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Introduction to interferometry with bolometers:

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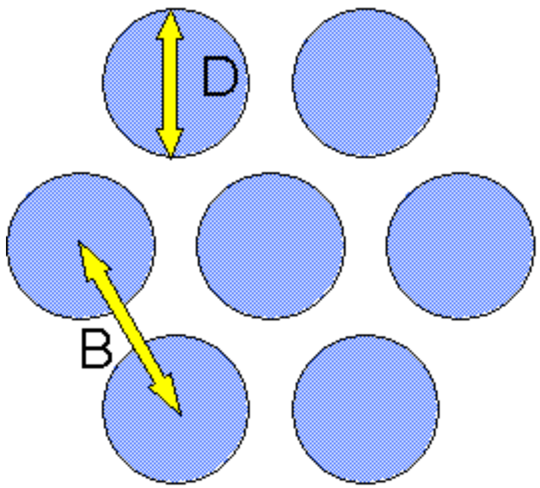
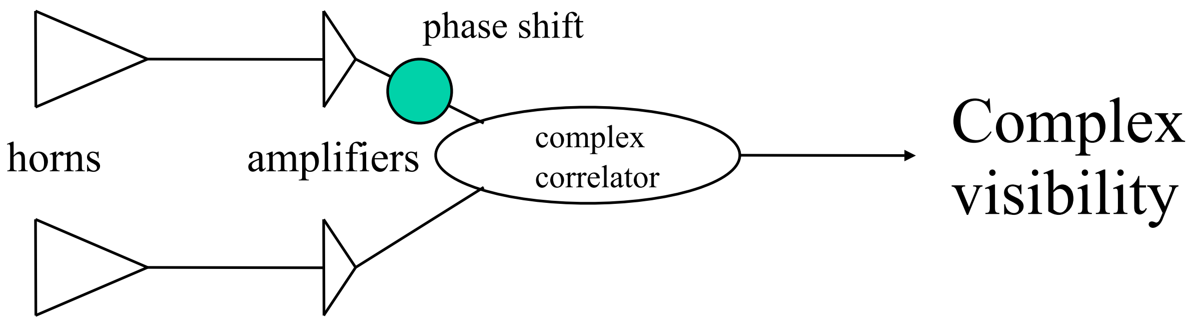


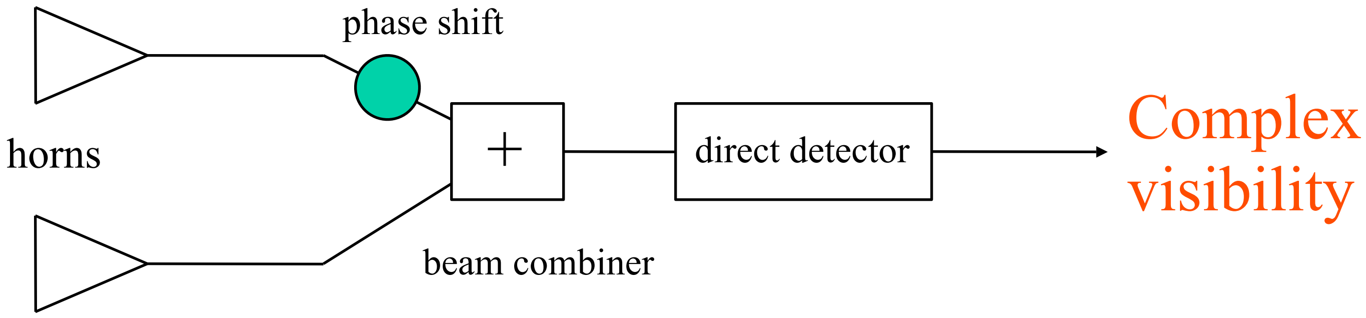
Interferometry (heterodyne)

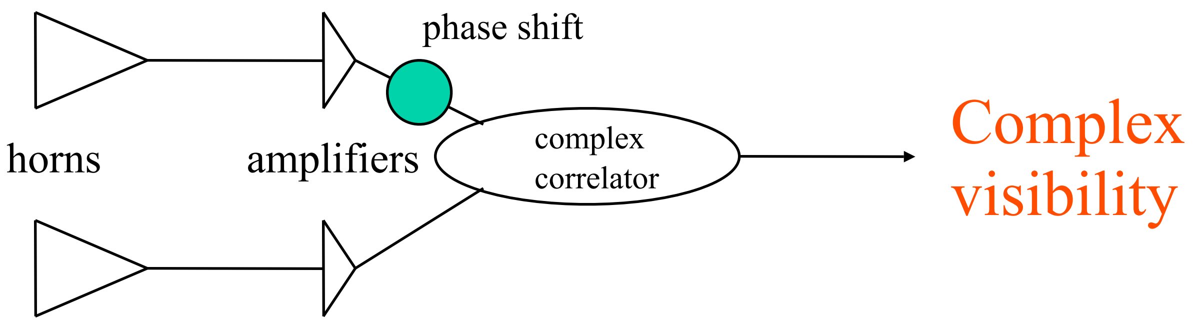
- In general we have i=1,...,n single dishes (with a single or dual receiver)
- Telescopes of diameter D spaced with baselines B_{ij}
- Form n(n-1)/2 baselines each requiring a correlator to recover the visibility



Bolometric interferometers are forced to use ***passive correlators*** and ***direct detectors***

∴ Have to use **adding** interferometry.

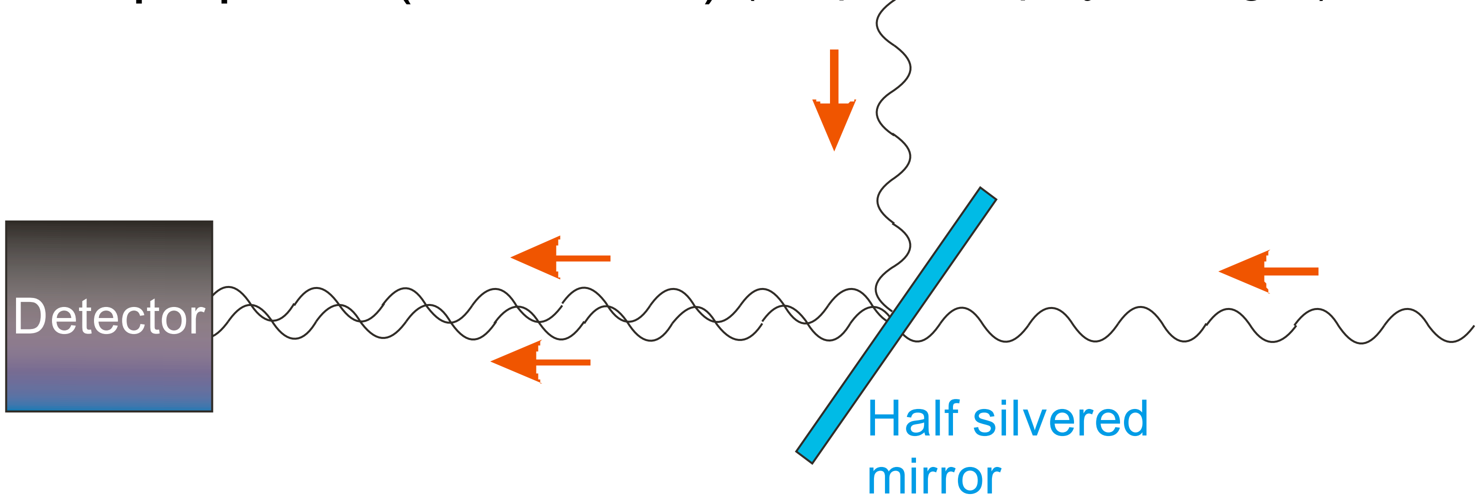




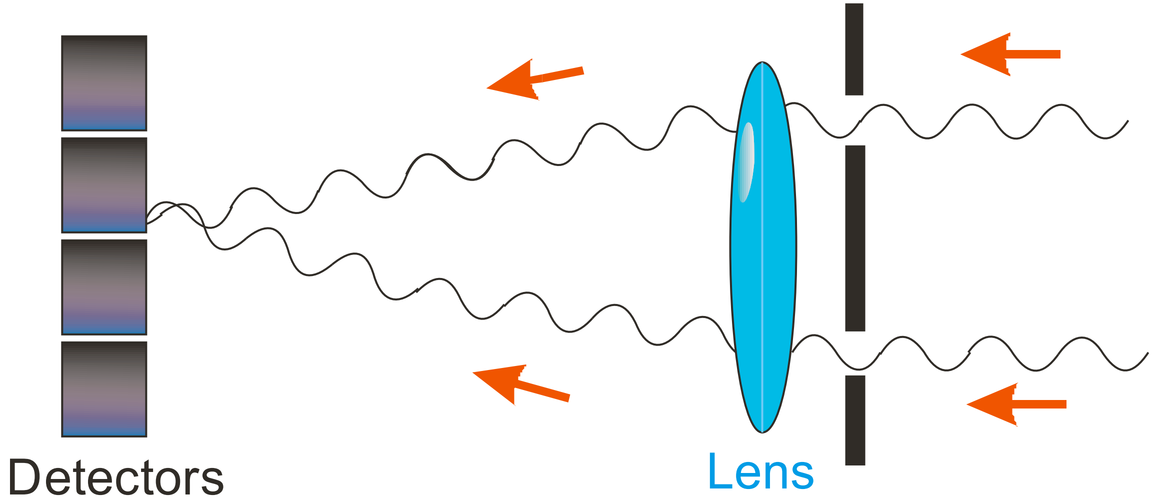
Heterodyne	**Bolometric**
Amplifier/mixer (low noise elem.)	Nothing
Digital/analogue correlator	Beam combiner (passive)
Diodes	Direct detectors (low noise elem.)

Two ways of doing adding interferometry:

1. Pupil-plane (Michelson) (temporal display of fringes)



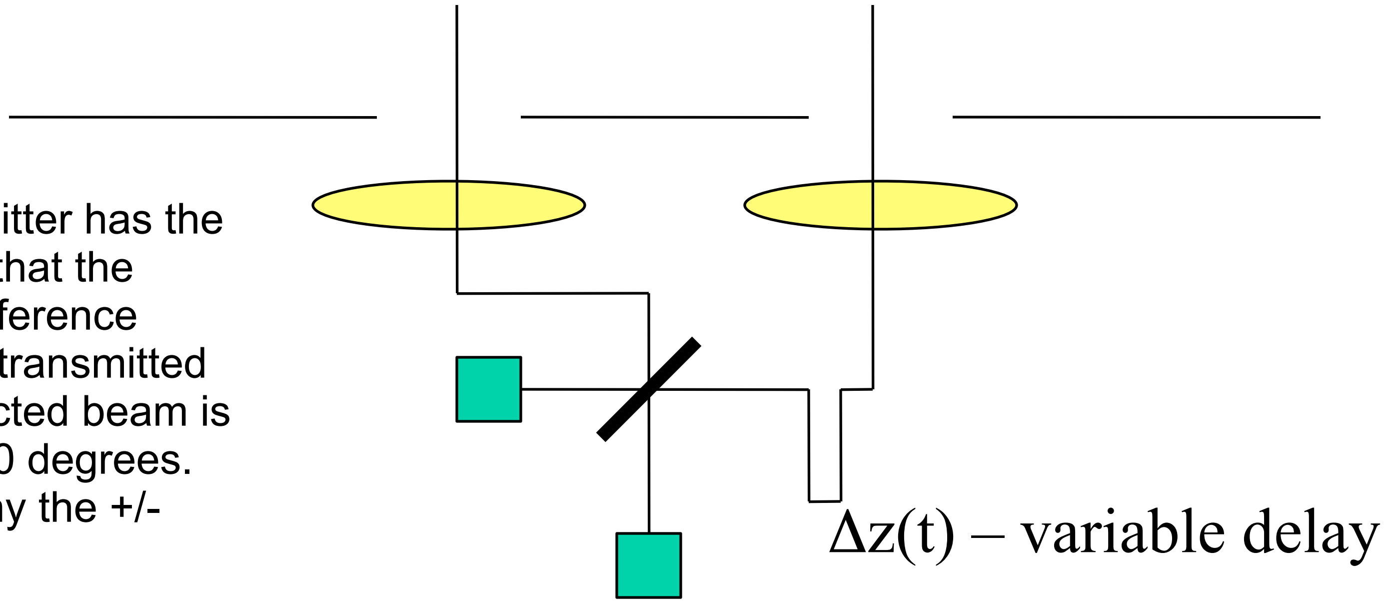
2. Image-Plane (Fizeau) (spatial display of fringes)



Pupil plane interferometry (fringes are temporally displayed)

Method of combining the two beams using polarizers (or half-silvered mirrors) and then focusing on a single pixel detector. Also called Michelson interferometer.

Beam splitter has the property that the phase difference between transmitted and reflected beam is exactly 90 degrees. That's why the +/-



$$I_{\mathrm{int}}(t) = 2I_{tel}(\theta)\left[1 \pm V\sin(2\pi\Delta z(t)/\lambda)\right]$$

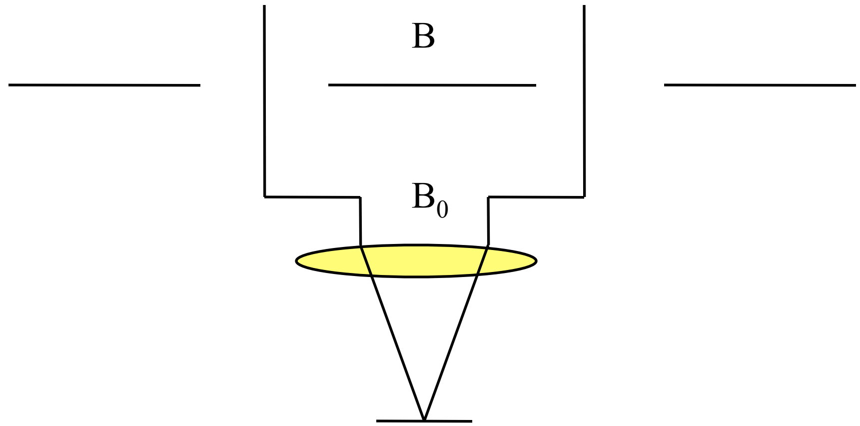
V is the visibility

Image plane combination (fringes are spatially displayed)

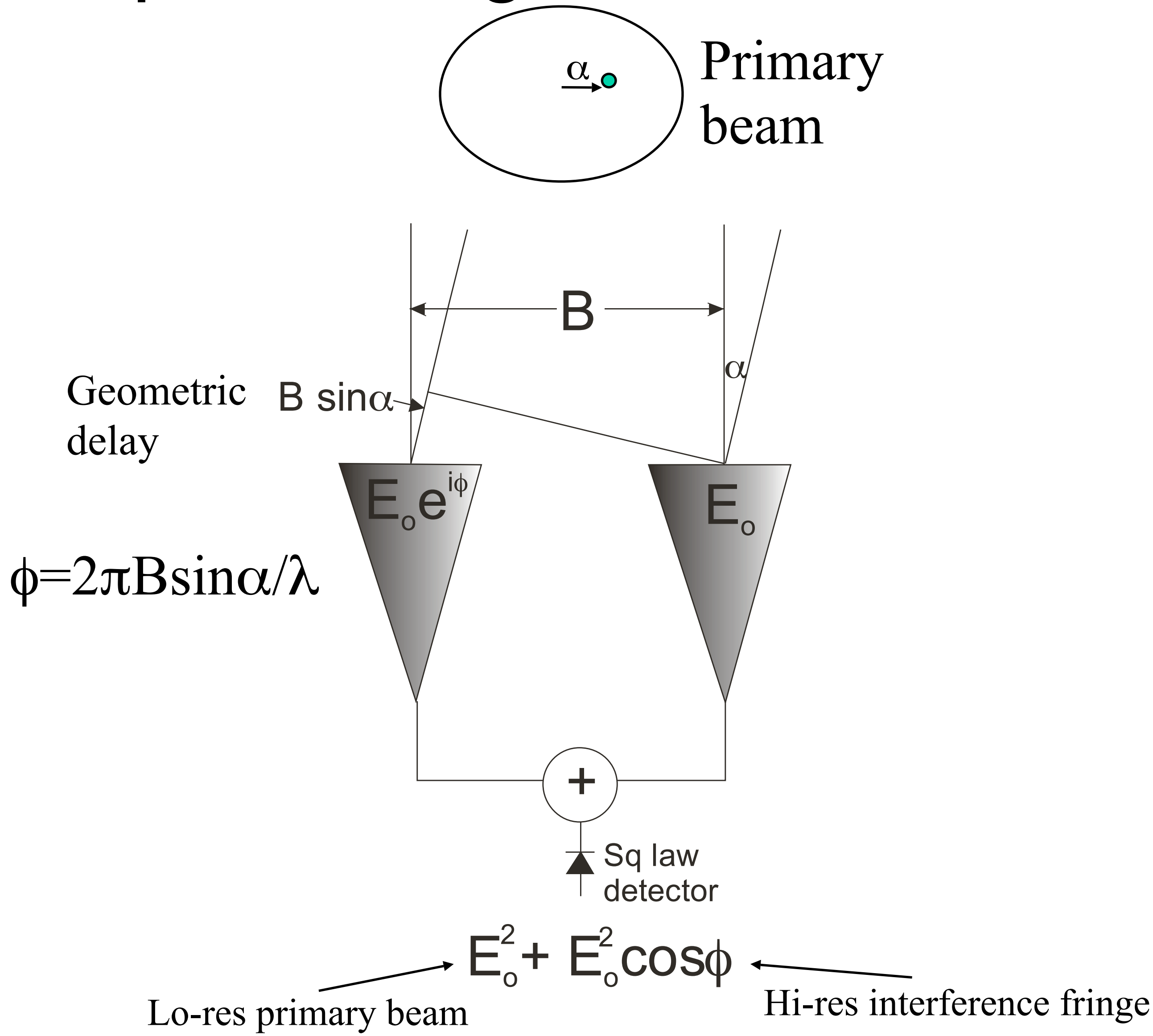
Method of combining the two beams in which each beam is focused to make an **image** of the sky. The images are superposed and interference fringes will form across the image. Also called **Fizeau** interferometer.

A phase difference (Δz) is artificially introduced to compensate for optical delays into the system or to modulate the signal into your detector

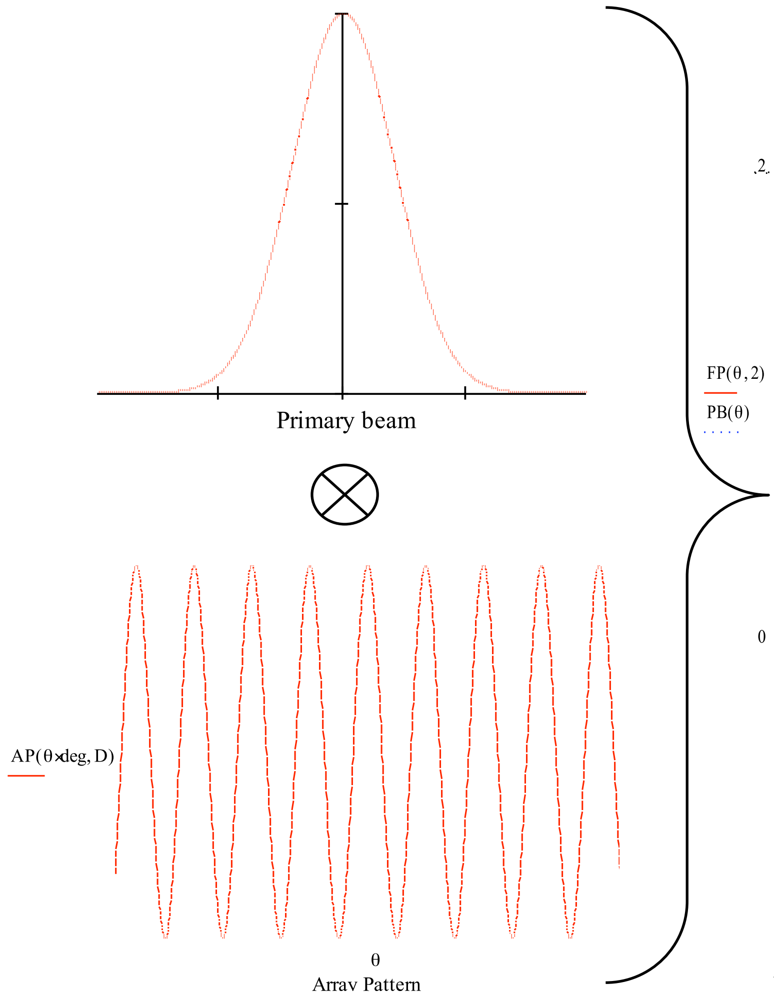
$$I_{\text{int}}(\theta) = 2I_{tel}(\theta)\left[1 + V\cos(2\pi(\theta B_0 + \Delta z)/\lambda\right]$$

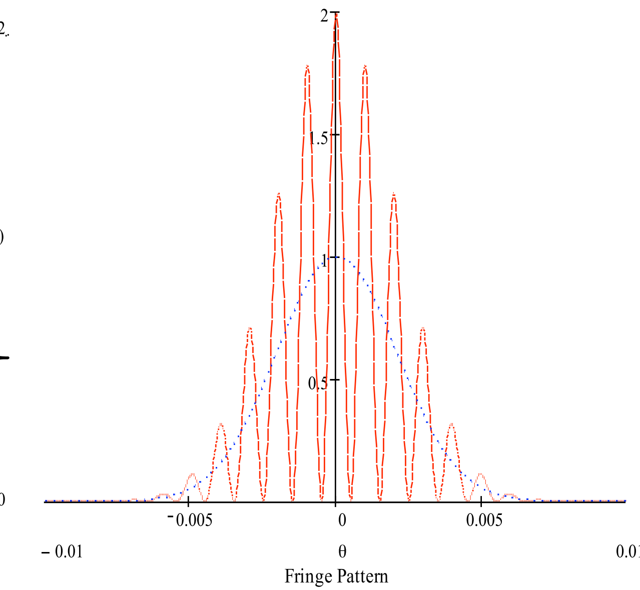


Simple adding interferometer



What signal we expect to see at the detector?

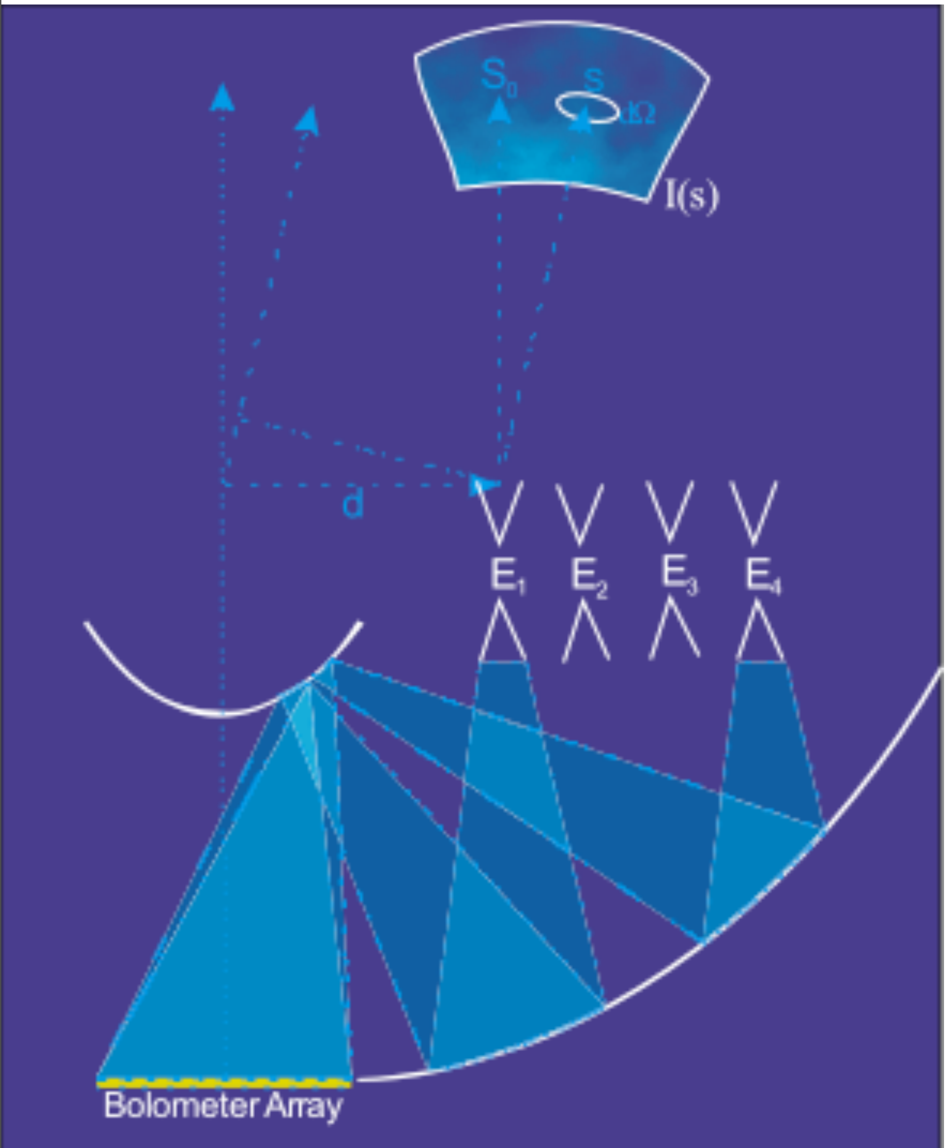




recovered with spatial or temporal chopping

An FFT will separate directly into visibilities, so one can multiplex different baselines simultaneously.

Fringe patterns caused sky brightness distribution



E_j is the electric vector collected by horn H_j from all elements $d\Omega$ of the sky brightness distribution $I(s)$ with horn beam $B(s)$.

Field at point x on the array due to horn j is

$E_j(x) = kE_j \exp(-2\pi i d_j x/ Dfr)$,

where k is the dilution factor, d_j is the offset of horn j, D is the diameter of the mirror and *fr* the f-ratio. The exponent term describes the complex phase gradient due to offset

$E(x) = \Sigma_j kE_j \exp(-2\pi i d_j x/Dfr)$

Power at x = $\langle E(x)E(x)^* \rangle$ =

$\Sigma_j k^2 \langle E_j E_j^* \rangle + \Sigma_{n<m} 2\mathrm{Re}[k^2 \langle E_n E_m^* \rangle \exp(-2\pi i(d_n-d_m)x/Dfr)]$

Coherence of E vectors due to brightness distribution in horns given by van Cittert-Zernike theorem.

$\langle EnEm \rangle = \int B(s)I(s)\exp(2\pi i u_b.s)\, d\Omega$,

Where u_b is baseline vector formed by horns n and m.

So array power distribution is given by

$N_h k^2 \int B(s)I(s) + \Sigma_{n<m} 2k^2\mathrm{Re}[\exp(-2pi(d_n-d_m)x/Dfr) \int B(s)I(s)\exp(2\pi i u_b.s)]$

First term is low resolution beam convolved response and the second term is the sum of the interference fringes with amplitude and phase determined by the brightness distribution within the beam.

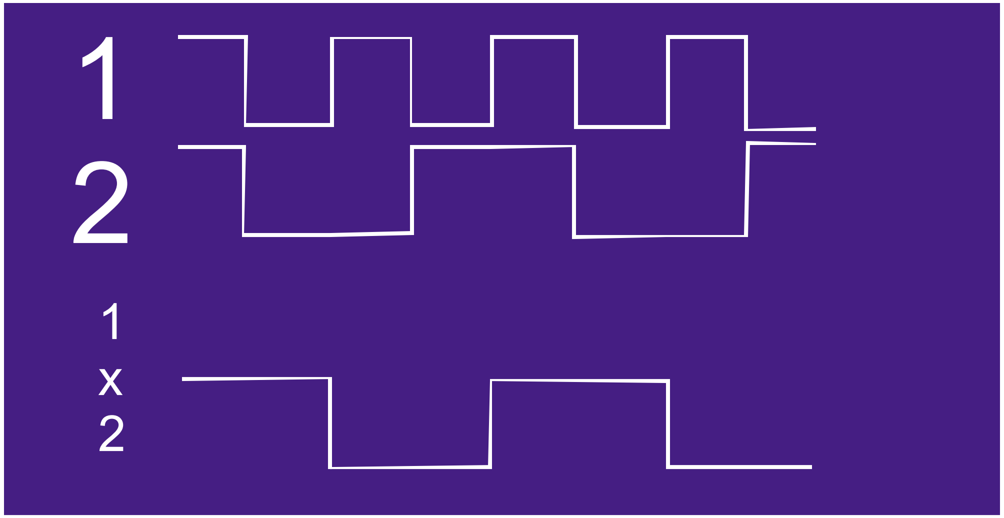
Use of phase switches in between receiver and transmit horns with orthogonal switch functions allows the extraction of the individual visibility fringes in parallel with only a √2 loss in sensitivity.

How to recover the visibilities:

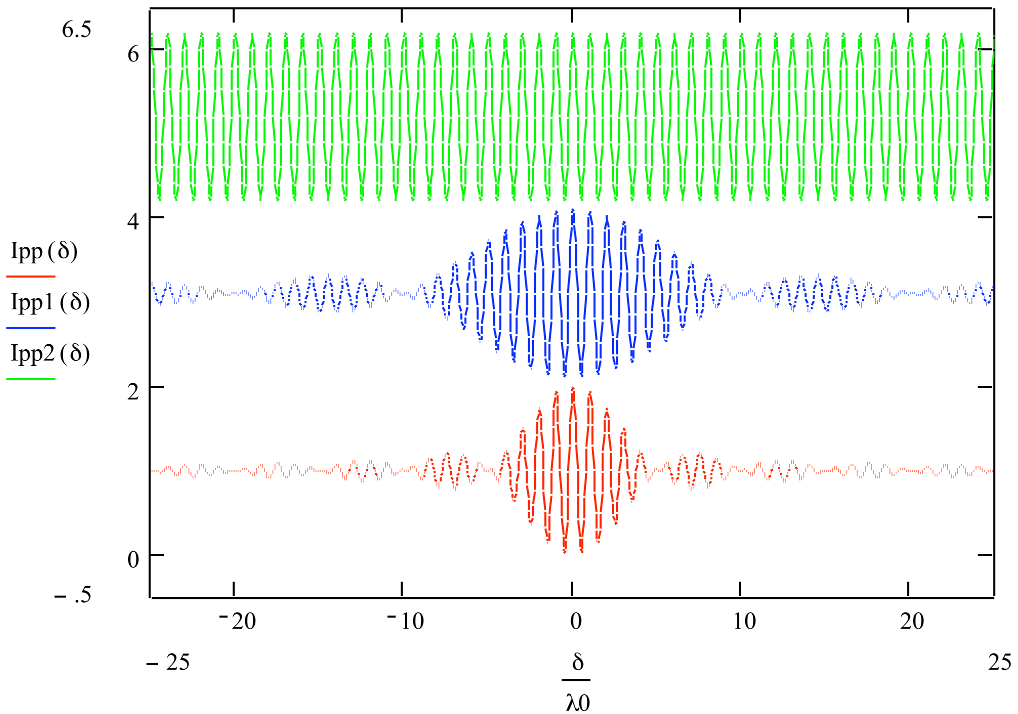
1. Each telescope/optical element has its own phase modulator.

2. Each phase modulator is switched with it's own unique sequence or 'Walsh function'.

3. The modulated baseline signal can be recovered by 'locking-in' with a new Walsh function which is the product of the two Walsh functions used on the phase modulators of the elements which the make up the baseline.



Bandwidth effect. (Green 0%, Blu 10%, Red 20%)



$$P \propto \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} [2 + 2\cos(kD)]\, d\nu$$

$$= \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} 2\,[1 + \cos(2\pi\nu\ D/c)]\, d\nu$$

$$= \Delta\lambda\left[1 + \frac{\sin \pi D\Delta\lambda/\lambda^2_0}{\pi D\Delta\lambda/\lambda^2_0}\cos k_0 D\right]$$

Fringes modulated with sinc function or ‘delay beam’

Spectral bandpass: the visibility decreases with distance from the zero-path difference (either spatial in image-plane or temporal in pupil-plane interferometer) by modulating the interferometer response with the Fourier Transform of the filter bandpass. The number of fringes is $N_f = 2\lambda/\Delta\lambda$.

Pupil-plane: *reduces the extent of useful delays*

Image-plane: *reduces the extent of the useful image in the focal plane*

For a compact array we want to use most of primary beam; then bandwidth is limited < array size/element size

Other visibility loss effects:

• Wavefront tilt: if the two wavefronts to be combined have a relative tilt of angle α, then the interference pattern will be smeared. Improper alignment of optical elements.

• Intensity mismatch: different optical transmission along different optical paths in the interferometer. If $I_1/I_2=\rho$ then

$$V_{\text{mismatch}} = \frac{2}{\rho^{1/2} + \rho^{-1/2}}$$

This effect is very tolerant. Example T=60%, R=40%, V_{mismatch}=0.98

Other visibility loss effects (continue):

• Optical surface figure errors. For an rms perturbation δ with respect to a perfect wavefront, then the Strehl ratio will be degraded by a factor:

$$V_{\text{surface}} \approx e^{-(2\pi\delta/\lambda)^2}$$

With N surfaces of rms δ_0 then $\delta = N^{1/2}\delta_0$.

• Polarization effects: can reduce visibility if not dealt with properly (it might even make the fringes disappear!).

Responsivity of an interferometer is like the number of photons collected from the source. If we want to estimate the number of photons collected by an interferometer we have to specify what is the source.

Source filling the beam (CMB)

- Use brightness units (W/m^2**sr**)
- No beam dilution effect because interferometer re-distribute the same photons on a different angular pattern in the sky.
- Receives twice as much photons because there are 2 horns/mirrors)
- BUT laws thermodynamics wins out to ensure signal on the detector is also the same uniform brightness

Compact object (point source)

- Use flux units (W/m^2Hz)
- Beam dilution problem. Less photons collected from a point-like source with respect to aperture filled telescopes
- Unless use compact array with filling factor near unity.

Instrument response to power spectrum ⇒ Window function

A single beam (total power) experiment:

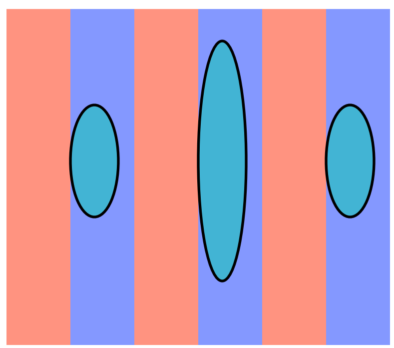
1. Gaussian beam ⇒ Gaussian window function (amplitude=1)

2. Beam is azimuthally symmetric - Doesn’t care about the orientation of the signal on the sky

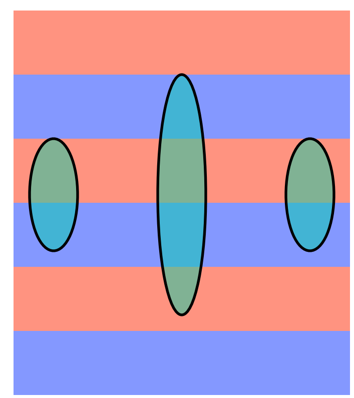
An interferometer

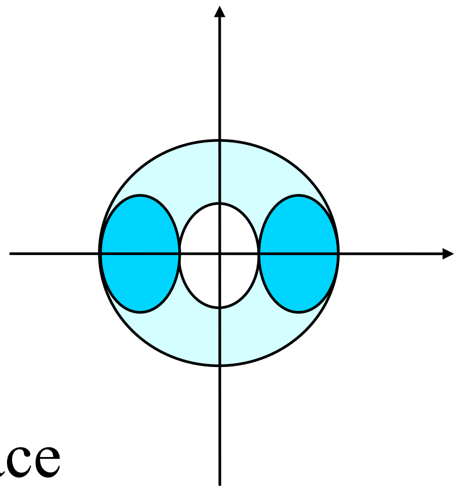
The beam corresponding to a baseline is not azimuthally symmetric
Therefore, the response to C_l gets diluted…have to average over angles

Maximum response

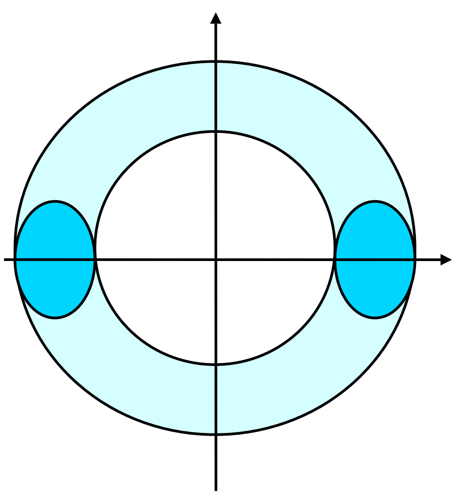


Minimum response





u-v space



Peak of window function proportional to the ratio of blue area over the annulus area

$\propto \sim 1/\ell$

So in going to higher ℓ C_ℓ response becomes weaker

One needs to observe equivalent baselines with different orientations to *fill in* the UV annulus.

Noise estimates

HEMT:

Antenna temperature: T_{ant}
Bandwidth: $\Delta\nu$
$NET_{R\text{-}J} \sim T_{ant} / \eta\sqrt{\Delta\nu}$

Bolometer:

Noise Effective Power: NEP
$dP/dT_{ant} = 2k_B\Delta\nu\eta$
$NET_{R\text{-}J} = NEP/2k_B\Delta\nu\eta$

$\Rightarrow$ Effective antenna temperature of bolometer:

$$T_{\text{ant}} = \frac{NEP}{2k_B\eta\sqrt{\Delta\nu}}$$

Noise in a bolometer

$$NEP^2 = \overbrace{NEP^2_{photon} + \frac{4kT^2G}{\eta^2}}^{\text{BLIP: these two guys dominate (hopefully)}} + \overbrace{\frac{(4kTR + V_A^2 + I_A^2 R)}{|S|^2}}^{\text{Johnson + electronics noise}} \quad \frac{W^2}{Hz}$$

where

$$NEP^2_{photon} = \left(\frac{4k^5}{c^2h^3}\right)\frac{T^5 A\Omega\varepsilon\tau}{\eta}\left(\int_{x_1}^{x_2}\frac{x^4dx}{e^x-1} + \frac{\varepsilon\tau\eta}{q}\int_{x_1}^{x_2}\frac{x^4dx}{\left(e^x-1\right)^2}\right)$$

$$q = \frac{2A\Omega\Delta\nu t}{\lambda^2}$$ Number of modes in one pol detected in time t and BW $\Delta\nu$

ε : emissivity

τ : filter transmission

η : optical efficiency

Noise of an Incoherent Interferometer

An incoherent interferometer has the disadvantage that the signal from each horn/telescope must be divided before it is detected.

Simple sensitivity calculation (give or take a √2):

$NEP^2_{total} = NEP^2_{BLIP} + NEP^2_{detector}$

Total loading on each detector:

$P_{total} = (N/n)\, P_{sys}$

$NEP^2_{BLIP} = 2(N/n)P_{sys}h\nu$

$NEP^2_{detector} = \gamma 4kT^2_{det}G$

for optimized bolometer: $G = P_{total}/2T_{det} = (N/n)P_{sys}/2T_{det}$

$\Rightarrow NEP^2_{detector} = \gamma 2(N/n)P_{sys}(kT_{det})$

so:

$NEP^2_{total} = 2(N/n)P_{sys}(h\nu + \gamma\, kT_{det})$

Conclusion: Detector NEP $\propto \sqrt{(2(N/n)P_{sys})}$

i.e. **more detectors, lower NEP**

Sensitivity of Interferometer: Conclusions

Coherent vs. Incoherent:

Factors lost/gained in integration time:

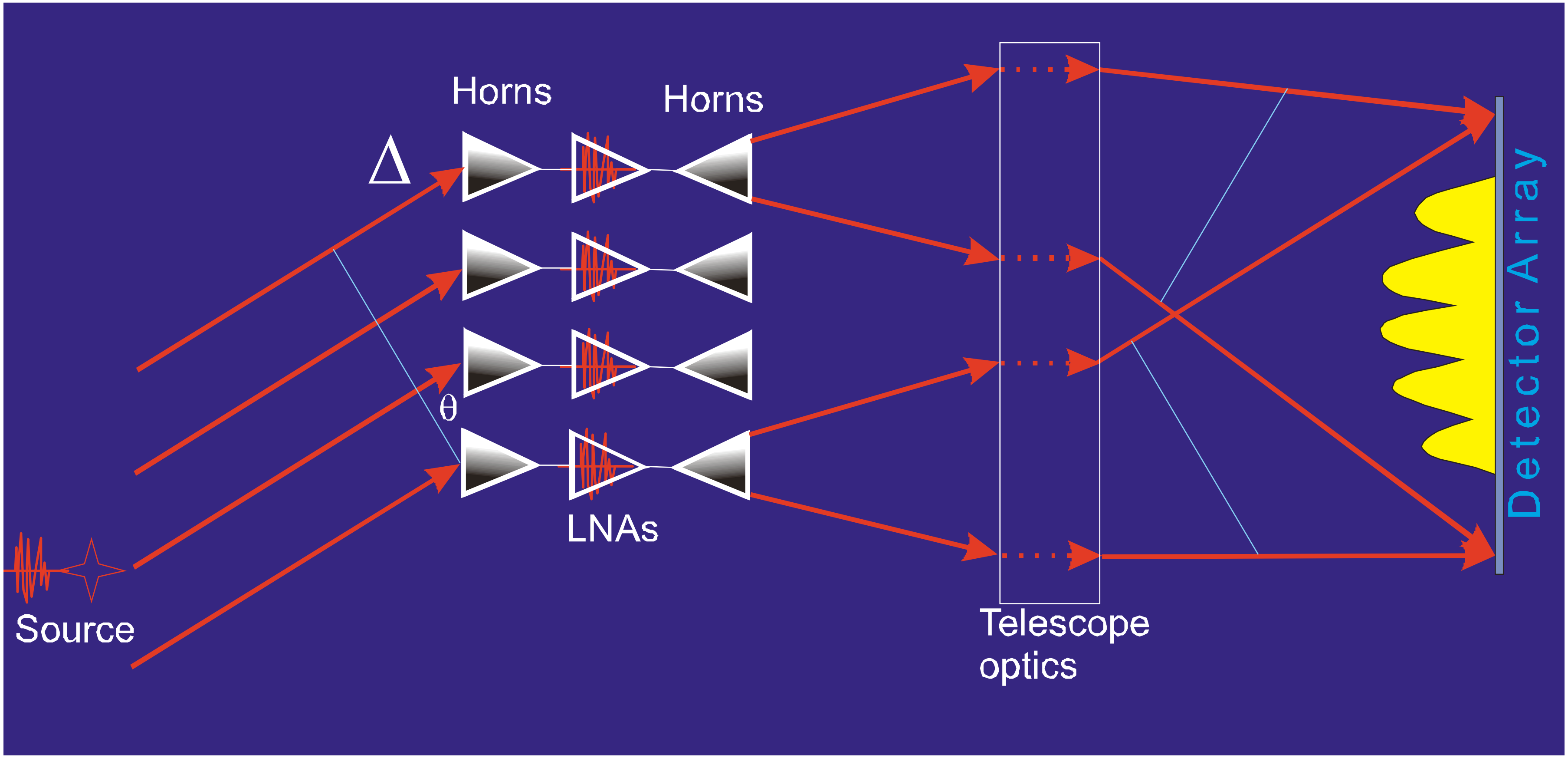
- Incoherent has to divide signals to correlate - lose $f_{combine} \sim N_{tel}$
- Incoherent has to chop phases to do correlation - lose $f_{chop} = 2$
- Incoherent can have both polarisations simultaneously - gain $f_{pol} = 2$
- Incoherent can have wide bandwidth - gain f_{bw}
- Incoherent can have BLIP limited sensitivity - gain depends on freq.
- Incoherent can have imaging array interferometry - gain $f_{image} \sim N_{pix}$

Other factors:

- **Incoherent correlator is simple**

- **Incoherent interferometer can work at high frequencies**

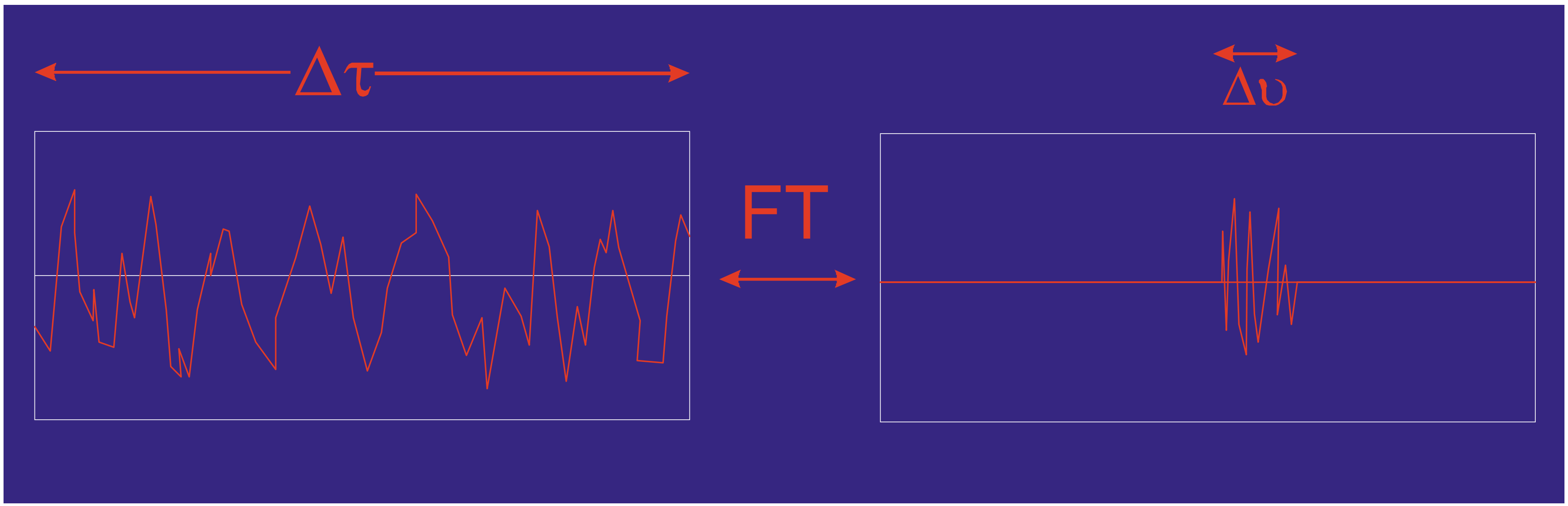
Model of Quasioptical correlator



Geometric delays can be easily modelled as a complex phase rotation
Different sources just described by different sets of random complex amplitudes.
Correlations and random partial correlations correctly taken into account.

Noise Simulations (Coherent)

- Instead of modelling noise sources as voltages over time E(t) I use sets of complex oscillators A(ν) as they need less numbers to store and are easier to use.



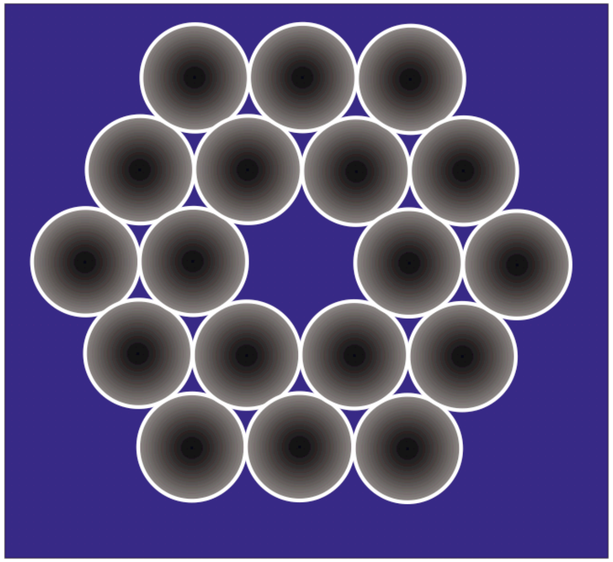
1μs at 30GHz needs 60,000 samples

1μs at 30GHz in 1GHz bandwidth only needs 1000 complex numbers - A_i

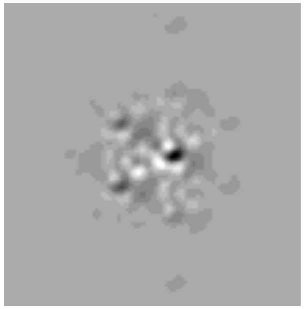
Number of oscillators comes from Nyquist sampling
Nosc = B.τ
so ΔT = T/√(B.τ) for total power rms variations is a

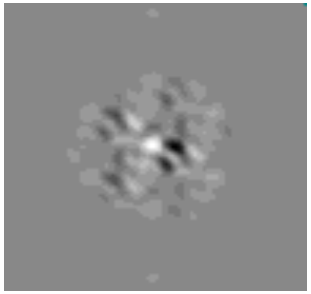
Noise Fringes on Array

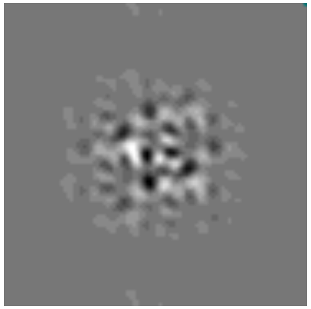
18 horn hexagonal array



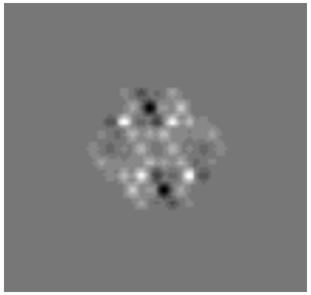
Fringes from LNA noise after phase switching

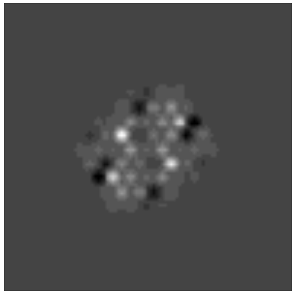


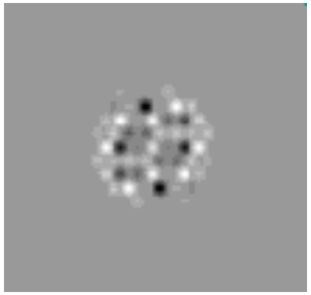




FFT of array data





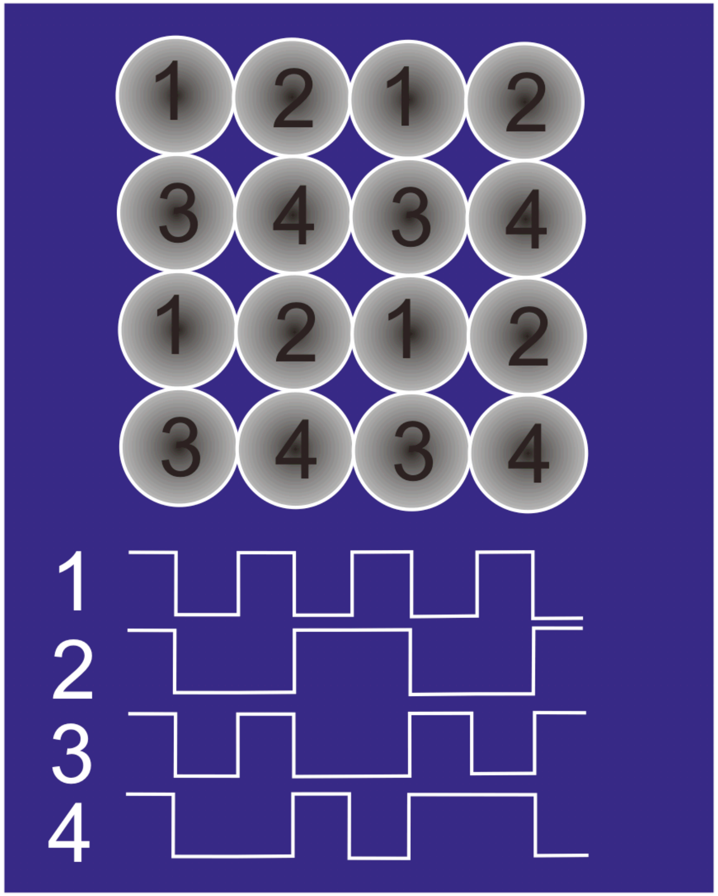


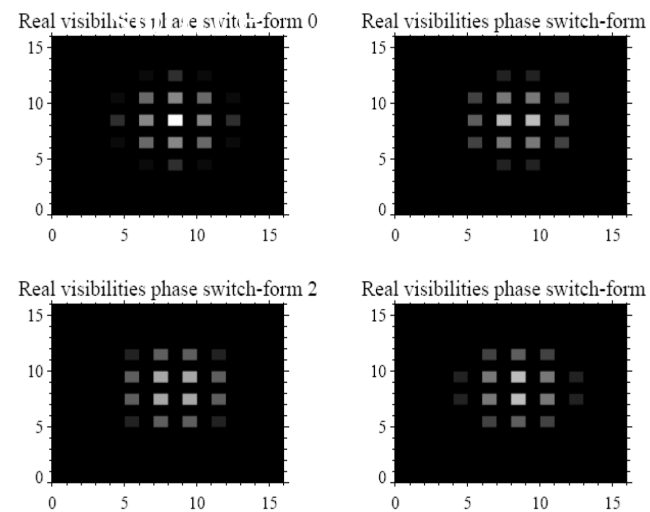
The noise from different LNAs causes random instantaneous fringes at level of $T_{sys}/\sqrt{(B.\tau)}$.

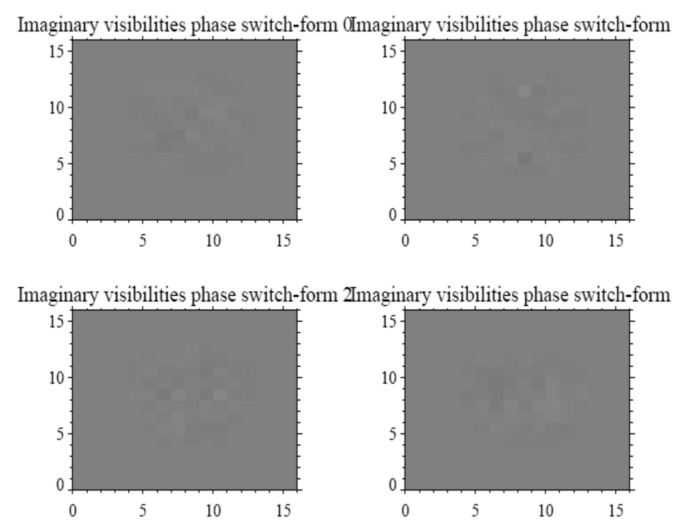
FFT of array gives power just at UV point where you expect signal to appear.

Noise Simulation for 4x4 square array

But for a coherent detector array!

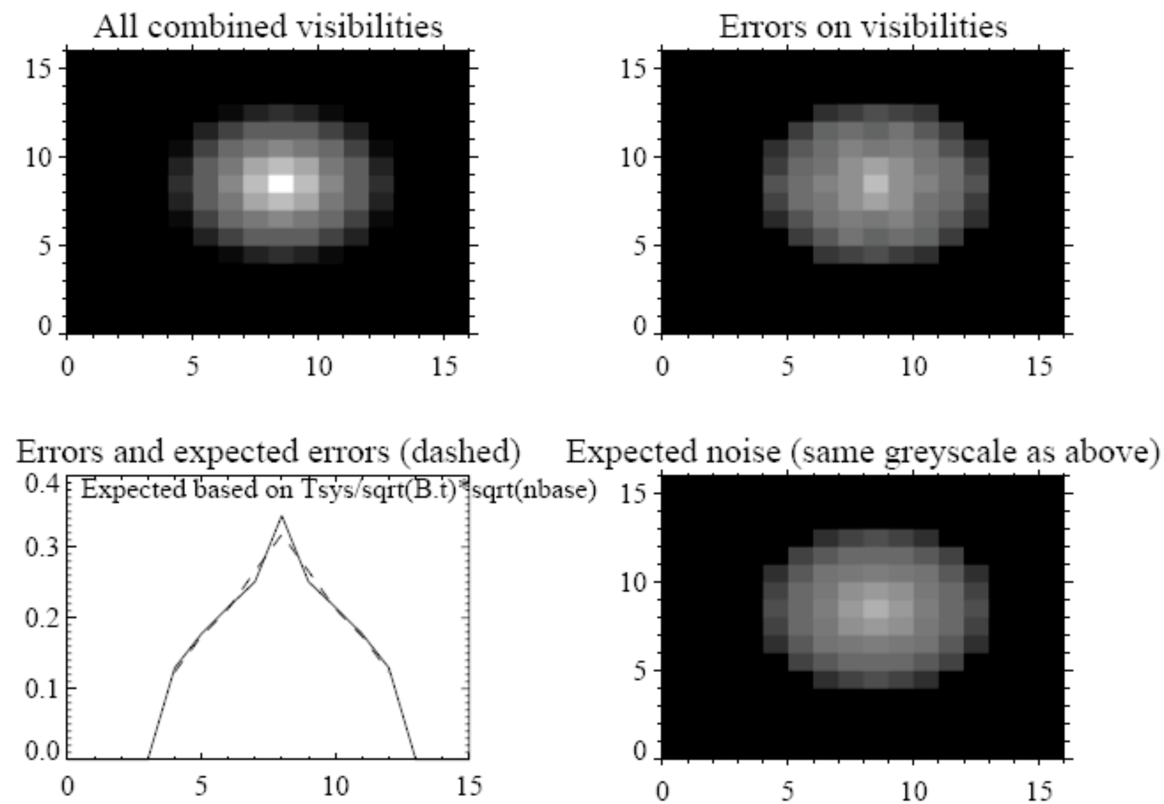






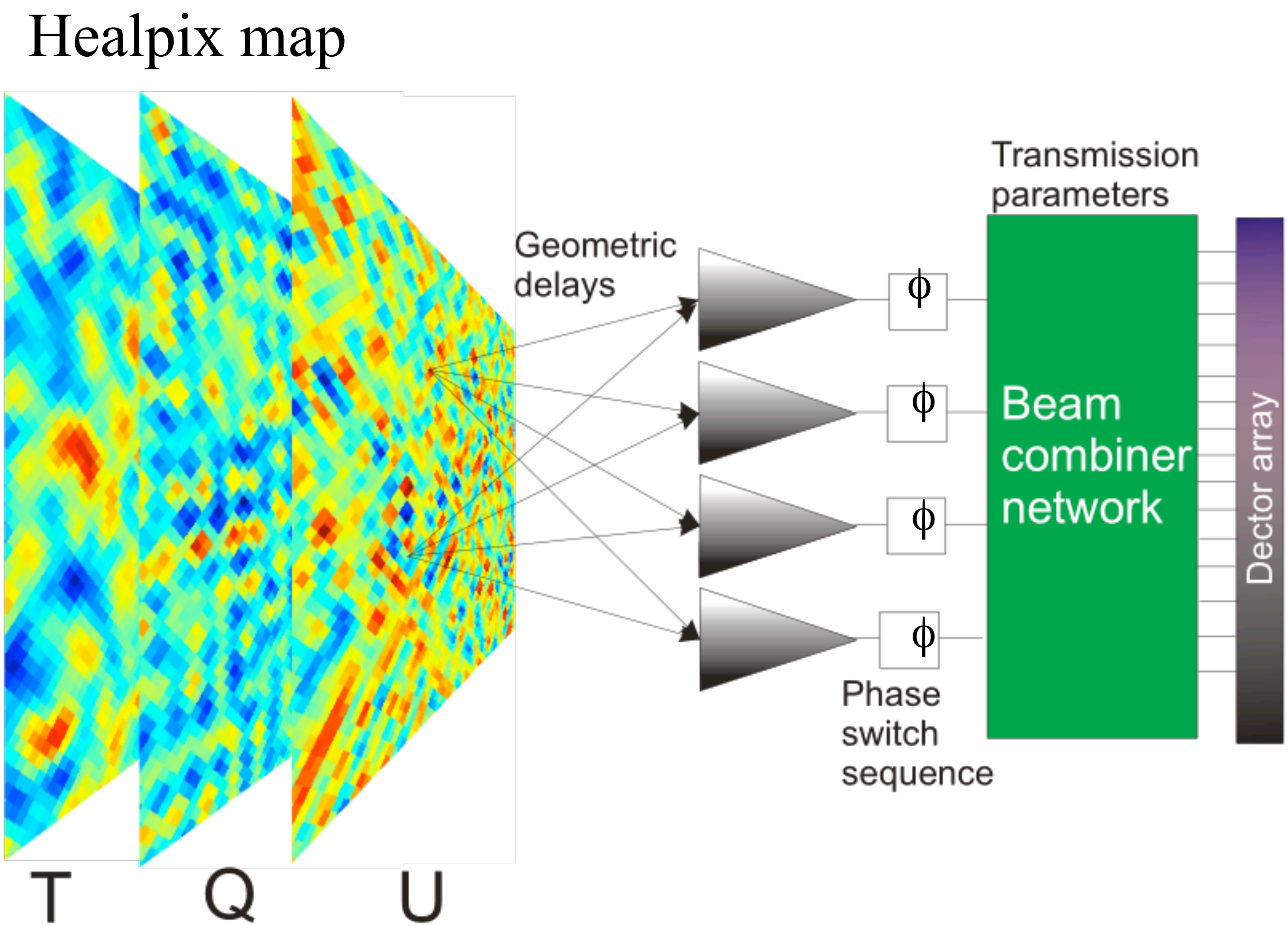
Need detector array twice as big in both axises in order to Nyquist sample fringes and recover phase. So need at least a 8x8 array.

Noise Results on 100 simulations



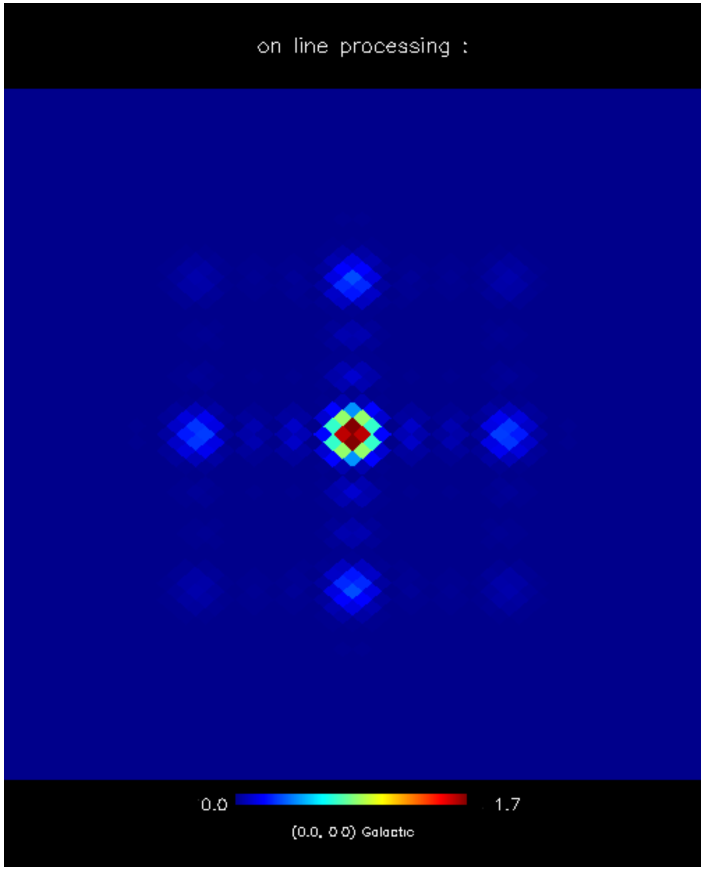
Due to phase switch scheme used common redundant baseline automatically visibilities lie on top of each and S/N boosted by sqrt(nbase).

Simple model of a bolometric polar

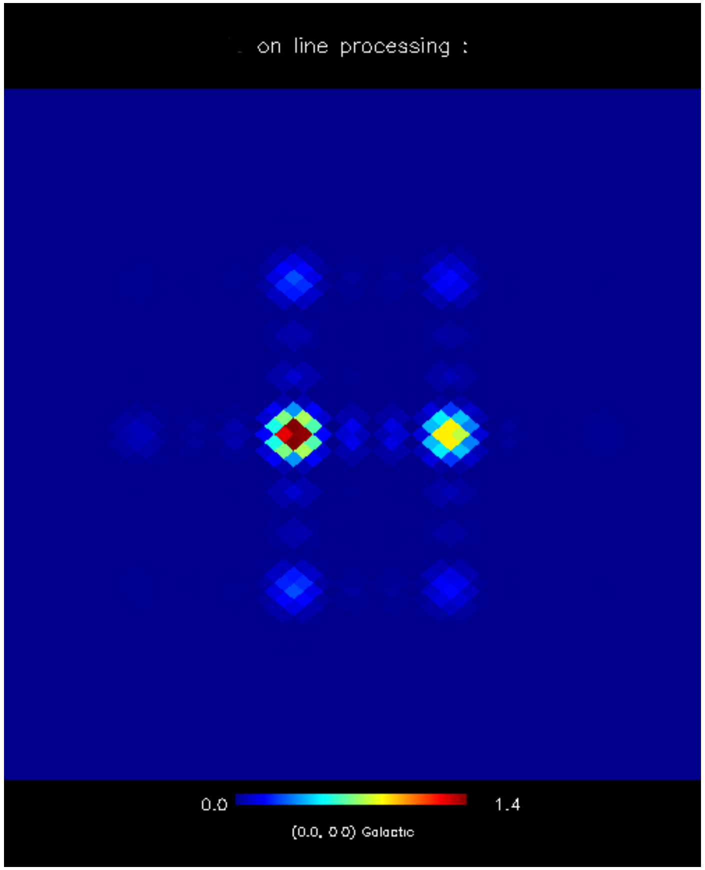


Can back track to see where power on detector came from

Can see the "gratting array" expected from phased array.

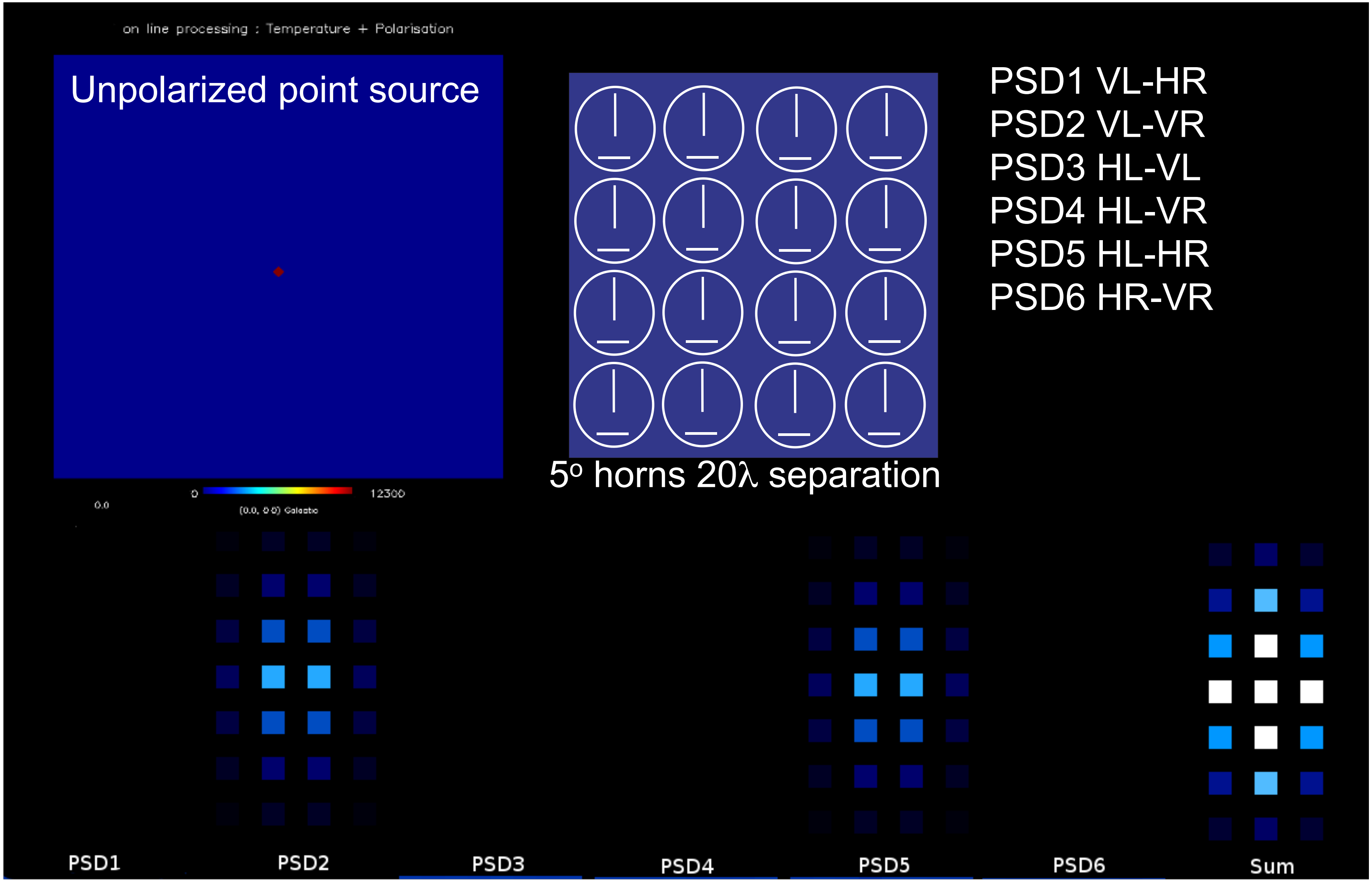


Centre of array

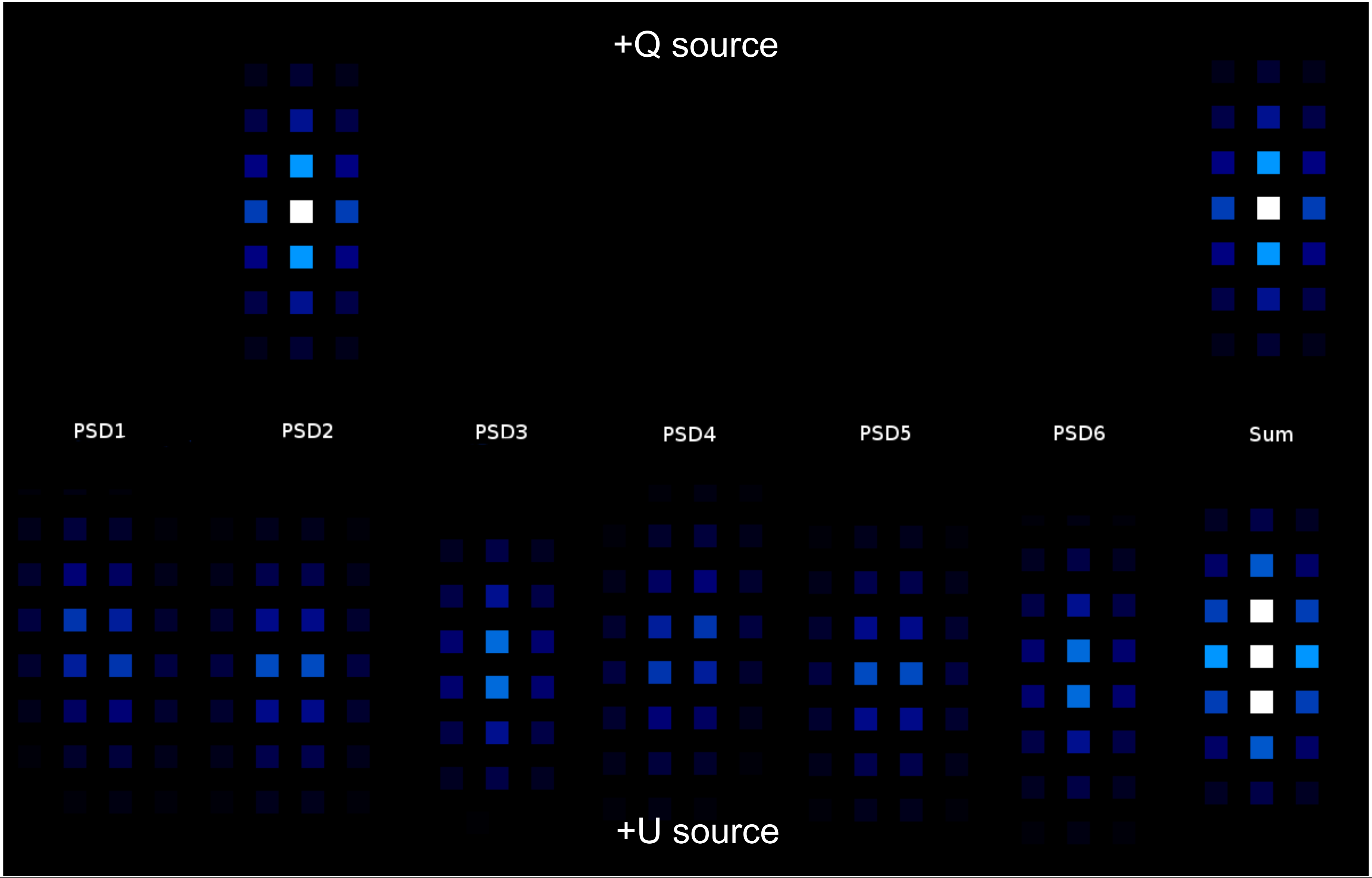


Towards edge

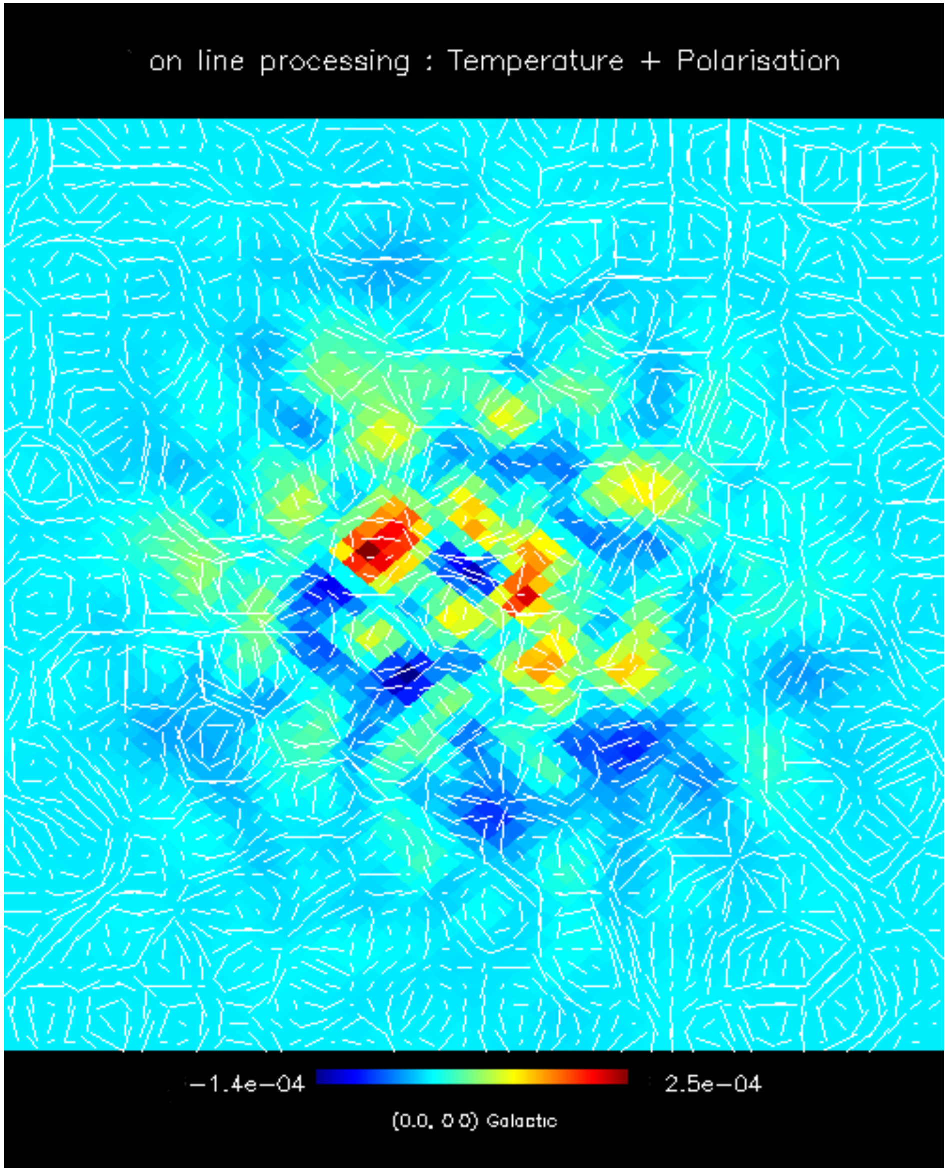
Dual polarization Simulation with HEALPix map input



Polarized point source

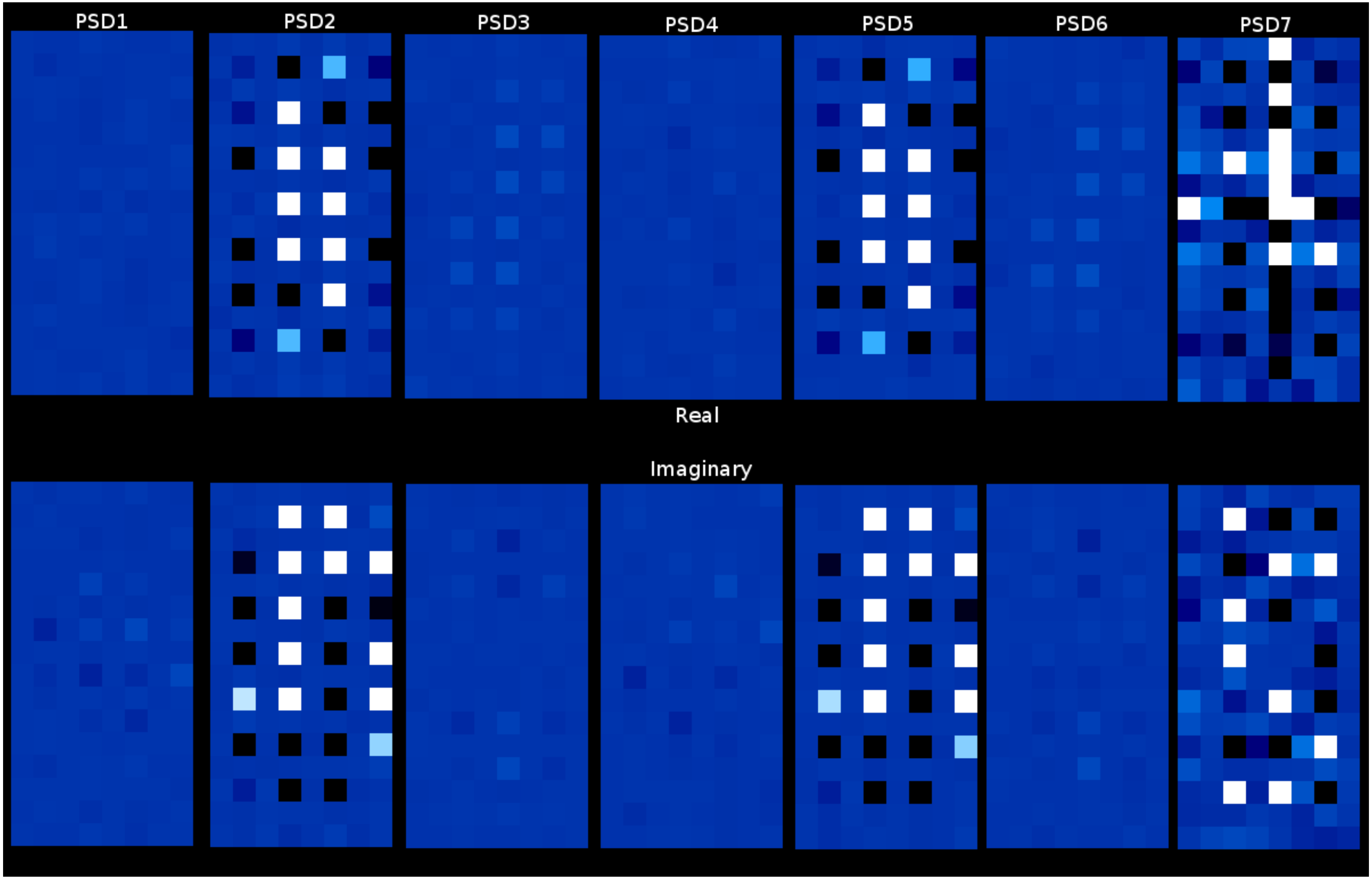


Use HEALPix CMB simulation with E & B modes



TQU input CMB map – nside 256

Visibilities for CMB simulation



Strong unpolarized signal with weaker polarized U visibilities at 10%