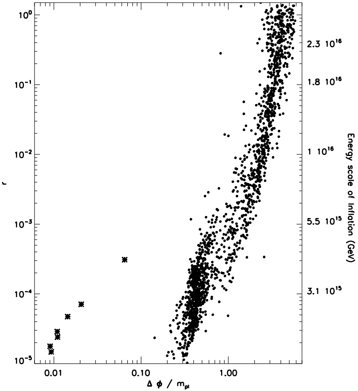
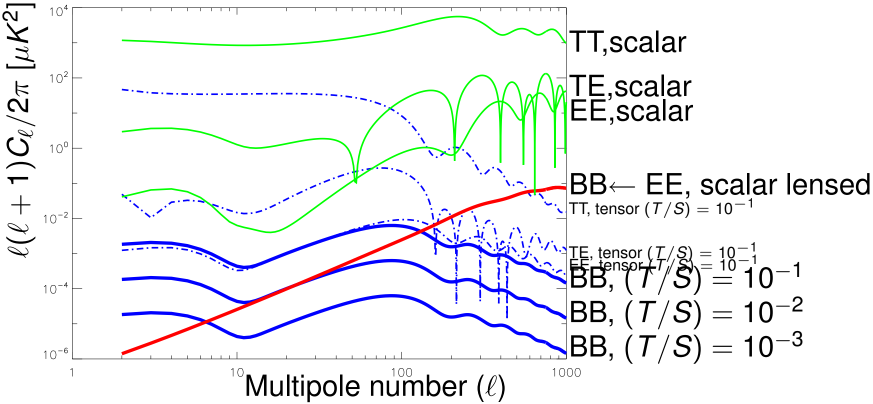
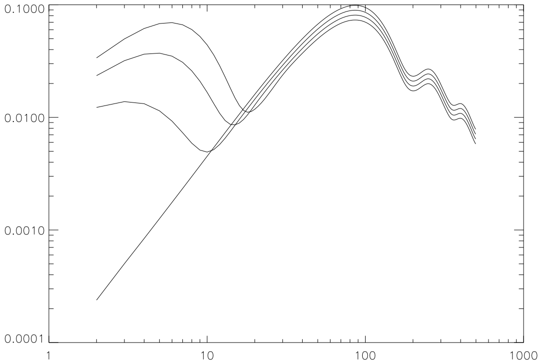


Figure produced by L. Verde, following closely the method of W. Kinney et al., Phys. Rev. D74, 023502 (2006) (astro-ph/0605338).

Inflationary Prediction for Scalar & Tensor Anisotropies

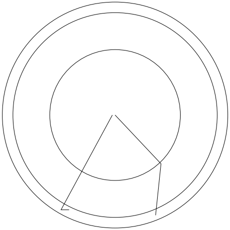


The Reionization Bump (I)



$\tau = 0.0, 0.5, 0.10, 0.15$ (bottom $\rightarrow$ top)

The Reionization Bump (II)



It turns out that

$$P \propto (1-\tau) d^2_{lastscatter} \frac{\partial^2 T}{\partial x^2}$$

is small compared to

$$P \propto \tau d^2_{reion} \frac{\partial^2 T}{\partial x^2}$$

even when τ is small.

Lensing of the E mode into the B mode — $(E^{scalar} + \Phi \rightarrow B^{scalar})$

(Flat sky approximation : $(\ell m) \rightarrow \boldsymbol{\ell}, \quad \boldsymbol{\theta}, \boldsymbol{\ell} \in \mathcal{R}^2$.)

$$\delta\boldsymbol{\theta} = (\boldsymbol{\nabla}\Phi), \qquad\qquad \delta T(\boldsymbol{\theta}) = (\boldsymbol{\nabla}\Phi)\cdot(\boldsymbol{\nabla}T).$$

$$\delta T(\boldsymbol{\ell}_F) = \int \frac{d^2\boldsymbol{\ell}_L}{(2\pi)^2}\,(-\boldsymbol{\ell}_L)\cdot(\boldsymbol{\ell}_F-\boldsymbol{\ell}_L)\;\Phi(\boldsymbol{\ell}_L)\;T(\boldsymbol{\ell}_F-\boldsymbol{\ell}_L).$$

$$\left\langle T(\boldsymbol{\ell})\;T(\boldsymbol{\ell}')\right\rangle = (2\pi)^2\;\delta^2(\boldsymbol{\ell}+\boldsymbol{\ell}')\;C^{TT}(\ell)$$

$$C^{TT}(\boldsymbol{\ell}_F) = \int \frac{d^2\ell_L}{(2\pi)^2}\,[\boldsymbol{\ell}_L\cdot(\boldsymbol{\ell}_F-\boldsymbol{\ell}_L)]^2\;C^{\Phi\Phi}(\boldsymbol{\ell}_L)\;C^{TT}(\boldsymbol{\ell}_I=|\boldsymbol{\ell}_F-\boldsymbol{\ell}_L|)$$

$$C^{BB}(\boldsymbol{\ell}_B) = \int \frac{d^2\ell_L}{(2\pi)^2}\,[\boldsymbol{\ell}_L\cdot(\boldsymbol{\ell}_F-\boldsymbol{\ell}_L)]^2\;\sin^2[2\Theta(\boldsymbol{\ell}_B,\boldsymbol{\ell}_E)]C^{\Phi\Phi}(\boldsymbol{\ell}_L)\;C^{EE}(\boldsymbol{\ell}_E=|\boldsymbol{\ell}_B-\boldsymbol{\ell}_L|)$$

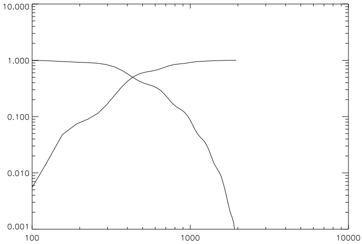
Lensing of the E mode into the B mode (II)

For small values of ℓ_B,

$$C^{BB}(\ell_B \approx 0) \sim \int_0^{\infty} \frac{d\ell}{\ell}\ \ell^6 C^{\Phi\Phi}(\ell)\ C^{EE}(\ell)$$

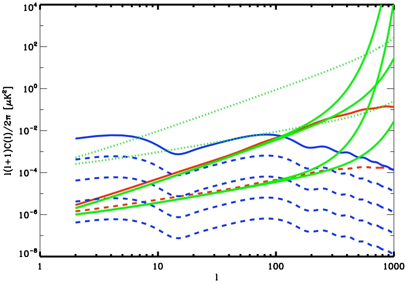
The bulk of the integral is concentrated around $\ell \approx 300$.
White noise spectrum up to $\ell \lesssim 300$

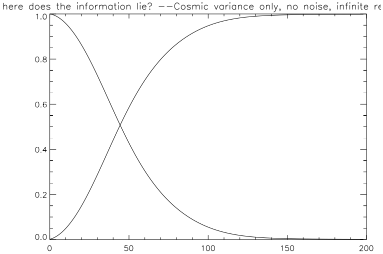
$$F(\ell_{max}) = \frac{\int_0^{\ell_{max}} \frac{d\ell}{\ell}\ \ell^6 C^{\Phi\Phi}(\ell)\ C^{EE}(\ell)}{\int_0^{\infty} \frac{d\ell}{\ell}\ \ell^6 C^{\Phi\Phi}(\ell)\ C^{EE}(\ell)}$$



Where does the information on (T/S) lie ?

$$\delta c_{\ell,measurable} \sim \frac{c_{\ell,parasite} + n_\ell}{\ell}$$

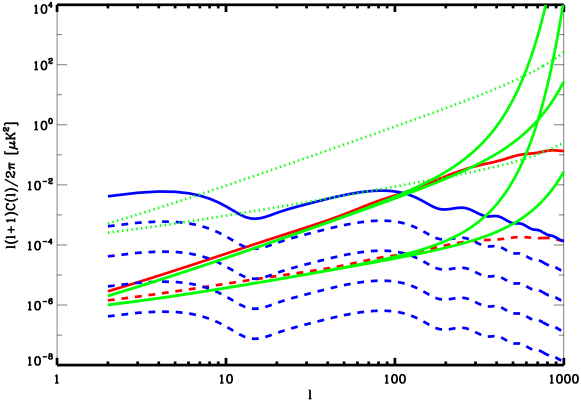




Conclusion : Approx. 80 % of the information (excluding the reionization bump) lies between $\ell = 20$ and $\ell = 80$.

The detection of B modes

The B mode is that component that cannot be represented as a double gradient on the celestial sphere. In the linear approximation there is no B mode component arising from scalar degrees of freedom. The presence of the B mode would unambiguously signal the presence of primordial gravitational waves.



BPol Capabilities : Fisher matrix analysis

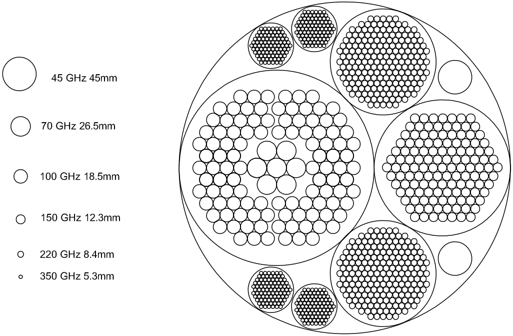
	Fiducial model	BPol 20′ fwhm, $(5\mu\mathrm{K}\cdot\mathrm{arcmin})^2$ (No reionization)					
		TT	TE	EE	BB	BB+EE	All
r	0.0	1.57×10^{-1}	7.19×10^{-2}	1.32×10^{-2}	1.58×10^{-3}	6.71×10^{-4}	6.60×10^{-4}
$\delta A_S/A_S$	1.00×10^{0}	3.14×10^{-1}	6.15×10^{-3}	3.81×10^{-3}	2.70×10^{0}	3.57×10^{-3}	1.27×10^{-3}
H	7.20×10^{1}	9.96×10^{-2}	6.62×10^{-2}	9.91×10^{-2}	5.94×10^{0}	8.06×10^{-2}	3.93×10^{-2}
Ω_b	5.00×10^{-2}	3.43×10^{-4}	2.68×10^{-4}	6.01×10^{-4}	3.91×10^{-2}	4.88×10^{-4}	1.39×10^{-4}
Ω_c	2.50×10^{-1}	3.73×10^{-5}	5.03×10^{-5}	1.34×10^{-4}	3.45×10^{-2}	1.28×10^{-4}	2.68×10^{-5}
n_s	1.00×10^{0}	3.47×10^{-3}	6.69×10^{-3}	3.98×10^{-3}	1.49×10^{-1}	3.74×10^{-3}	1.84×10^{-3}
Ω_k	0.0	5.53×10^{-4}	4.37×10^{-4}	3.08×10^{-4}	3.04×10^{-2}	3.01×10^{-4}	1.87×10^{-4}
τ	0.0	1.73×10^{-1}	8.00×10^{-4}	1.19×10^{-5}	1.60×10^{0}	5.96×10^{-6}	5.96×10^{-6}

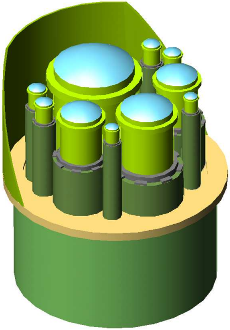
TAB.: 1σ errors resulting from the fit of an eight parameter family of cosmological models for a detector rms white noise amplitude of $5\mu\mathrm{K}\cdot\mathrm{arcmin}$ and a resolution of 20 arc minute. (Note that $\delta A_S/A_S$ denotes the variation of the normalization of the scalar power spectrum as compared to that inferred from COBE data.) This table

Requirements for a B Mode Polarization Mission

- **Sensitivity in the neighborhood of** $5\mu K \cdot arcmin$. Chosen equal to contaminant lensing signal. Approximately 13 times more sensitive than PLANCK.
- **Angular resolution of** $\approx 30 - 60'$ Lower than PLANCK ; therefore, one does not necessarily need a large mirror deployed in space.
- **Excellent control of systematic errors.** (These should not exceed the intrinsic random detector noise.)
- **Full sky coverage.** In particular important for mapping the low-ℓ modes of the re-ionization bump.
- **Sufficiently broad frequency coverage to remove galactic foreground components (synchrotron radiation, spinning dust (?), polarized dust emission.**

The B-Pol Instrument



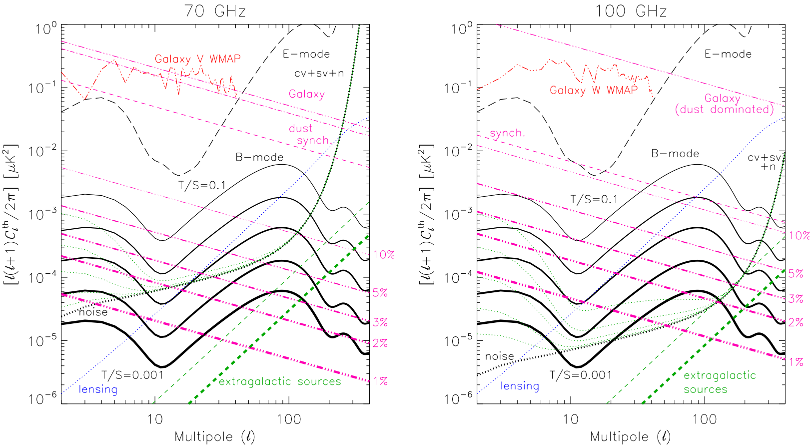


B-Pol Characteristics Summary

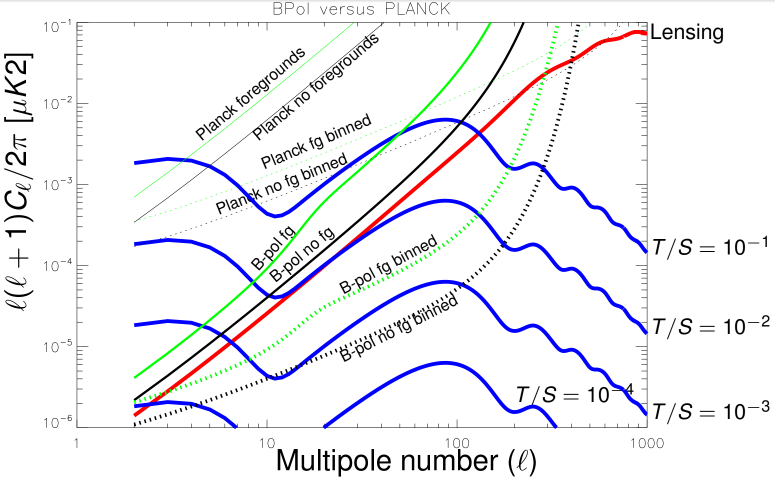
Freq. band (GHz)	45	70	100	143	217	353
$\Delta\nu$	30%	30%	30%	30%	30%	30%
ang. res.	15°	68'	47'	47'	40'	59'
# horns	2	7	108	127	398	364
det. noise ($\mu K \cdot \sqrt{s}$)	57	33	53	53	61	119
Q & U sens. ($\mu K\cdot$ arcmin)	33	23	8	7	5	10
Tel. diam. (mm)	45	265	265	185	143	60

$$c_{noise}^2 = 4\times\frac{4\pi}{t_{miss}N_{det}}\times s_{det}^2 = (5\mu K\cdot arcmin)^2\times\left(\frac{2\ years}{t_{miss}}\right)\times\left(\frac{s_{det}}{50\mu K\sqrt{s}}\right)^2 \quad (1)$$

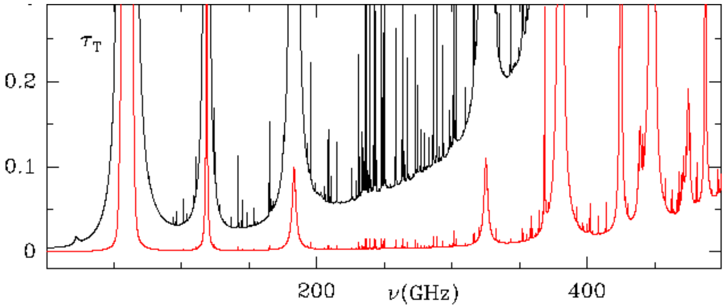
Foregrounds relative to signal



(Courtesy of Carlo Burigana)


BPol versus PLANCK
Lensing
Planck foregrounds
Planck no foregrounds
Planck fg binned
Planck no fg binned
B-pol fg
B-pol no fg
B-pol fg binned
B-pol no fg binned
$T/S = 10^{-1}$
$T/S = 10^{-2}$
$T/S = 10^{-3}$
$T/S = 10^{-4}$
$\ell(\ell+1)C_\ell/2\pi\ [\mu K2]$
Multipole number (ℓ)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1
10
100
1000

Observations from the ground (I)



Atmospheric interference. Calculated optical depth through the atmosphere for a good ground-based site like the South Pole or Dome-C in Winter (black) and at balloon altitude (red). Frequency bands for sub-orbital experiments must be carefully chosen to avoid the emission by molecular lines. Moreover, emission from oxygen lines is circularly polarized and care must be taken to avoid a significant polarized signal from the tails of these lines.

Observations from the ground (II)

Numerous CMB polarization experiments from the ground and balloons at various stages : QUaD, BICEP ; BRAIN, CLOVER, EBEx, PAPPA, PolarBear, QUIET and Spider

- Far side lobes
- Scanning strategy (must scan at constant zenith angle)
- Polarization from interaction of Zeeman splitting by earth's magnetic field of oxgen lines. Atmospheric backscattering (very polarized) could be a serious problem. [See L. Pietranera et al., "Observing the CMB polarization through ice," MNRAS 346, 645 (2007)].
- Lack of stability and partial sky coverage