Component Separation

Building a multi-component sky model from multi-frequency observations

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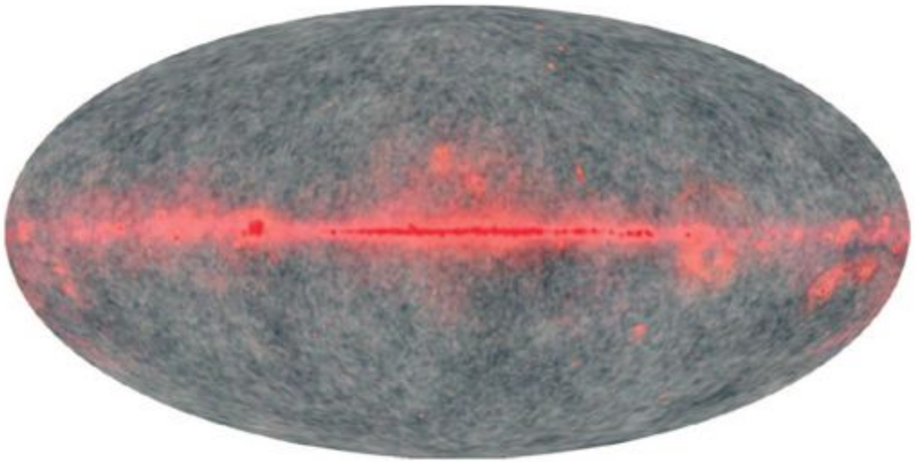
Outline

- Multi-component sky emission
- The Planck sky model
- Topics in component separation
- Validation
- Conclusion

CMB contamination by foregrounds

- The sky emission, at a given frequency, is a superposition of emissions from different sources
  - Different emission processes (thermal, synchrotron, Bremsstrahlung, …)
  - Different media/objects (Milky way ISM, CMB, clusters of galaxies)
- Has always been an issue for CMB observations, since early measurements of CMB anisotropies!

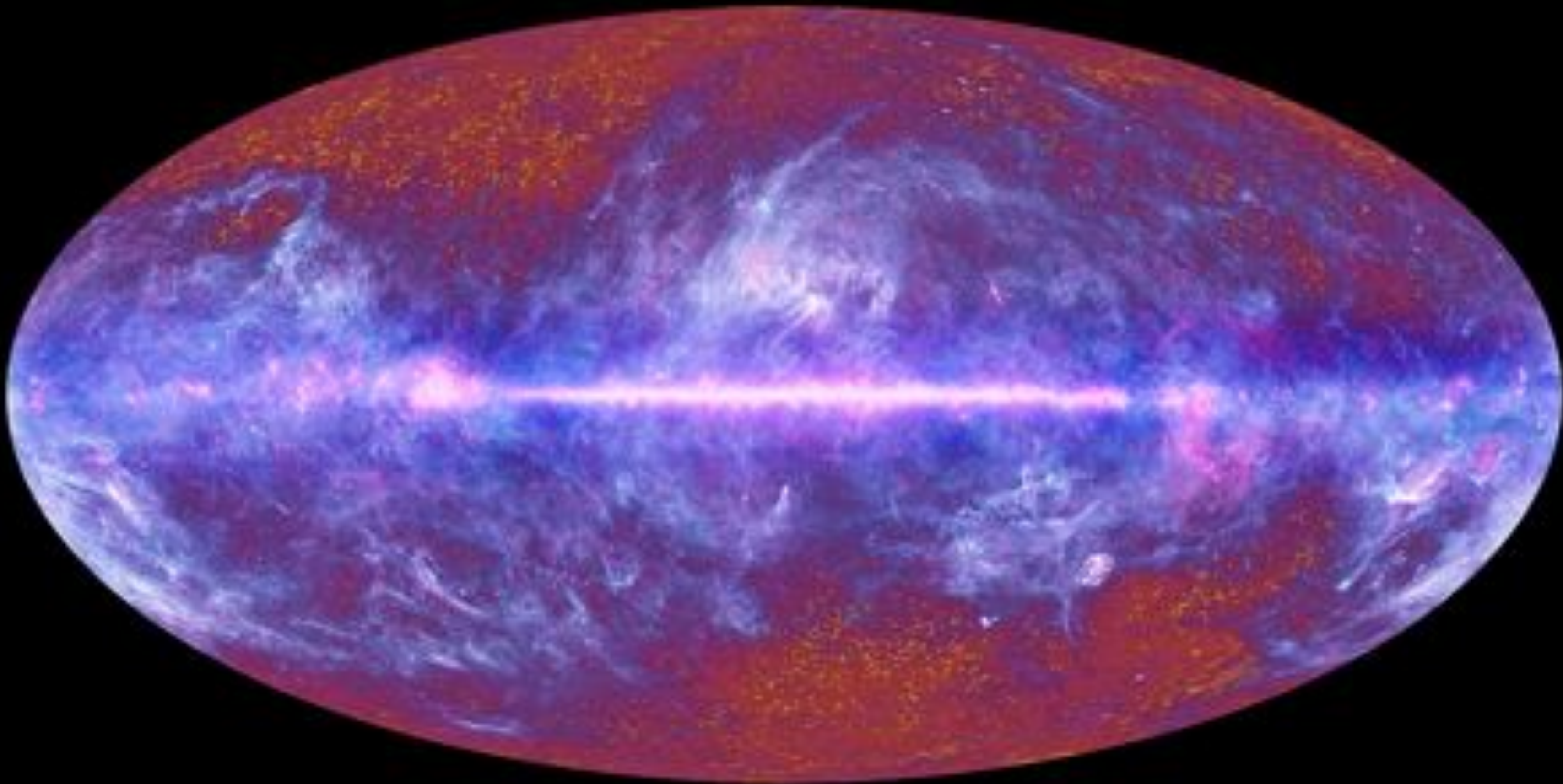
Multi-frequency observations allow us to check that observed anisotropies have the correct emission law

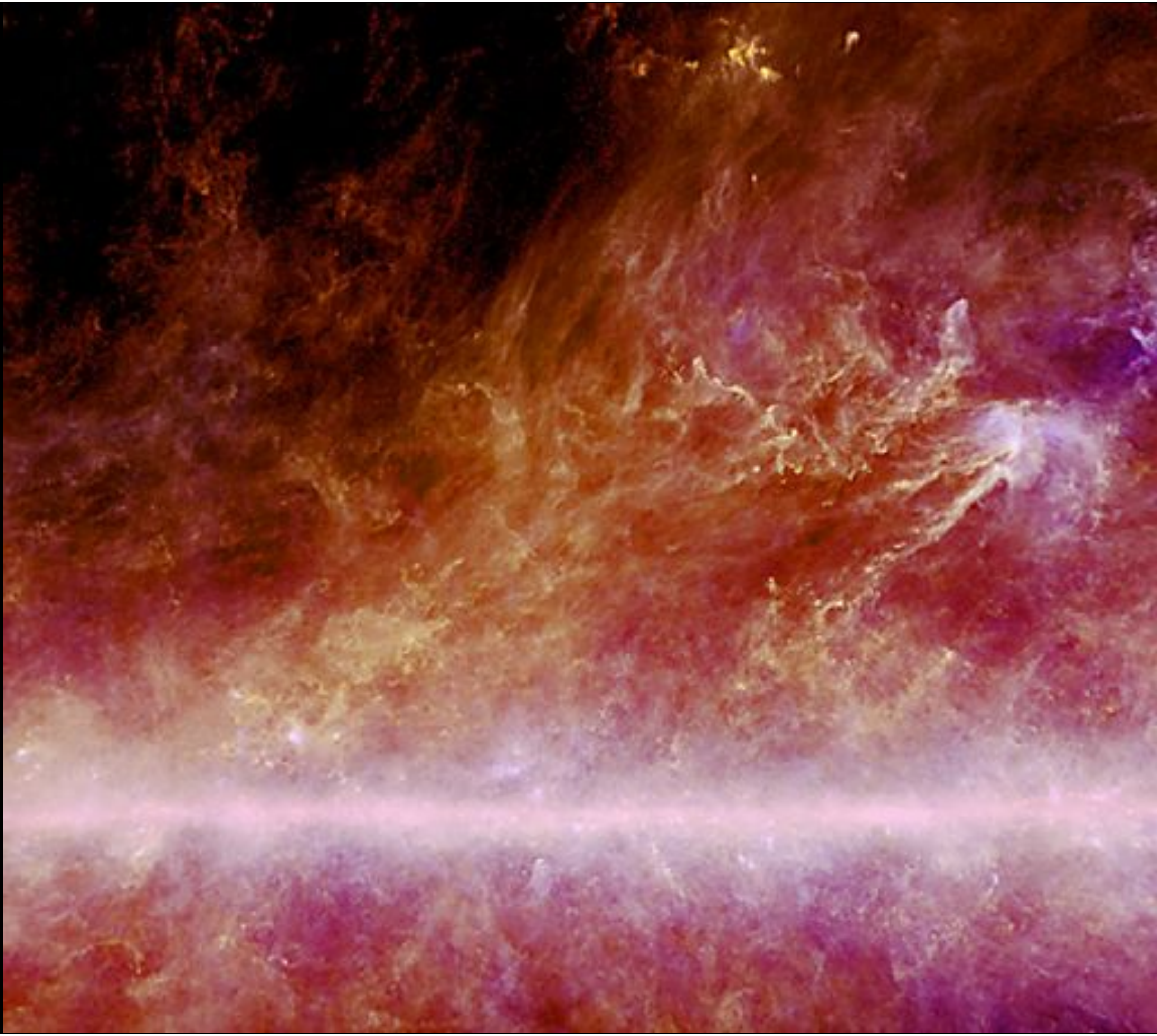


Bennett et al., ApJSS Volume 148, Issue 1, pp.97-117 (2003)

(false) color image of the submm sky

Planck observations



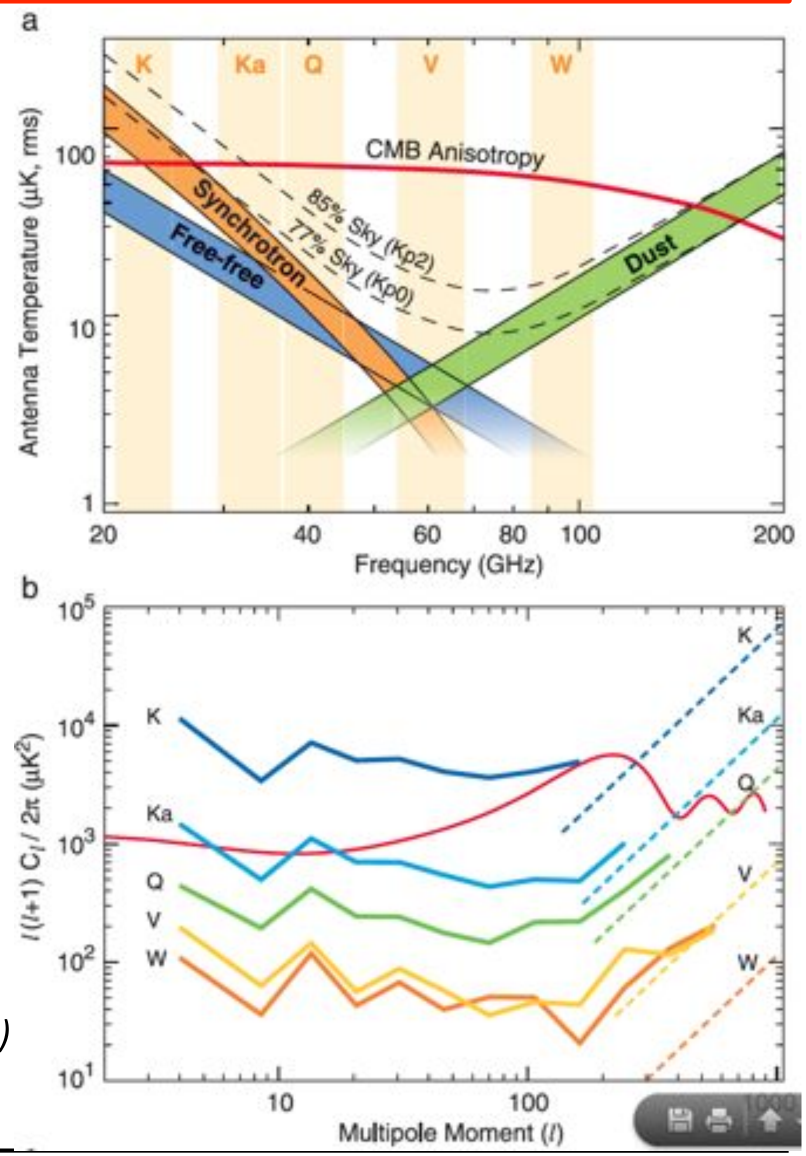


The ultimate frontier

- Astrophysical confusion is the ultimate frontier when instrumental noise becomes vanishingly small…
- Are there any components besides the obvious ones that dominate in one region of the sky?

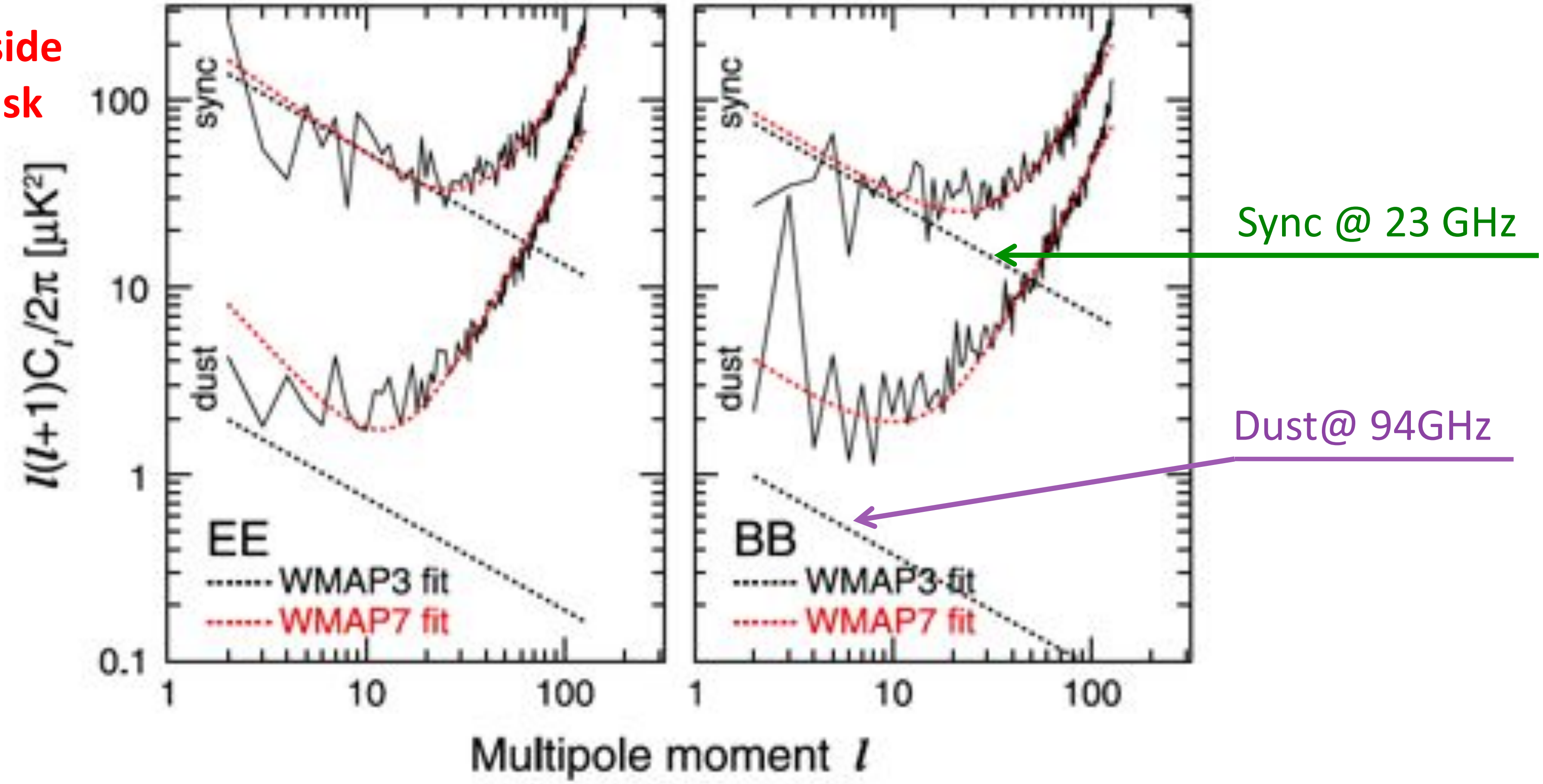
- Foreground level with WMAP kp2 mask as compared to CMB temperature anisotropies

Bennett et al., ApJSS Volume 148, Issue 1, pp.97-117 (2003)



The ultimate frontier

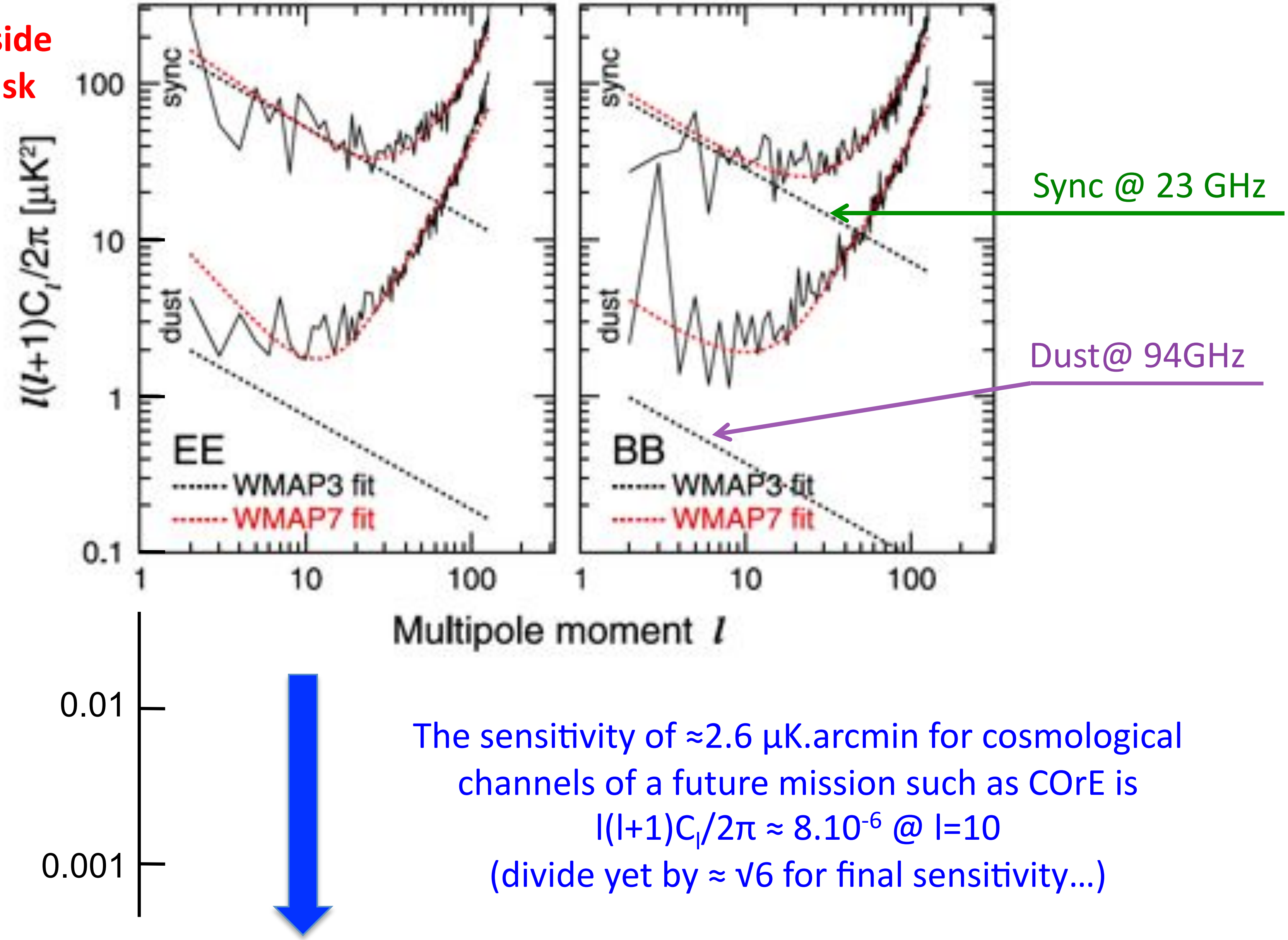
Spectra outside the P06 mask



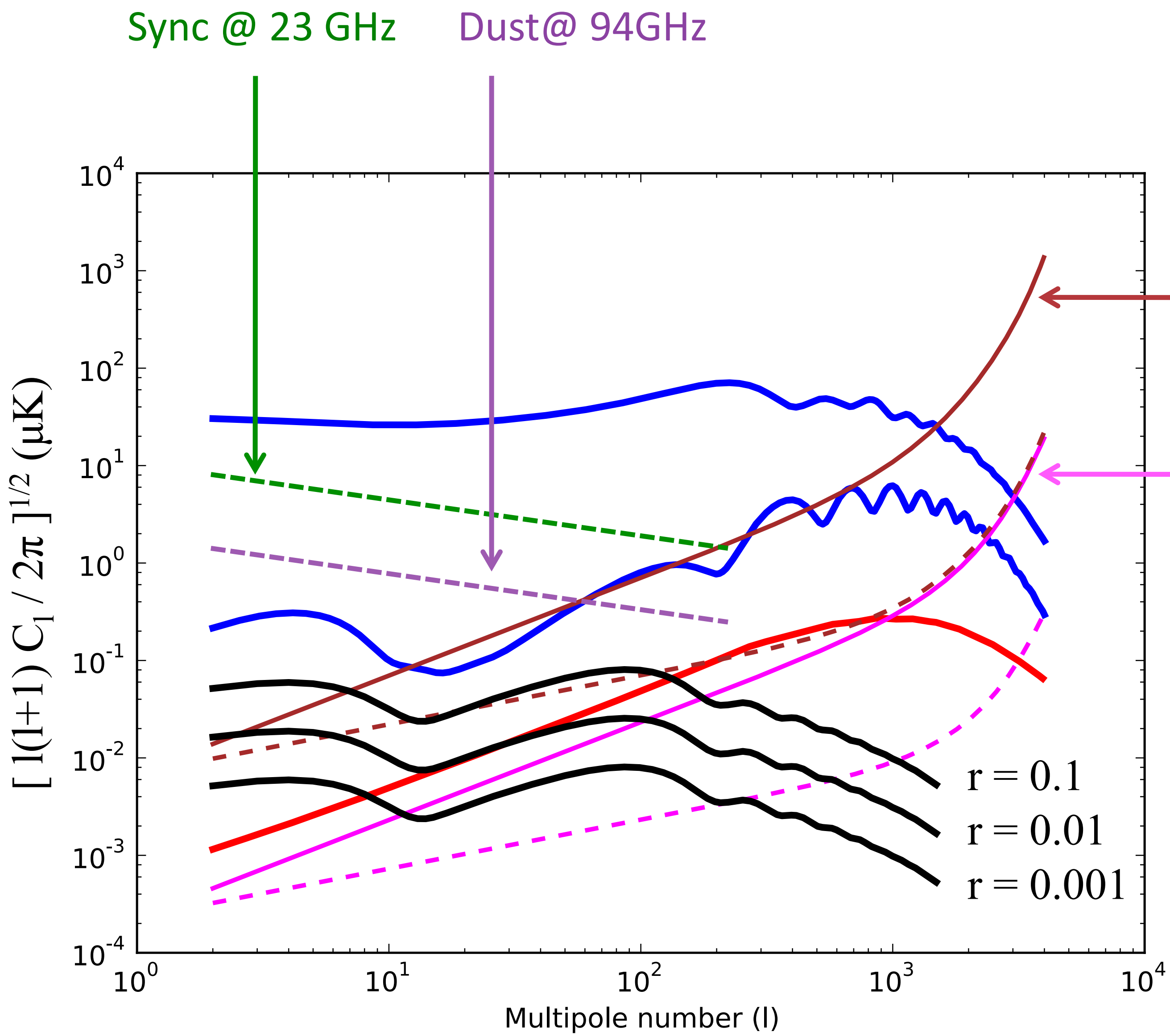
Gold et al., ApJS Volume 192, Issue 2, article 15 (2012)

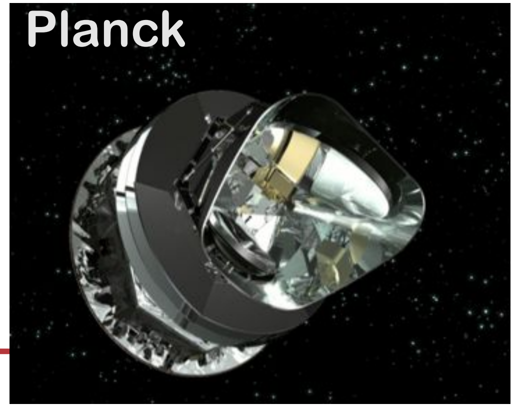
The ultimate frontier

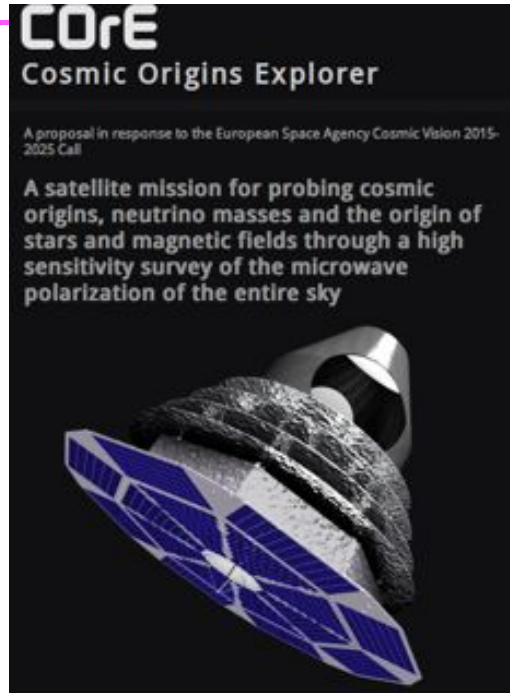
Spectra outside the P06 mask



The sensitivity of ≈2.6 μK.arcmin for cosmological channels of a future mission such as COrE is $l(l+1)C_l/2\pi \approx 8.10^{-6}$ @ l=10
(divide yet by ≈ √6 for final sensitivity…)







Need a complete, multi-component, polarised model of sky emission: the PSM

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- Multi-component sky emission
- The Planck sky model
- Topics in component separation
- Validation
- Conclusion

A multi-component model of sky emission

- The ultimate objective of all of our observations!
  - Our understanding of any emission is summarised in a model
  - The model can be physical, descriptive, parametric…

- The Planck Sky Model (PSM) is a global model of multi-component sky emission, in intensity and polarisation, over the frequency range ≈ 3GHz to ≈ 3THz, currently being developed in the Planck collaboration.

- It comprises
  - An underlying cosmological model (standard Big-Bang scenario) with associated cosmological parameters H_0, Ω_M, Ω_B, Ω_{DE}, n_s, …
  - A model of CMB emission and growth of structure (density contrast in the linear regime);
  - A model of Sunyaev-Zel'dovich emission from the hot gas in clusters of galaxies;
  - Models for the emission of the galactic ISM: Synchrotron, Dust (thermal and spinning), Free-free, some molecular lines (CO);
  - Models for extragalactic emission from compact sources (IR galaxies and CIB, radio galaxies).

A model that does not use yet Planck observations

The pre-launch *Planck Sky Model*: a model of sky emission at submillimetre to centimetre wavelengths

J. Delabrouille[1*], M. Betoule[2,3], J.-B. Melin[4], M.-A. Miville-Deschênes[5,6], J. Gonzalez-Nuevo[7], M. Le Jeune[1], G. Castex[1], G. de Zotti[7,8], S. Basak[1], M. Ashdown[9,10], J. Aumont[5], C. Baccigalupi[7], A. Banday[11], J.-P. Bernard[11], F.R. Bouchet[12], D.L. Clements[13], A. da Silva[14], C. Dickinson[15], F. Dodu[1], K. Dolag[16], F. Elsner[12], L. Fauvet[17], G. Faÿ[18,1], G. Giardino[17], S. Leach[7], J. Lesgourgues[19,20,21], M. Liguori[22,12], J.-F. Macías-Pérez[23], M. Massardi[8,24], S. Matarrese[22], P. Mazzotta[25], L. Montier[11], S. Mottet[12], R. Paladini[26], B. Partridge[27], R. Piffaretti[4], G. Prezeau[28,29], S. Prunet[12], S. Ricciardi[30], M. Roman[1], B. Schaefer[31], and L. Toffolatti[32]

(Affiliations can be found after the references)

To be submitted soon to A&A

ABSTRACT

We present the *Planck Sky Model* (PSM), a parametric model for the generation of all-sky, few arcminute resolution maps of sky emission at submillimetre to centimetre wavelengths, in both intensity and polarisation. Several options are implemented to model the cosmic microwave background, Galactic diffuse emission (synchrotron, free-free, thermal and spinning dust, CO lines), Galactic H II regions, extragalactic radio sources, dusty galaxies, and thermal and kinetic Sunyaev-Zeldovich signals from clusters of galaxies. Each component is simulated by means of educated interpolations/extrapolations of data sets available at the time of the launch of the *Planck* mission, complemented by state-of-the-art models of the emission. Distinctive features of the simulations are: spatially varying spectral properties of synchrotron and dust; different spectral parameters for each point source; modeling of the clustering properties of extragalactic sources and of the power spectrum of fluctuations in the cosmic infrared background. The PSM enables the production of random realizations of the sky emission, constrained to match observational data within their uncertainties, and is implemented in a software package that is regularly updated with incoming information from observations. The model is expected to serve as a useful tool for optimizing planned microwave and sub-millimetre surveys and to test data processing and analysis pipelines. It is, in particular, used for the development and validation of data analysis pipelines within the *Planck* collaboration. A version of the software that can be used for simulating the observations for a variety of experiments is made available on a dedicated website.

Key words. Cosmology: cosmic background radiation – Interstellar medium (ISM), nebulae – Galaxies: clusters: general – Galaxies: general – Infrared: diffuse background

The PSM in practice

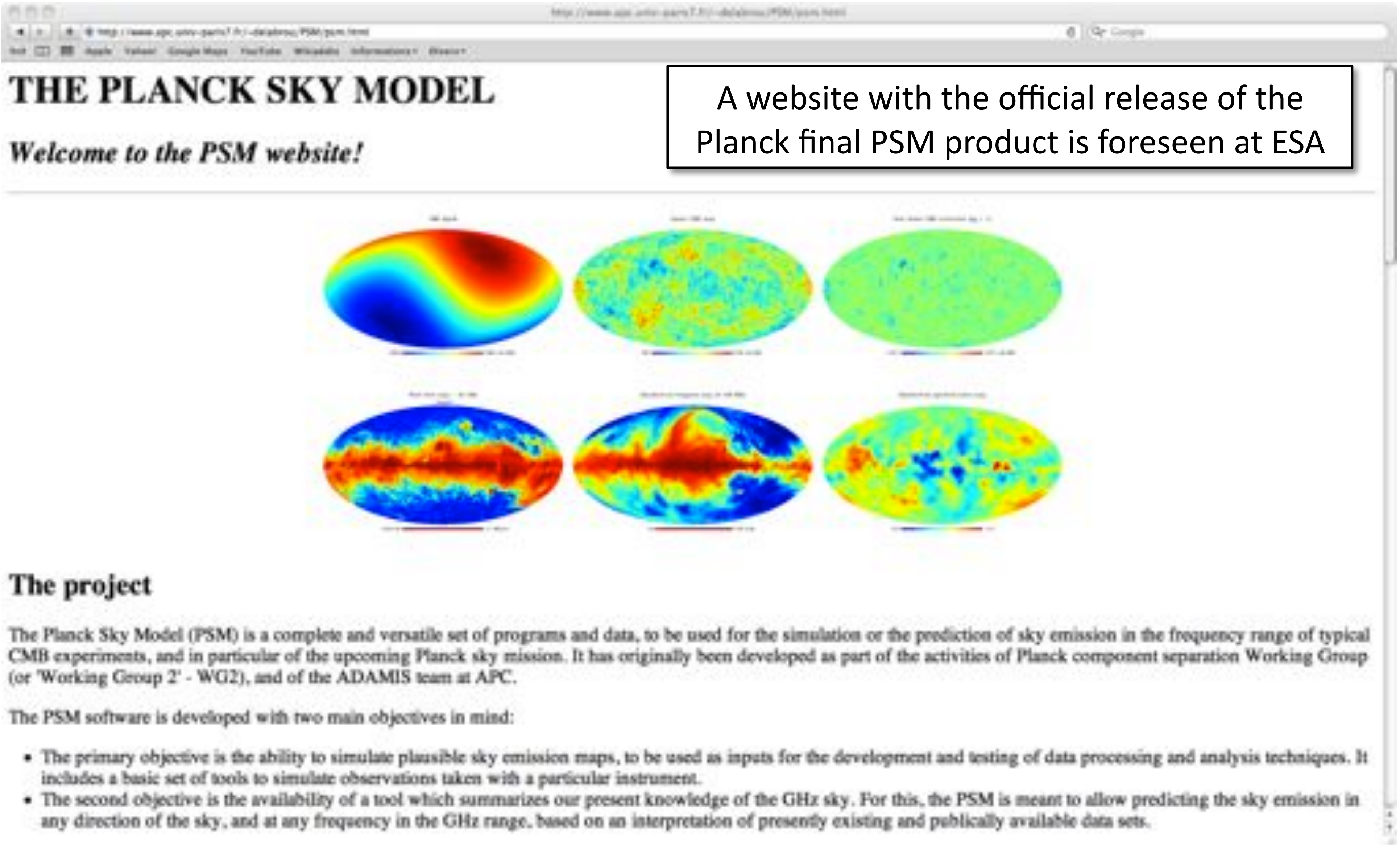
- DATA
  - Maps of observed or simulated diffuse components (CMB, synchrotron intensity, synchrotron spectral index, CO-line emission…)
  - Constraints on model parameters (e.g. cosmological parameters and errors)
  - Catalogues of known clusters of galaxies
  - Catalogues of know point sources

- SOFTWARE
  - Implements the model in a (hopefully) user-friendly way
  - *Input*: choice for global parameters (e.g. cosmological parameters, choice among alternate options for modelling each component) in the form of a configuration file
  - *Output*: maps and catalogues of modelled (simulated or predicted) sky emission for each component
  - *Output*: sky emission in user-defined frequency channels (total emission, and emission per component, in intensity and polarisation)

Keyword: Flexibility

The PSM website

A website with the official release of the Planck final PSM product is foreseen at ESA



THE PLANCK SKY MODEL

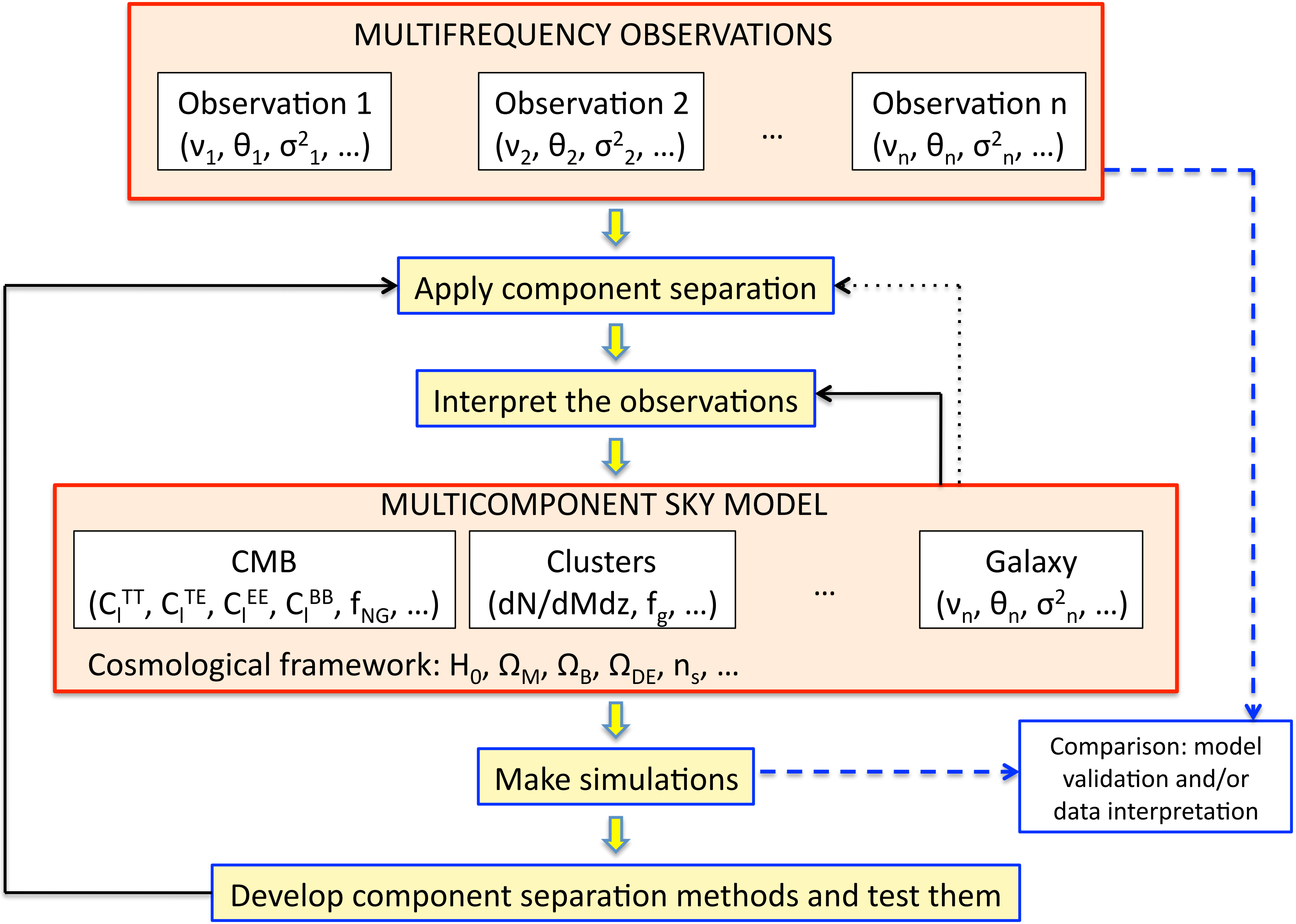
Welcome to the PSM website!

The project

The Planck Sky Model (PSM) is a complete and versatile set of programs and data, to be used for the simulation or the prediction of sky emission in the frequency range of typical CMB experiments, and in particular of the upcoming Planck sky mission. It has originally been developed as part of the activities of Planck component separation Working Group (or 'Working Group 2' - WG2), and of the ADAMIS team at APC.

The PSM software is developed with two main objectives in mind:

- The primary objective is the ability to simulate plausible sky emission maps, to be used as inputs for the development and testing of data processing and analysis techniques. It includes a basic set of tools to simulate observations taken with a particular instrument.
- The second objective is the availability of a tool which summarizes our present knowledge of the GHz sky. For this, the PSM is meant to allow predicting the sky emission in any direction of the sky, and at any frequency in the GHz range, based on an interpretation of presently existing and publically available data sets.


MULTIFREQUENCY OBSERVATIONS
Observation 1 (ν_1, θ_1, σ^2_1, …)
Observation 2 (ν_2, θ_2, σ^2_2, …)
…
Observation n (ν_n, θ_n, σ^2_n, …)
Apply component separation
Interpret the observations
MULTICOMPONENT SKY MODEL
CMB (C_l^{TT}, C_l^{TE}, C_l^{EE}, C_l^{BB}, f_{NG}, …)
Clusters (dN/dMdz, f_g, …)
…
Galaxy (ν_n, θ_n, σ^2_n, …)
Cosmological framework: H_0, Ω_M, Ω_B, Ω_{DE}, n_s, …
Make simulations
Comparison: model validation and/or data interpretation
Develop component separation methods and test them

Outline

- Multi-component sky emission
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- Topics in component separation
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Basic ideas for component separation

Consider a single pixel p. The signal observed at frequency ν, in pixel p, is

$$x(\nu, p) = \sum_i x_i(\nu, p) + n(\nu, p)$$

$$= \sum_i a_i(\nu, p)\, s_i(p) + n(\nu, p),$$

or, in vector-matrix format

$$\boldsymbol{x}(p) = \mathsf{A}(p)\, \boldsymbol{s}(p) + \boldsymbol{n}(p).$$

If we know the frequency dependence of each foreground in that particular pixel, then we can invert the system perfectly.

Model known *a priori* ?
$dB_\nu/dT.s_{CMB} + s_{dust}.\nu^\beta B_\nu(T_d) + s_{sync}.\nu^\alpha$

Eriksen et al. 2006
Stivoli et al. 2006
Stompor et al. 2009

At some frequency ν_0

Blind ICA and more

- Classically assumes a linear mixture model

$$x(p) = A.s(p) + n(p)$$

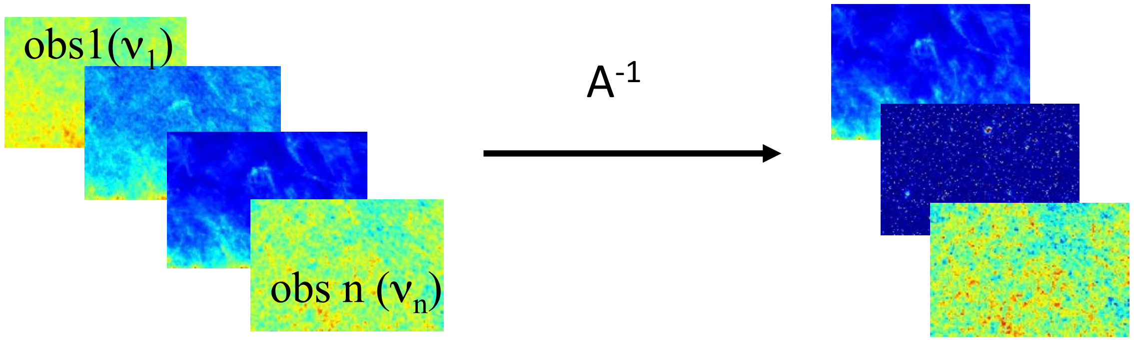
- Estimate A so that we can invert the linear system
  - FastICA : Find the transformation matrix W that makes Wx as non-gaussian as possible: *Hyvärinen 1999; Baccigalupi et al. 2000; Maino et al. 2002*
  - Other idea: train a neural network to find W for you: *Nørgaard-Nielsen et al. 2008, 2009, 2012*
  - SMICA: Compute the second-order statistics of x(p) over independent domains (spectral bands, needlets, or even independent regions of sky), and fit for A and second order statistics of s and n, *Delabrouille et al. 2003*
  - GMCA: find W that allows the sparsest representation of the data, *Bobin et al 2008*

- Main problems
  - In reality A=A(p) for some components
  - Some components are not independent

- This has motivated developments such as
  - Extended SMICA: *Cardoso et al. 2008*
  - CCA: *Bedini et al. 2005, Bonaldi et al. 2006*
  - Local GMCA: *Bobin et al 2012*

Linear inversion

$x(p) = A.s(p) + n(p)$



Wiener filter

$$\hat{s} = [A^{\dagger}R_n^{-1}A + R_s^{-1}]^{-1} A^{\dagger}R_n^{-1} x$$

Unknown

Unknown

Not so sure…

Spectral Matching ICA

Delabrouille, Cardoso & Patanchon 2003
Cardoso et al 2008

If $x = A.\,s + n$

then

$$\langle xx^t \rangle = A \langle ss^t \rangle A^\dagger + \langle nn^t \rangle$$

$$\begin{pmatrix} \langle x_1 x_1 \rangle & \langle x_1 x_2 \rangle & \dots & \langle x_1 x_m \rangle \\ \langle x_2 x_1 \rangle & \langle x_2 x_2 \rangle & & \\ : & & & \\ \langle x_m x_1 \rangle & \dots & & \langle x_m x_m \rangle \end{pmatrix}$$

Mixing matrix

$$\begin{pmatrix} C_l(\text{CMB}) & 0 & 0 \\ 0 & C_l(\text{dust}) & 0 \\ 0 & 0 & C_l(\text{SZ}) \end{pmatrix}$$

$$\begin{pmatrix} N_1 & 0 & & 0 \\ 0 & N_2 & & \\ : & & & 0 \\ 0 & \dots & 0 & N_m \end{pmatrix}$$

- minimise the mismatch between $\hat{R}_x(l)$ and $AR_s(l)A^\dagger + R_n(l)$

Many applications !

- Multi-component C_l estimation

$$R_x(l) \;=\; A \,.\, \begin{pmatrix} C_l(\mathrm{CMB}) & 0 & 0 \\ 0 & C_l(\mathrm{dust}) & 0 \\ 0 & 0 & C_l(\mathrm{sync}) \end{pmatrix} .\, A^{\dagger} + R_n(l)$$

Can estimate noise levels as well...

- Multi-component BLIND component separation

$$R_x(l) = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \\ A_{41} & A_{42} & A_{43} \end{pmatrix} . \begin{pmatrix} C_l(\mathrm{CMB}) & 0 & 0 \\ 0 & C_l(\mathrm{dust}) & X_l \\ 0 & X_l & C_l(\mathrm{sync}) \end{pmatrix} .\, A^{\dagger} + R_n(l)$$

Least Square linear combination

Suppose that we know only the frequency dependence of one component of interest,

$$x(\nu, p) = a(\nu)\, s(p) + r(\nu, p)$$

Then the 'optimal' reconstructed signal by linear combination of the input maps is

$$\widehat{s}(p) = \frac{\boldsymbol{a}^t \mathsf{R}_r^{-1}}{\boldsymbol{a}^t \mathsf{R}_r^{-1} \boldsymbol{a}} \boldsymbol{x}(p)$$

Other idea: the ILC

- Usually derived as the (internal) linear combination of the input maps which minimizes the variance of the output, with unit response to the CMB

$$\widehat{s}_{\mathrm{ILC}}(p) = \sum_i w_i x_i(p) = \mathbf{w}^t \mathbf{x}(p)$$

$$\sum_i w_i a_i = \mathbf{w}^t \mathbf{a} = 1$$

$$\text{minimize } \sum_p |\widehat{s}(p)|^2$$

- Actual implementation

$$\widehat{s}_{\mathrm{ILC}}(p) = \frac{\mathbf{a}^t \widehat{\mathbf{R}}_x^{-1}}{\mathbf{a}^t \widehat{\mathbf{R}}_x^{-1} \mathbf{a}} \mathbf{x}(p)$$

  – Uses the empirical covariance matrix of the observations (and this is a very important distinction)

Woodbury matrix identity and consequences

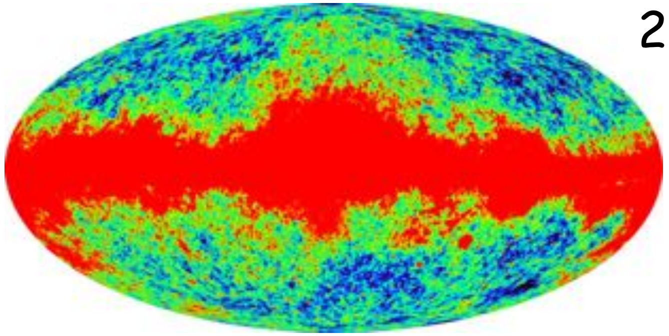
$$R_x^{-1} = \left[\boldsymbol{a}\boldsymbol{a}^t\sigma_{\rm cmb}^2 + R_n\right]^{-1}$$
$$= R_n^{-1} - \sigma_{\rm cmb}^2 \frac{R_n^{-1}\boldsymbol{a}\boldsymbol{a}^t R_n^{-1}}{1+\sigma_{\rm cmb}^2 \boldsymbol{a}^t R_n^{-1}\boldsymbol{a}}$$

$$\boldsymbol{a}^t R_x^{-1} = \boldsymbol{a}^t R_n^{-1} - \sigma_{\rm cmb}^2 \frac{\boldsymbol{a}^t R_n^{-1}\boldsymbol{a}\,\boldsymbol{a}^t R_n^{-1}}{1+\sigma_{\rm cmb}^2 \boldsymbol{a}^t R_n^{-1}\boldsymbol{a}}$$

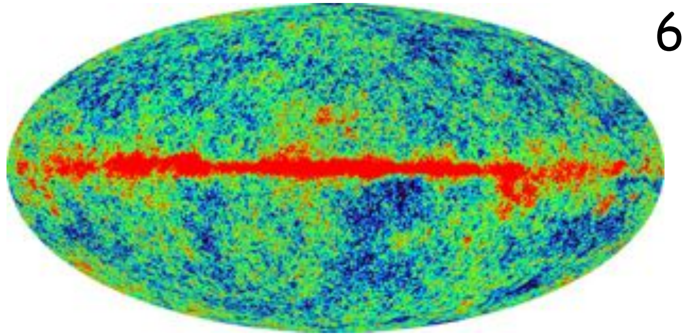
$$\boldsymbol{a}^t R_x^{-1} \propto \boldsymbol{a}^t R_n^{-1}$$

ILC ≈ GLS But with empirical statistics R_x

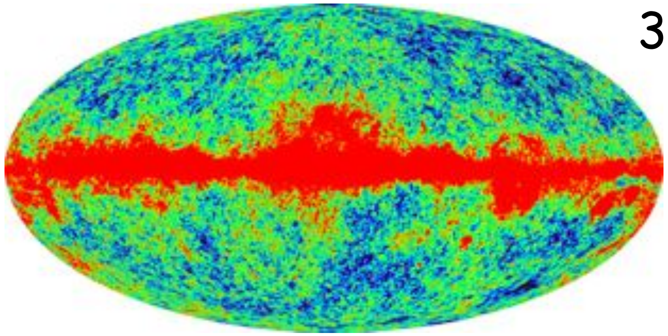
The ILC on real data



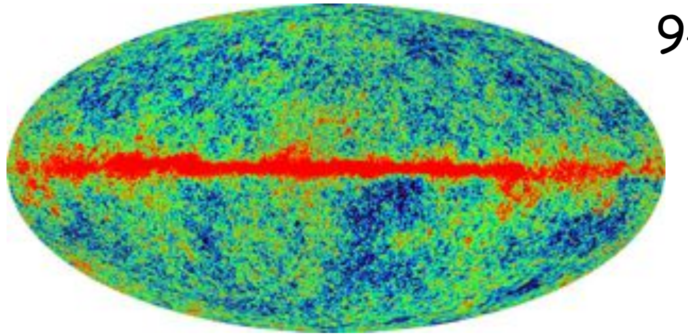
20 GHz



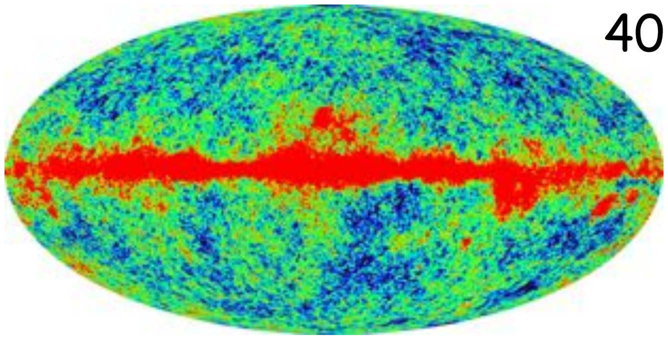
60 GHz



30 GHz



94 GHz



40 GHz

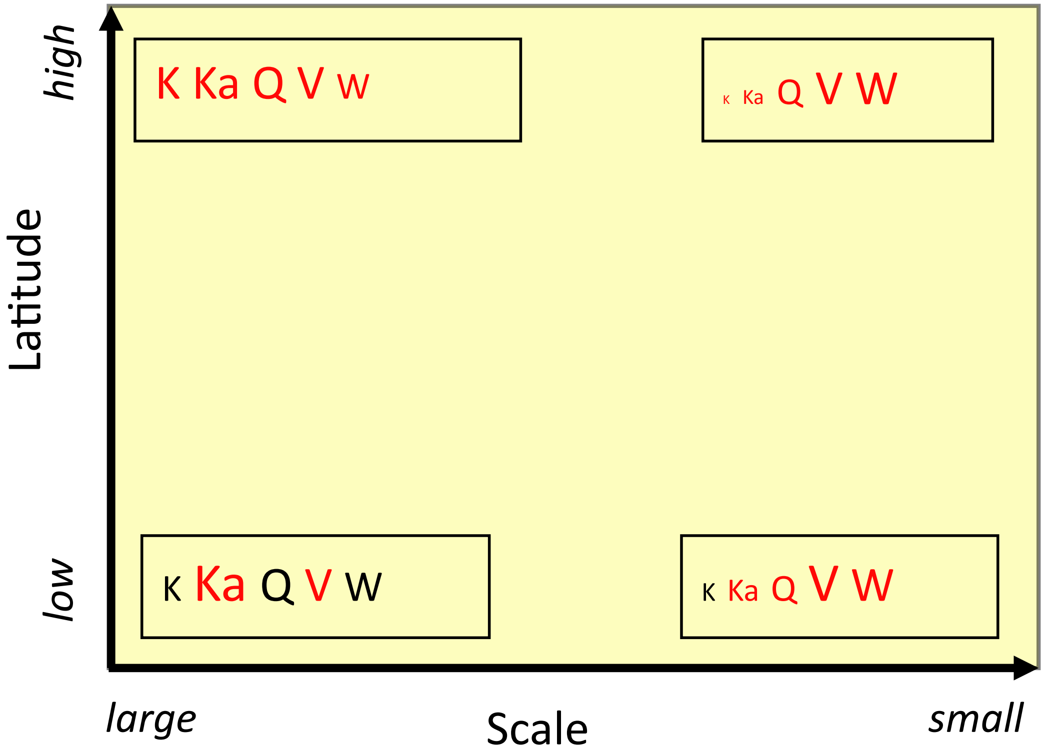
ILC implemented by many different authors/teams

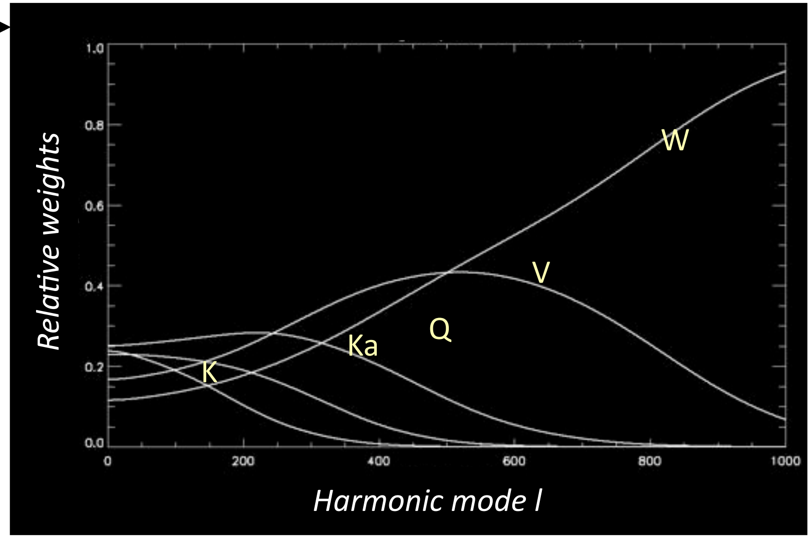
- *WMAP team (2003)*
- *Tegmark et al. (2003)*
- *Eriksen et al. (2004)*
- *Park et al. (2007)*
- *Saha et al. (2008)*
- *Delabrouille et al. (2009)*
- *Kim et al. (2009)*
- *Samal et al. (2010)*

Optimisation ?

Delabrouille, Cardoso, Le Jeune et al. 2009, A&A, 493, 835

In the absence of foregrounds





The Needlet ILC

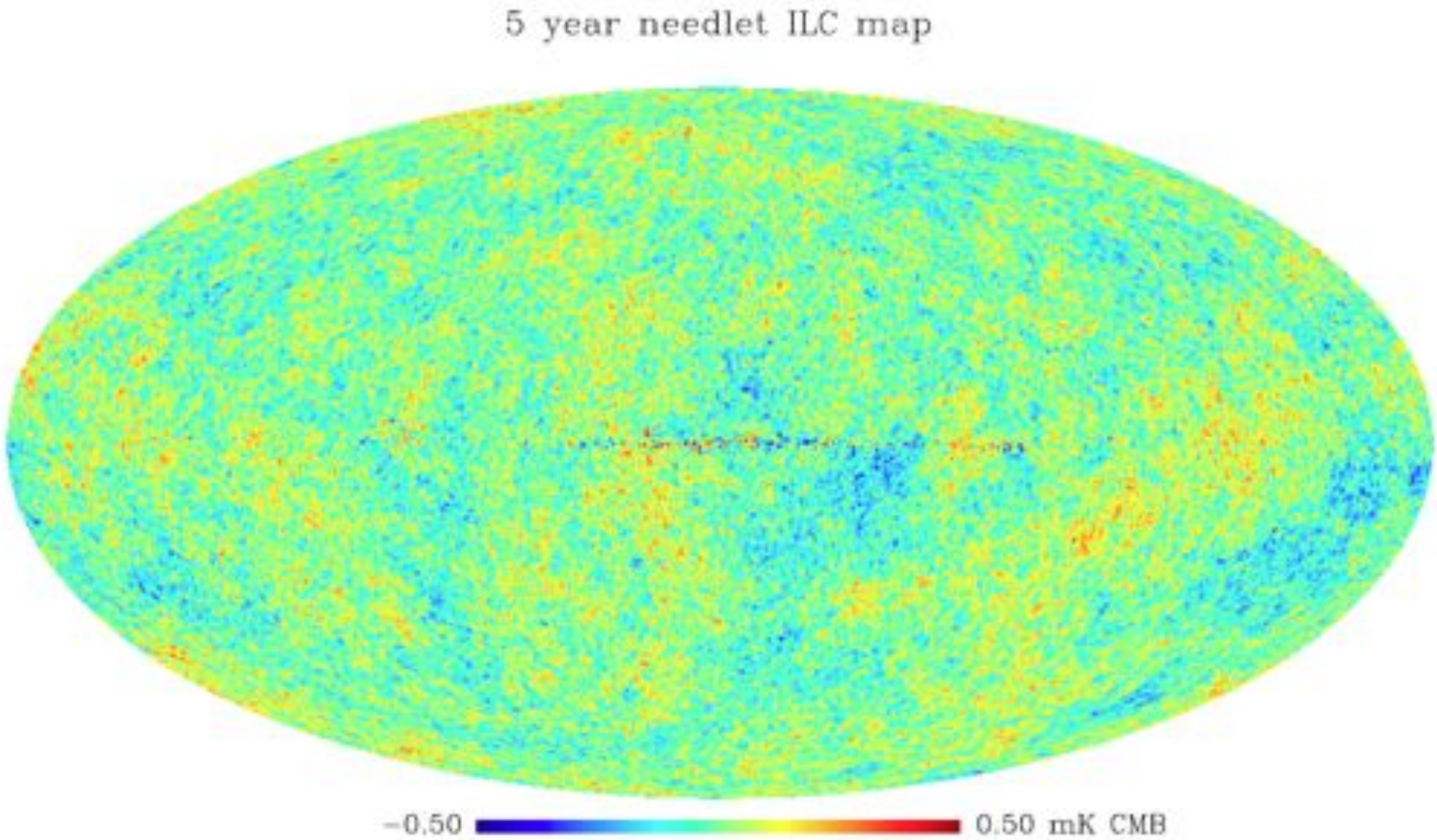
- Use needlet (wavelet) decompositions for defining domains of the data set that are localised in both space and harmonic domain;
- Implement an ILC independently in each needlet domain;
- Reconstruct the final CMB map from CMB needlet coefficients obtained by the local ILCs.

Used on WMAP data →

Delabrouille et al., A&A 493, 835 (2009)
Basak & Delabrouille, MNRAS, 419, 1163 (2012)
Basak & Delabrouille, arXiv 1204:0292

Used for CMB subtraction in Planck
Used for COrE proposal and white paper

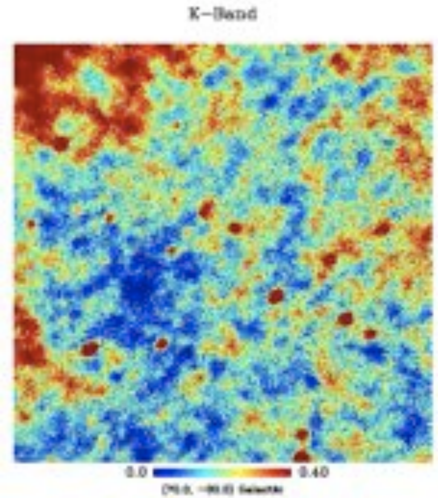
The CMB



CMB and foreground emission

Around *l,b = (70°,-30°)* – Moderate galactic latitudes

Original map



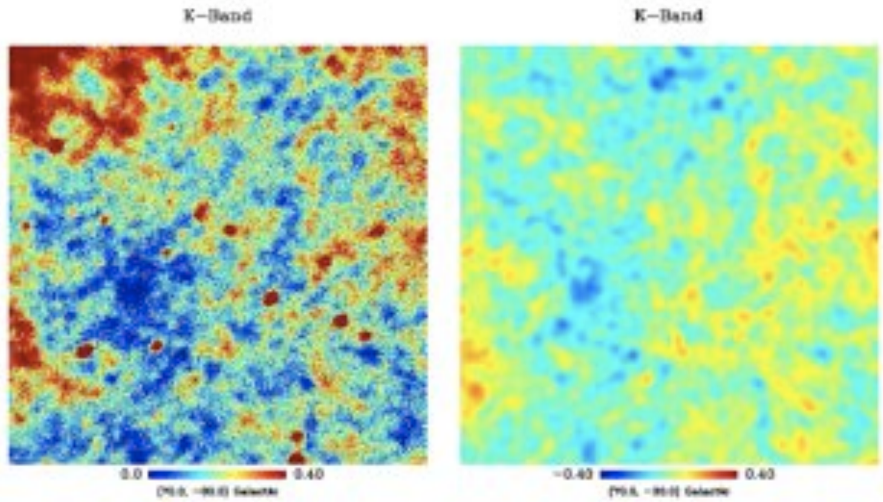
WMAP K band at 23 GHz dominated by galactic synchrotron

In a given pixel:
- CMB?
- galactic ISM?
- radio source ?

CMB and foreground emission

Delabrouille, Cardoso, Le Jeune et al., 2009, A&A 493, 835

Original map



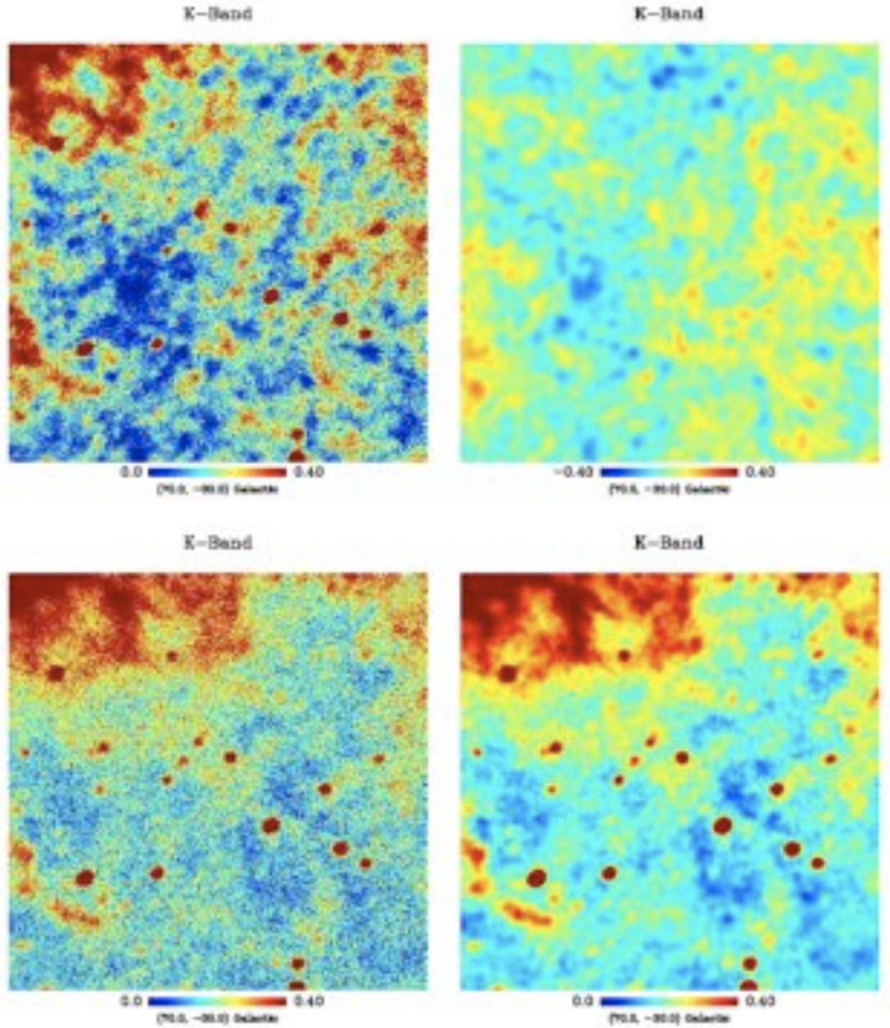
Estimated CMB

CMB and foreground emission

Delabrouille, Cardoso, Le Jeune et al., 2009, A&A 493, 835
Ghosh, Delabrouille, Remazeilles et al., 2011, MNRAS, 412, 883

Original map

Estimated CMB

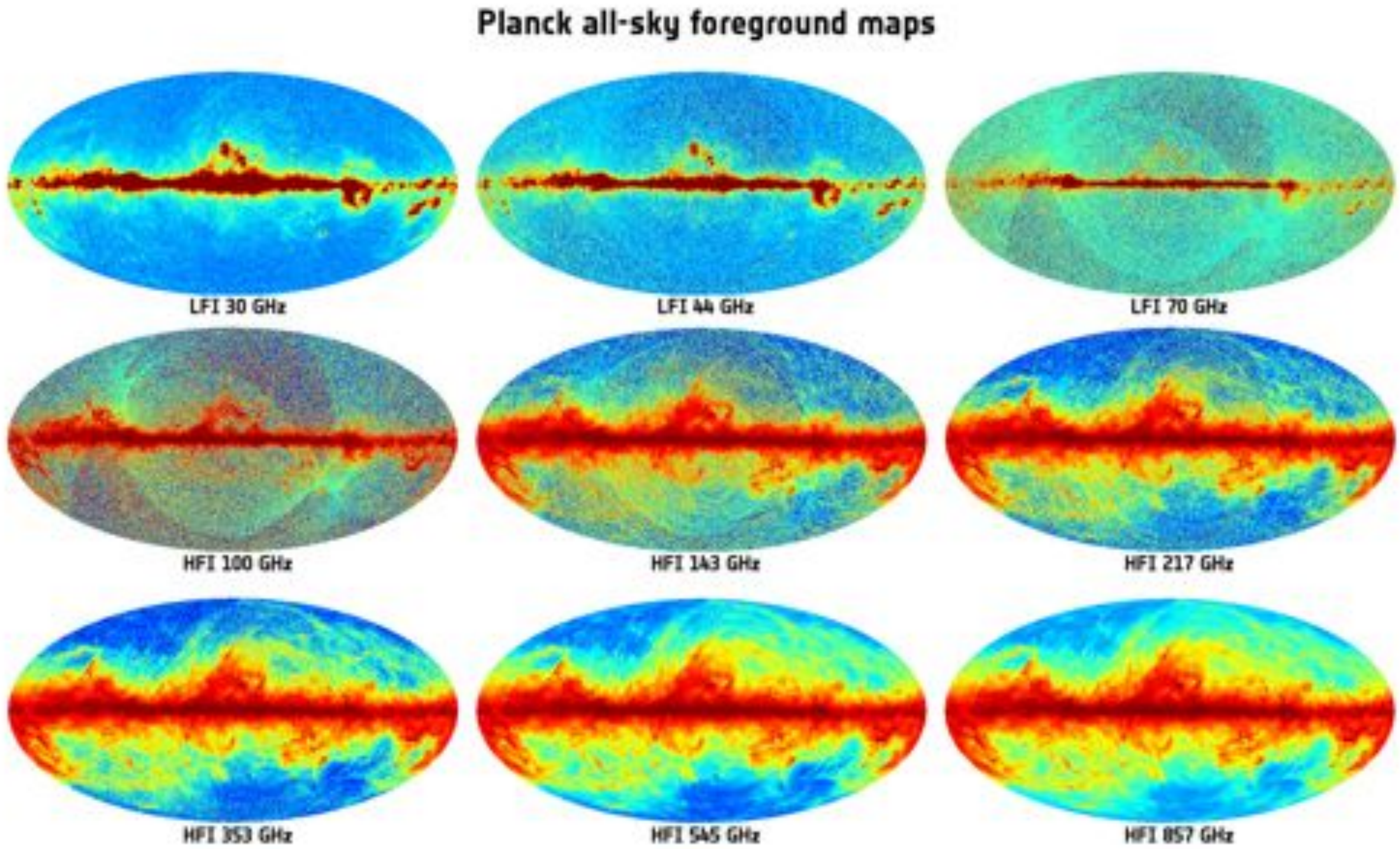


CMB subtraction

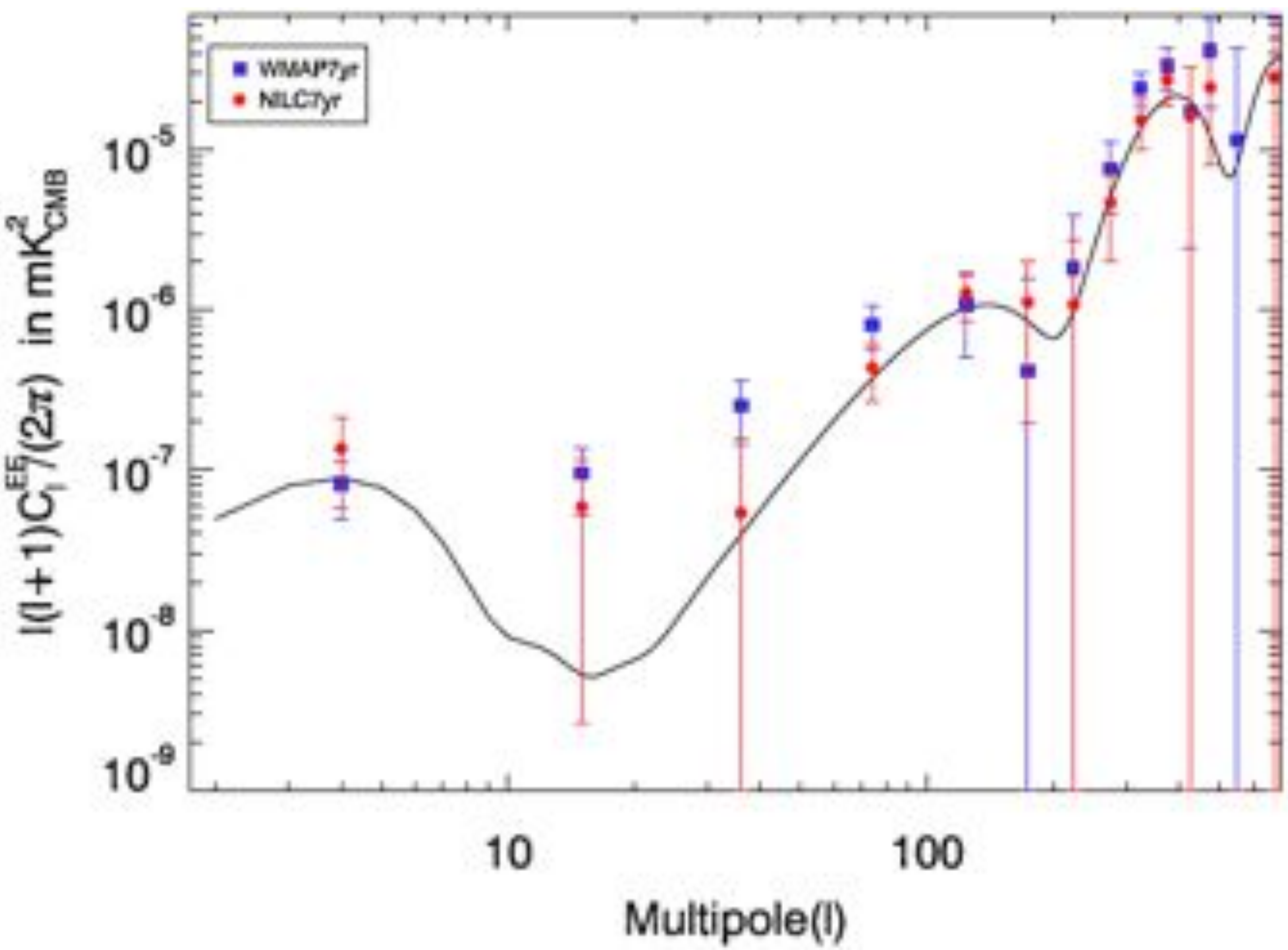
Map with no CMB

Map after Additional filtering

Planck Maps after subtraction of the CMB



WMAP - C^{EE}



Basak & Delabrouille, 2012

Limitations of the standard ILC

- You can only recover a component which
  - Has a known emission law — see however *Remazeilles, Delabrouille & Cardoso 418, 467 (2011)*
  - Is not correlated with anything else

- Empirical correlations do matter (they induce a bias)
  - A fraction of the CMB modes are "killed" by the minimization ($n_{channels}$-1 modes)
  - See appendix of *Delabrouille et al., A&A 493, 835 (2009)*
  - Compromise between localisation and number of modes used to compute statistics

- Calibration accuracy does matter
  - If the emission law is not well known (or the instrument is not well calibrated) then the ILC constraint does not preserve the CMB well enough
  - The minimization then "conspires" to kill away some of the CMB
  - This happens for high S/N measurements
  - *Dick, Remazeilles & Delabrouille, MNRAS 401, 1602 (2010)*

- Minimum variance criterion (does not guarantee vanishing FG contamination)

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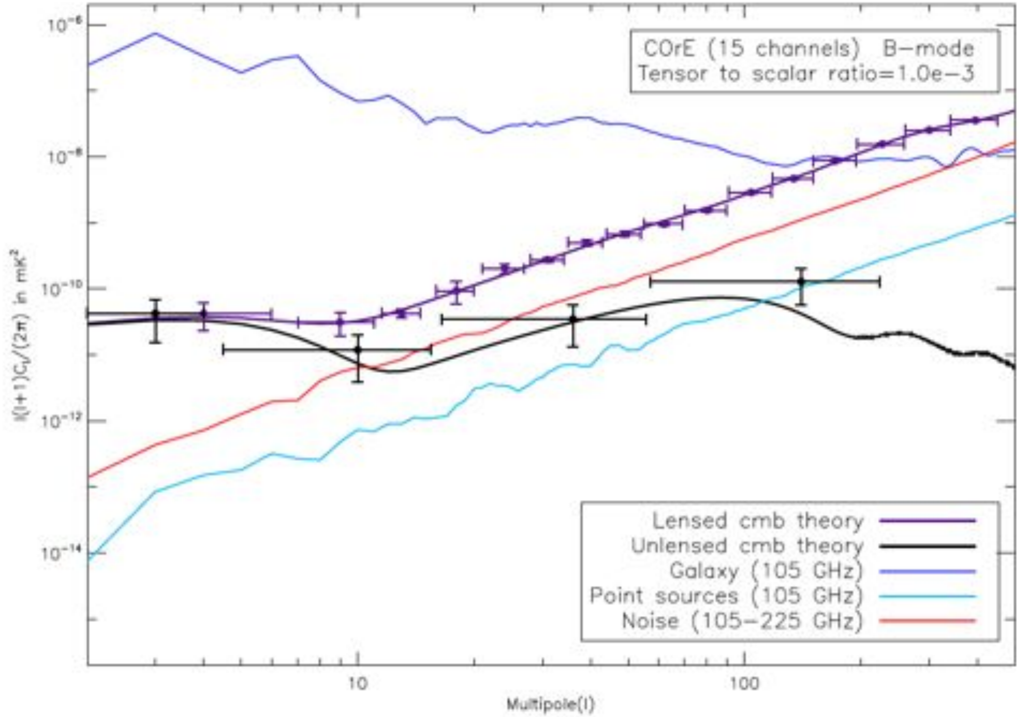
How do we know that it works ?

Solution 1:
Simulations

COrE: *COrE collaboration, arXiv, 2011*

Planck: *Leach et al. A&A, 2008*



Obvious caveat: are the simulations representative ?

How do we know that it works ?

Solution 2: The dimension of the signal subspace

CMB Foregrounds Noise

$$x(\nu, p) = s(p) + u(\nu, p) + n(\nu, p)$$

We suppose that the three terms are pairwise decorrelated. Then we have

$$R_x = R_s + R_u + R_n$$

We suppose that R_n is known. Then we can whiten the data by multiplication by $R_n^{-1/2}$. For the new data set,

$$R_x = R_s + R_u + \mathrm{Id}$$

We can diagonalise R_x. In the basis of diagonalisation, the covariance becomes

$$R_x = \Delta + \mathrm{Id}$$

Remazeilles, Delabrouille & Cardoso, MNRAS 418, 467 (2011)

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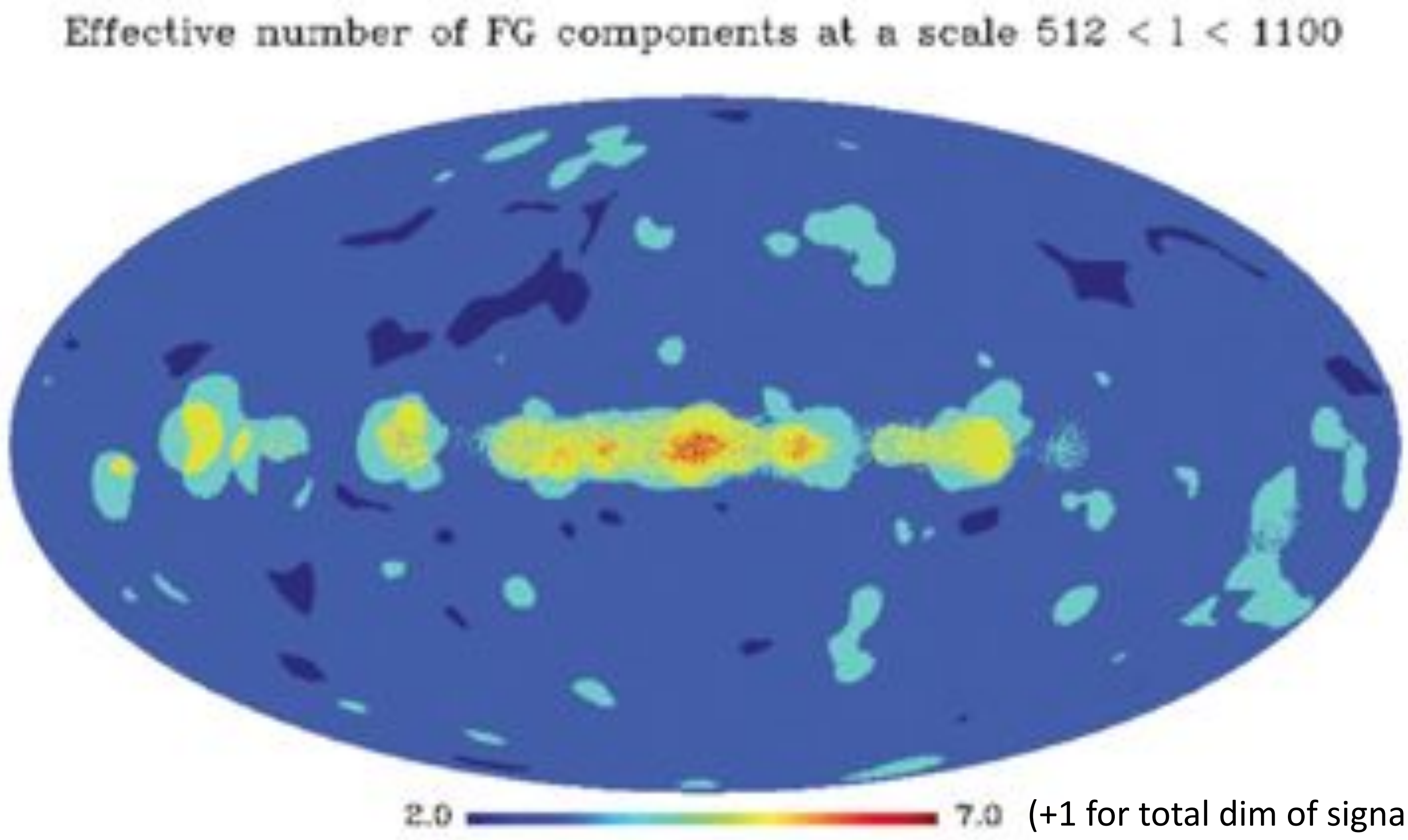
$$\begin{pmatrix} 1+\delta_1 & & & \\ & 1+\delta_2 & & \\ & & \ddots & \\ & & & 1+\delta_n \end{pmatrix}$$

Remazeilles, Delabrouille & Cardoso, MNRAS 418, 467 (2011)

Rank of the signal subspace: example



(+1 for total dim of signal subspace)

We know a posteriori where to mask !

Remazeilles, Delabrouille & Cardoso, MNRAS 418, 467 (2011)

Outline

- Multi-component sky emission
- The Planck sky model
- Topics in component separation
- Validation
- ➡ Conclusion

Summary on component separation

- Increasingly important (the ultimate challenge !)
  - Significant progress in the past 15 years
- Number of bands AND angular resolution are important
  - The future CMB space mission must have both
- The EASY things (at least conceptually)
  - Fit for parameter values of a parametric model, and propagate errors
  - get an ILC map of a component that has a known fixed emission law and is not correlated with other emissions
  - Do a blind ICA assuming a perfect linear mixture
- The NOT SO EASY things
  - Optimize the above approaches
  - Get reliable errors (contamination, biases)
  - Interpret ICA outputs
- The DIFFICULT (and interesting) things
  - Learn the model from the data
  - Make a universal pipeline that works for all components

The future

- Progress towards a coherent, global model of multi-component sky emission
  - A pre-launch PSM is about to be published
  - Extensively used as a simulation tool for Planck
  - Being enriched by the Planck observations

- Tight connection between sky model and component separation
  - Learn and develop methods on simulated data sets
  - The sky model summarises our understanding in a single coherent picture

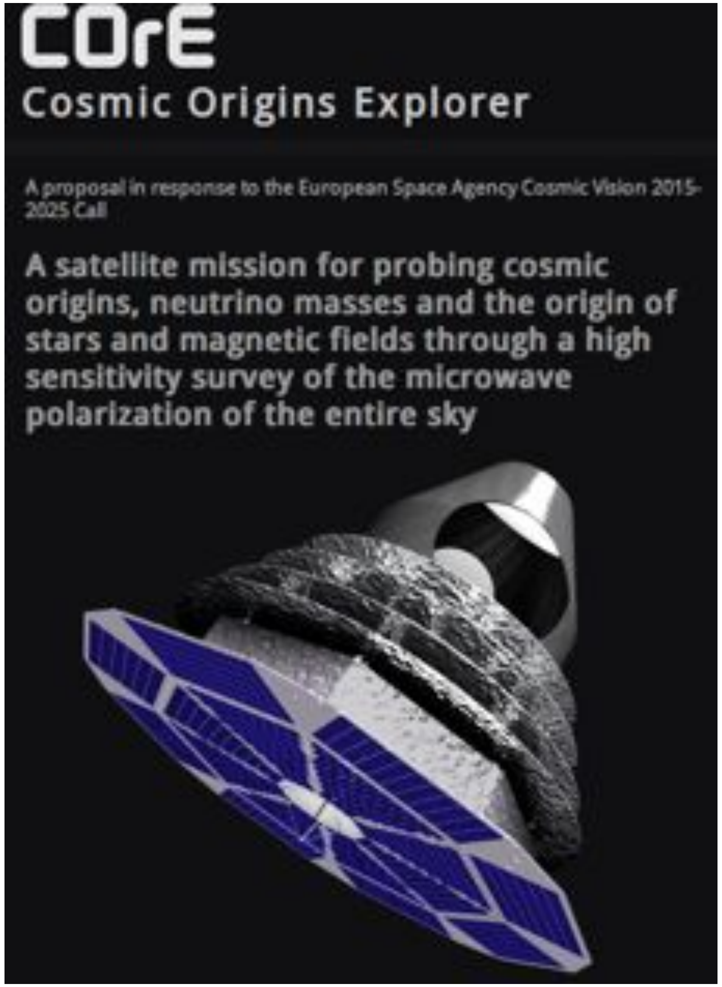
- Crucial research for preparing the future CMB / millimeter-wave space mission

For the future

- More channels for future experiments
  - COBE-DMR had 3 channels
  - WMAP had 5
  - Planck has 9

ν	$(\Delta\nu)$	n_{det}	θ_{fwhm}	Temp (I) μK-arcmin		Pol (Q,U) μK-arcmin	
GHz	GHz		arcmin	RJ	CMB	RJ	CMB
45	15	64	23.3	4.98	5.25	8.61	9.07
75	15	300	14.0	2.36	2.73	4.09	4.72
105	15	400	10.0	2.03	2.68	3.50	4.63
135	15	550	7.8	1.68	2.63	2.90	4.55
165	15	750	6.4	1.38	2.67	2.38	4.61
195	15	1150	5.4	1.07	2.63	1.84	4.54
225	15	1800	4.7	0.82	2.64	1.42	4.57
255	15	575	4.1	1.40	6.08	2.43	10.5
285	15	375	3.7	1.70	10.1	2.94	17.4
315	15	100	3.3	3.25	26.9	5.62	46.6
375	15	64	2.8	4.05	68.6	7.01	119
435	15	64	2.4	4.12	149	7.12	258
555	195	64	1.9	1.23	227	3.39	626
675	195	64	1.6	1.28	1320	3.52	3640
795	195	64	1.3	1.31	8070	3.60	22200

COrE summary (4 year mission)

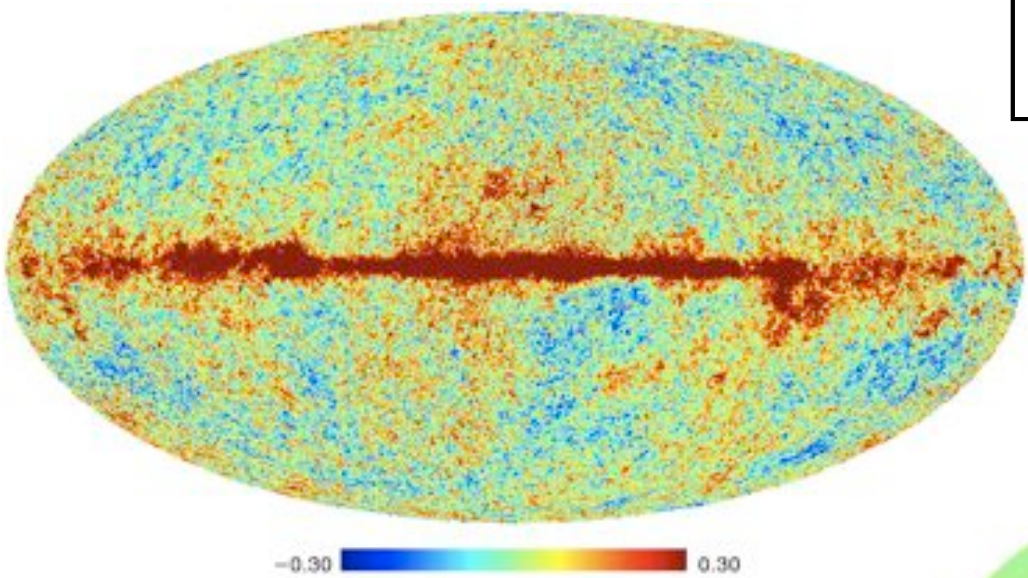


- Further investigation requires simulations and a realistic model of sky emission: the Planck Sky Model…

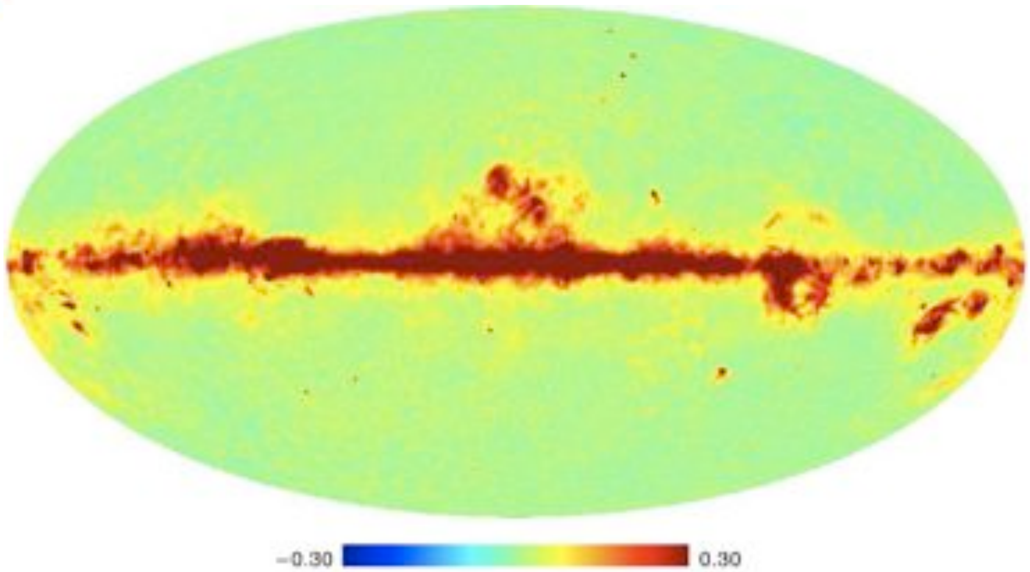
The END

Units in mK

V band (61 GHz)
Original map



V band (61 GHz)
Processed map



The CMB subtraction implemented by the DPCs for preparing the data used in the Planck early papers is very similar to this (except for the final localized filtering)

Constrained ILC

$$\mathbf{x}(p) = \boxed{\mathbf{a}s(p)} + \boxed{\mathbf{b}z(p)} + \boxed{\mathbf{n}(p)}$$

CMB

Thermal SZ

Everything else

$$\widehat{s}_{\mathrm{ILC}}(p) = \sum_i w_i x_i(p) = \mathbf{w}^t \mathbf{x}(p)$$

$$\text{minimize } \sum_p \left|\widehat{s}(p)\right|^2$$

Two constraints

$$\sum_i w_i a_i = \mathbf{w}^t \mathbf{a} = 1$$

$$\sum_i w_i b_i = \mathbf{w}^t \mathbf{b} = 0$$

$$\widehat{s}_{\mathrm{constr.ILC}}(p) = \frac{(\mathbf{b}^t \widehat{\mathbf{R}}_x^{-1} \mathbf{b}) \mathbf{a}^t \widehat{\mathbf{R}}_x^{-1} - (\mathbf{a}^t \widehat{\mathbf{R}}_x^{-1} \mathbf{b}) \mathbf{b}^t \widehat{\mathbf{R}}_x^{-1}}{(\mathbf{a}^t \widehat{\mathbf{R}}_x^{-1} \mathbf{a})(\mathbf{b}^t \widehat{\mathbf{R}}_x^{-1} \mathbf{b}) - (\mathbf{a}^t \widehat{\mathbf{R}}_x^{-1} \mathbf{b})^2} \mathbf{x}(p)$$

M. Remazeilles, J. Delabrouille, J.-F. Cardoso, MNRAS 408, 2481 (2011)