Measurement of Electrostatic Dissipation on GRS for LISA / LPF


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Content

1. Electrostatic dissipation noise on GRS
2. New measurement challenge with two techniques
3. Discussion and implication to LISA
1.1 Electrostatic dissipation-related noise

- Two sources to produce electrostatic dissipation: [1, 2]
  1. Ohmic delay $\tau$ arising from sensor circuitry (for instance due to the LPFilter effect)
     \[
     \delta_{\text{Ohm}} = \omega \tau
     \]
  2. Freq-independent electrostatic loss arising from conductor surface
     \[
     \delta_{\text{ES}} : \text{more dangerous at low freq}
     \]

- Dissipation contributes potential fluctuation $V_n$:
  mixing with TM potential $\delta V$ to produce force noise on TM

- LISA requires $\delta < 10^{-5}$ [3]

$$S_a^{1/2}(f) \sim 3 \times 10^{-15} \text{ m}^2/\sqrt{\text{Hz}} \left( \frac{\delta}{10^{-5}} \right)^{1/2} \left( \frac{10^{-4} \text{ Hz}}{f} \right)^{1/2} \left( \frac{Q_M}{10^7 \text{ e}} \right)$$

1.2 Dissipation model

Actuation / Sensing Circuitry (ohmic losses $\delta_{\text{ohm}}$)

Intrinsic surface losses (model for freq-ind $\delta_{\text{ES}}$) *

Ideal test mass (electrode capacitance $C$)

$\tau \approx (R_1 + R_2)C_2 + R_1C_1$

$\delta_{\text{Ohm}} = \omega \tau$

$Z_\delta = \frac{1}{i \omega C \ln(\tau_{\text{MAX}} / \tau_{\text{MIN}})} \left\{ i \tan^{-1} \omega \tau_{\text{MAX}} - \tan^{-1} \omega \tau_{\text{MIN}} \right\} + \frac{1}{2} \ln \left( \frac{1 + (\omega \tau_{\text{MAX}})^2}{1 + (\omega \tau_{\text{MIN}})^2} \right)$

$\approx \frac{1}{i \omega C} \left[ \frac{C \pi}{2 \Delta \ln(\tau_{\text{MAX}} / \tau_{\text{MIN}})} \right] \left( i + \frac{2}{\pi} \ln \omega \tau_{\text{MAX}} \right)$

$\tau_{\text{MIN}} \ll \tau = R_i C_\Delta \ll \tau_{\text{MAX}}$

$\frac{1}{i \omega C_e} = \frac{1}{i \omega C} + Z_\delta$

$\delta_{\text{ES}} = \frac{\text{Im}(C_e)}{\text{Re}(C_e)} \approx \frac{C \pi}{2 \Delta \ln(\tau_{\text{MAX}} / \tau_{\text{MIN}})} \propto \frac{1}{d}$

$F_{\text{ele}} \propto \frac{1}{d^2}$


1.3 Recent measurements on electrode noise (2012)

- In general, noisy electrostatic potentials mix with DC voltage differences – such as that caused by TM charging – to create force noise

\[
S_{F}^{1/2} = \left( \frac{q}{C_{r}} \right) \frac{\partial C}{\partial x} S_{\Delta x}^{1/2} \approx 1.3 \text{ fN/Hz}^{1/2} \times \left( \frac{q}{10^7 e} \right) \times \left( \frac{S_{\Delta x}^{1/2}}{100 \mu V/Hz^{1/2}} \right)
\]

- Recent experimental upper limit on \( D_{x} \) of 80 mV / Hz\(^{1/2} \) at 1 mHz *
  \( \Rightarrow \) marginally OK for LISA

- \( \delta = 10^{-5} \) would give 20 mV / Hz\(^{1/2} \) at 0.1 mHz

* F. Antonucci et al, PRL 108 181101 (2012)
1.4 Previous measurements on surface loss (2005)

Object: A GRS prototype, Au-coated Mo electrodes, 2 mm capacitive gap

Tech: directly measure transient force due to dissipation by square wave modulation [1, 2]

Square mod and Comp voltages

Transient force and loss extraction

Results:
For 1W2E pair, \( \delta = ( -0.3 \pm 1.0 ) \times 10^{-7} \)
For 2W1E pair, \( \delta = ( 10.6 \pm 0.6 ) \times 10^{-7} \)

Motivation of next step:
investigate dielectric loss for LPF GRS flight model with higher precision

### 2.1 Measurement with ringdown tech (2010~2012)

**Object:** A LPF flight model, Au-coated electrodes, 4 mm capacitive gap along x

**Tech1:** measure pendulum amplitude decay as a function of electrode voltages

\[
\frac{1}{\tau_{\text{exp}}} = \frac{1}{\tau_v} + \frac{1}{\sqrt{1/(\Gamma_F + \Gamma_{\text{ES}})}} \cdot (\Gamma_F \cdot \delta_F - \Gamma_{\text{ES}} \cdot \delta_{\text{ES}})
\]

Viscous damping \( x 10^6 \)  
Fiber damping  
Electrostatic damping

![Graph showing decay time extraction](image)

**Table:**

<table>
<thead>
<tr>
<th>Fit condition</th>
<th>( \delta_{\text{ES}} ) (10^{-7})</th>
<th>( \delta_F ) (10^{-7})</th>
<th>( 1/\tau_v ) (10^{-8} /s)</th>
<th>( \chi^2 )</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 free parameters</td>
<td>7.6 (7.5)</td>
<td>-4 (12)</td>
<td>1.1 (1.6)</td>
<td>1.2 (20 DOF)</td>
<td>Physical</td>
</tr>
<tr>
<td>Assuming ( 1/\tau_v ) only from gas</td>
<td>4.5 (1.3)</td>
<td>8.7 (.4)</td>
<td>0.43</td>
<td>1.2 (21 DOF)</td>
<td>Physical</td>
</tr>
<tr>
<td>Assuming ( \delta_{\text{ES}} = 0 )</td>
<td>- -</td>
<td>16 (.03)</td>
<td>-0.5 (-.3)</td>
<td>1.2 (21 DOF)</td>
<td>Not physical</td>
</tr>
</tbody>
</table>

**Result:** upper limit of \( \delta_{\text{ES}} 1.5 \times 10^{-6}. \)
2.2 Measurement with modulation tech (2011~2012)

Tech2: Use perfect Square Wave modulation (ideally gives constant torque proportional to $V^2$), delays due to lossy elements cause force transients proportional to delta at every square wave transition

![Diagram of Square mod and Com voltages and Circuit and surface losses (τ & δ_ES)]

- +/- $V_{com}$ and out of phase square wave applied to diagonal pairs, to avoid the change of TM potential
- enable to measure circuit loss and surface loss $\tau$ & $\delta_{ES}$ together
- Huge DC torque ($\sim 10^{-10} \text{ N m}$) cancelled, small transient torque left ($\sim 10^{-17} \text{ N m} \leftrightarrow \delta_{ES} \sim 10^{-7}$) for losses extraction

Ideal SW to one diagonal pair

$$V_{MOD} = \sum_{j \text{odd}} \frac{4V_M}{j\pi} \sin(\omega_M t)$$

Compensation voltages to the other pair

$$N_{DC} \equiv \frac{\partial C}{\partial \phi} V_{COM}^2$$

Huge DC torque cancelled
Small transient torque left

* L. Carbone et al, 6th Amaldi, Japan, 2005
2.3 Illustration of measured signal

Electrode voltage:  

Resulting torque ($\propto V^2$):

No losses

Ohmic delay

Frequency independent $\delta$

• 2f signal (+ other even harmonics)
• Nearly “Dirac $\delta$-function” for ohmic delay (all cosine)
• Longer lived signal for frequency independent $\delta_{ES}$ (both sine and cosine)

\[
\begin{align*}
\frac{-N_{2f}}{N_{DC}} & \approx \left[ \delta_{ES} \cdot \left( \frac{4}{\pi} \right)^2 \frac{2}{\pi} \left( \sum_{j, \text{odd}} \frac{\ln(j+2)}{j(j+2)} \right) + 8 f_M \tau \right] \cos (2\omega_M t) + \left[ \delta_{ES} \cdot \left( \frac{4}{\pi} \right)^2 \right] \sin (2\omega_M t) \\
& \approx 0.681 \\
\frac{-N_{4f}}{N_{DC}} & \approx \left[ \delta_{ES} \cdot \left( \frac{4}{\pi} \right)^2 \frac{2}{\pi} \left( \sum_{j, \text{odd}} \frac{\ln(j+4)}{j(j+4)} - \frac{\ln 3}{3} \right) + 8 f_M \tau \right] \cos (4\omega_M t) + \left[ \delta_{ES} \cdot \left( \frac{2}{3} \right) \left( \frac{4}{\pi} \right)^2 \right] \sin (4\omega_M t) \\
& \approx 0.574 \\
& \approx 0.67
\end{align*}
\]
2.4 Sensitivity requirements

For resolution $\delta_{ES} = 10^{-7}$

$\Rightarrow$ with $V_M = 8$ V need 2f torque resolution 0.02 fNm

Noise of current TP $\sim 1$ fNm/Hz$^{1/2}$ @ 1mHz

10000 s statistical resolution $\Rightarrow 0.01$ fNm @ 1mHz

External torque noise of TP with fused silica fiber
2.5 Experimental noises and suppression

Challenges and solutions:

• **Actuation filter gives 400 μs ohmic delay (δ 10^{-5} at 3 mHz)**
  → Short filter (and also 100 kHz injection, readout transformer circuitry, also responsible for 2f force transients)

• **Large DC bias torque signals at odd harmonics (non-linear coupling into 2f/4f/6f signal)**
  → Correct mod electrode potential difference to < 1 bit resolution
    +/− 150 μV still gives 3 fNm signal
  → Compensation of mean electrode potential to null translation dependence

• **AC detector non-linearities, semi-periodic jumps of up to 100 nrad (1 fN m)**
  → Calibrate and correct in data pre-processing

• **Modulation of electrostatic stiffness at 1f mixes with DC bias signals to give 2f signal**
  → Measure period difference, correct stiffness in time domain torque extraction

• **V_m modulates V_TM when off-centred, fake delta signal due to switch of V_TM**
  (Δx = 50 um ↔ δ = 3e-8)
  → Calibrate and correct in data post-processing
2.6 Experimental transient torque

- Tests for both pairs: $f_M = 1.25 \text{ mHz}$, $V_M = 8 \text{ V}$, integral time $> 80 \text{ h}$;
- Get the averaged transient torque and compare with theoretical prediction;
- The signal form is as expected for model!

(model works well, $\delta_{ES} \sim 3e-7$)
2.7 Torque demodulation and $\delta$ extraction

Data Process:

(1) Several tests at different freq in [0.2 2] mHz,
(2) For each run, demod sin/cos torque for 2f/4f/6f
(3) Fit to sin torque of one group to get $\delta_{ES}$ and fit to cos torque to get both $\delta_{ES}$ and $\tau$

Result: $\delta_{ES}$ and $\tau$ from different harmonics consistent in 2$\sigma$
2.8 Investigation of dependence of delta on gap

- Purpose for further study:
  1. Experimentally study if $\delta_{ES}$ proportional to C, compare with theoretical analysis
     - Rotate housing (or TM) to change capacitance
  2. Test $\delta_{ES}$ in different freq to see if $\delta_{ES}$ freq-ind
     - test at different $f_{MOD}$ as wider as our experimental resolution allows

\[ \delta_{ES} \approx \frac{C\pi}{2C_{\Delta} \ln\left(\frac{\tau_{MAX}}{\tau_{MIN}}\right)} \propto \frac{1}{d} \]

For each diagonal pair, test $\delta_{ES}$ at three configurations:

- $d_{0}=4\text{mm}$
- $\Delta d \approx 0.4\text{mm}$

- $\varphi=-30\text{ mrad}$
- $\varphi=0$
- $\varphi=30\text{ mrad}$

rotating housing clockwise
2.9 Result of residual $\tau$

For each pair, fit all data from all three configurations:

<table>
<thead>
<tr>
<th>Pair</th>
<th>$\tau_1$ (μs)*</th>
<th>$\tau_e$ (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1N/2S</td>
<td>3.4</td>
<td>-1.6(0.7)</td>
</tr>
<tr>
<td>2N/1S</td>
<td>3.4</td>
<td>3(1)</td>
</tr>
</tbody>
</table>

* residual ohmic delay from slew rate of mod switch

Result:
- residual $\tau$ consistent with theoretical expectation in few $\sigma$;
- $\delta_{ES}$ is freq-independent in freq range tested
- for 1N2S, there still some systematic error

$$\Delta \delta \approx 2 \times 10^{-8} \left( \frac{f_M}{1 \text{ mHz}} \right) \left( \frac{\Delta \tau}{3.4 \text{ μs}} \right)$$
2.10 Result of $\delta_{ES}(X_n = 20 \, \mu \text{m})$

* After taking into account the translation readout uncertainty 20 $\mu$ m

<table>
<thead>
<tr>
<th></th>
<th>smaller gap</th>
<th>centered</th>
<th>Larger gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{ES}$</td>
<td>3.18(0.05)</td>
<td>2.83(0.07)</td>
<td>2.67(0.07)</td>
</tr>
<tr>
<td>Chi^2</td>
<td>1.3 (408 DOF)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>smaller gap</th>
<th>centered</th>
<th>Larger gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{ES}$</td>
<td>3.49(0.12)</td>
<td>2.95(0.07)</td>
<td>2.68(0.09)</td>
</tr>
<tr>
<td>Chi^2</td>
<td>1.1 (270 DOF)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) Feature proportional-to-$C$ verified experimentally, tests in a relative wider capacitance range may be also interesting;

(2) $\delta_{ES}$ from two pairs sensitive in our facility seem uniform!
3.1 Discussion: possible sources of $\delta_{ES}$

* Before venting: GRS kept staying in $3 \times 10^{-6}$ Pa for more than two years
* Venting: venting chamber to lab atmosphere for three days by a $\Phi$ 2.5 cm tube of length $\sim$ 30 cm (to absorb dielectric)
* After venting: pump chamber to pressure lower than $10^{-5}$ Pa

<table>
<thead>
<tr>
<th>Position</th>
<th>When TM is centered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2N/1S$</td>
<td>46 runs (before venting)</td>
</tr>
<tr>
<td></td>
<td>8 runs (after venting)</td>
</tr>
<tr>
<td>$\delta_{ES}$ (1.e-7)</td>
<td>$2.95(0.07)$</td>
</tr>
<tr>
<td>$\tau_e$ (μs)</td>
<td>3(1)</td>
</tr>
<tr>
<td>Chi^2</td>
<td>1.1 (270 DOF)</td>
</tr>
</tbody>
</table>

Result:
(1) Tests are basically reproducible, $\delta_{ES}$ before/after venting consistent in $2\sigma$ (difference only 5%)
(2) $\delta_{ES}$ should not arise from the surface layers condense quickly from lab atmosphere
3.1 Discussion: possible sources of $\delta_{ES}$

Assuming some dielectric materials absorbed by conductor surface:

$$\varepsilon_r = \varepsilon_{r0}(1 - j\delta_{\varepsilon})$$

$$\delta_{ES} = \frac{\delta_{\varepsilon}}{\varepsilon_r(d-t)+t} \cdot t$$

$$\approx \frac{\delta_{\varepsilon}}{\varepsilon_r} \cdot \frac{t}{d} \quad (t \ll d)$$

$\delta_{ES} = 2.9e-7$, $d = 4$ mm

$\varepsilon_r \sim 8$, $\delta_{\varepsilon} \sim 0.01$, $t \sim 1$ $\mu$m

Unlikely that a surface absorbate could be so thick! Need more study ...

<table>
<thead>
<tr>
<th>Materials *</th>
<th>$\varepsilon_r$</th>
<th>$\delta_{\varepsilon}(10^{-4})$</th>
<th>$t$ ($\mu$m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fused quartz</td>
<td>3.8</td>
<td>10 (@50Hz)</td>
<td>4.4</td>
</tr>
<tr>
<td>Alumina</td>
<td>8.5</td>
<td>20 (@50Hz)</td>
<td>4.9</td>
</tr>
<tr>
<td>Air</td>
<td>1</td>
<td>100?</td>
<td>0.12</td>
</tr>
</tbody>
</table>

*http://www.kayelaby.npl.co.uk/general_physics/2_6/2_6_5.html
3.2 Summary

(1) 1N2S pair: $\delta_{ES} = (2.83\pm0.07)\times10^{-7}$

2N1S pair: $\delta_{ES} = (2.95\pm0.07)\times10^{-7}$

Loss angle from delay: $\delta_{\text{ohm}} < 2.5\times10^{-8}$ (at 1 mHz)

For LISA, force noise due to the loss lower than 0.1 fm/s²/Hz¹/₂ near 1mHz

(2) Torque wave form due to the losses obeys model very well

(3) The tests varying gap: $\delta \propto C$ (match model well)

(4) $\delta$ is freq-independent in freq range tested (match model well)

(5) $\delta$ unchanged by venting to air for three days

(6) What causes $\delta$? Need more study...
Thank you!
Any questions are welcome!
2.5 Experimental noises and suppression

(1) Current actuation filter gives ohmic delay $\tau \sim 0.4$ ms ($\delta 10^{-5}$ at 3 mHz)
   - Short filters, reduce ohmic delay to $\tau \sim 4$ $\mu$ s, torque contribution from 1 fNm to 0.01 fNm (also 100 kHz injection, readout transformer circuitry, also responsible to 2f torque transient)

(2) AC detector non-linearities, semi-periodic jumps of up to 100 nrad (1 fN m)
   - Calibrate and correct in data pre-processing
2.5 Experimental noises and suppression

(3) Large DC bias torque signals at odd harmonics (non-linear coupling into 2f/4f/6f signal)

→ Carefully compensate stray DC bias of electrodes, minimize variable torque at odd harmonics (Correct mod electrode potential difference to < 1 bit resolution +/- 150 μV still gives 3 fNm signal; Compensation of mean electrode potential to null translation dependence)

\[ N(\omega t) \approx V_M \text{sqr}(\omega t) \cdot \left( \delta V_{1N} - \delta V_{2S} \right) \left( \frac{\partial^2 C}{\partial \varphi^2} \right) + \left( x - x_0 \right) \left( \delta V_{1N} + \delta V_{2S} - 2V_{TM} \right) \frac{\partial^2 C}{\partial \varphi \partial x} \]

(4) Modulation of electrostatic stiffness at 1f (\( \Delta \Gamma \sim 2e-11 \text{ Nm/ rad} \)) mixes with DC bias signals to give 2f signal

→ Measure period difference, correct stiffness in time domain torque extraction
2.5 Experimental noises and suppression

- TM translation has another effect: TM potential fluctuation $V_{TM}$ caused by $V_{MOD}$. The imperfect switch of $V_{TM}$ on transient moment also produces loss mixing with $\delta_{ES}$.

$\rightarrow$ calibrate tilt-charge effect, correct in data post-processing. (Near centred position: $x=50\mu m \leftrightarrow 10\% \delta$)

$$V_{T0} = \frac{4V_{M}}{d_0} \cdot \Delta x \cdot sq(\omega_M t)$$

$$N_{VT0} = \frac{\partial^2 C}{\partial \phi \partial x} \cdot \Delta x \left[ -2V_M \cdot sq(\omega_M t)V_{T0} \cdot sq(\omega_{VT0} t) \right] \propto \Delta x^2$$
Summary and discussion

(1) The study focuses on the dielectric dissipation of GRS flight model.
(2) Ringdown tech and square-wave mod tech employed. Final result of $\delta_{ES}$ is 2.9e-7, with uncertainty lower than 10%, for all electrodes tested. The value implies its contributed noise for LISA is lower than 0.1 fm/s²/Hz¹/².
(3) Features proportional-to-C and freq-ind are experimentally tested, and match model very well.
(4) Dielectric dissipation on the electrodes studied seem have a good uniformity.
(5) The physical sources and distributions on electrodes of $\delta_{ES}$ need more study.

Dissipations of 1TM torsion pendulum associated with GRS flight model surrounding (suspending by fused silica fiber, $V_{inj} = 3.5$ V)

<table>
<thead>
<tr>
<th>Sources</th>
<th>damping</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual gas viscous (fixed)</td>
<td>$d\beta/ dp = 5.6e-8$ Nm/s (at $p = 3.3e-6$ Pa)</td>
<td>3,100,000</td>
</tr>
<tr>
<td>Dielectric loss (fixed)</td>
<td>$\delta_i = 2.9e-7$</td>
<td>26,700,000</td>
</tr>
<tr>
<td>Fiber structural loss</td>
<td>$\delta_{ES} = 9.0(0.3)e-7$</td>
<td>1,020,000 $^*$</td>
</tr>
<tr>
<td>Totally</td>
<td></td>
<td>750,000</td>
</tr>
</tbody>
</table>

$^*$ The rest damping could also come from other sources like magnetic field.
### 3.2 Example of $\delta_{ES}$ and $\tau$ extraction

Demod even torques for each test (2N/1S pair, large gap, $f_M = 0.35$ mHz)

Fitting to all tests from one group to get $\tau$ & $\delta_{ES}$ (2N/1S pair, large gap)

- from cos torque: $\delta_{ES}$ & $\tau$
- from sin torque: only $\delta_{ES}$
- do the same analysis to 2f/4f/6f torque
### 3.3 Example of preliminary result: 2N1S pair

**Table: 2N1S Pair Measurements**

<table>
<thead>
<tr>
<th>C Gap</th>
<th>Smaller gap (group 203: 6 runs)</th>
<th>Centered (group 121: 7 runs)</th>
<th>Larger gap (group 191: 8 runs)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2f cosine</strong></td>
<td>$\delta_{ES} = 3.50(0.71)e^{-7}$ $\tau = 4.2(9.5) \mu s$ $\chi^2 = 2.0 (4 \text{ DOF})$</td>
<td>$\delta_{ES} = 3.05(0.36)e^{-7}$ $\tau = -2.1(5.3) \mu s$ $\chi^2 = 1.0 (5 \text{ DOF})$</td>
<td>$\delta_{ES} = 1.84(0.33)e^{-7}$ $\tau = 18.6(5.0) \mu s$ $\chi^2 = 2.1 (6 \text{ DOF})$</td>
</tr>
<tr>
<td><strong>2f sine</strong></td>
<td>$\delta_{ES} = 3.6(0.17)e^{-7}$ $\chi^2 = 0.5 (5 \text{ DOF})$</td>
<td>$\delta_{ES} = 3.14(0.12)e^{-7}$ $\chi^2 = 0.5 (6 \text{ DOF})$</td>
<td>$\delta_{ES} = 2.55 (0.13)e^{-7}$ $\chi^2 = 0.9 (7 \text{ DOF})$</td>
</tr>
<tr>
<td><strong>4f cosine</strong></td>
<td>$\delta_{ES} = 2.73(0.77)e^{-7}$ $\tau = 13.7(8.7) \mu s$ $\chi^2 = 0.8 (4 \text{ DOF})$</td>
<td>$\delta_{ES} = 1.50(0.70)e^{-7}$ $\tau = 16.3(7.1) \mu s$ $\chi^2 = 0.2 (5 \text{ DOF})$</td>
<td>$\delta_{ES} = 2.15(0.67)e^{-7}$ $\tau = 12.5(6.6) \mu s$ $\chi^2 = 1.2 (6 \text{ DOF})$</td>
</tr>
<tr>
<td><strong>4f sine</strong></td>
<td>$\delta_{ES} = 3.19(0.24)e^{-7}$ $\chi^2 = 0.5 (5 \text{ DOF})$</td>
<td>$\delta_{ES} = 3.05(0.17)e^{-7}$ $\chi^2 = 1.3 (6 \text{ DOF})$</td>
<td>$\delta_{ES} = 2.78(0.17)e^{-7}$ $\chi^2 = 1.4 (7 \text{ DOF})$</td>
</tr>
<tr>
<td><strong>6f cosine</strong></td>
<td>$\delta_{ES} = 3.82(1.49)e^{-7}$ $\tau = -8.9(13.8) \mu s$ $\chi^2 = 1.4 (4 \text{ DOF})$</td>
<td>$\delta_{ES} = 3.68(1.2)e^{-7}$ $\tau = -2.0(11.0) \mu s$ $\chi^2 = 1.0 (5 \text{ DOF})$</td>
<td>$\delta_{ES} = 2.14(0.92)e^{-7}$ $\tau = 1.0(8.8) \mu s$ $\chi^2 = 2.9 (6 \text{ DOF})$</td>
</tr>
<tr>
<td><strong>6f sine</strong></td>
<td>$\delta_{ES} = 3.56(0.29)e^{-7}$ $\chi^2 = 1.85 (4 \text{ DOF})$</td>
<td>$\delta_{ES} = 3.34(0.25)e^{-7}$ $\chi^2 = 0.7 (6 \text{ DOF})$</td>
<td>$\delta_{ES} = 2.64(0.22)e^{-7}$ $\chi^2 = 1.1 (7 \text{ DOF})$</td>
</tr>
<tr>
<td><strong>One-fit</strong></td>
<td>$\delta_{ES} = 3.49(0.12)e^{-7}$ $\tau = 3.1(2.6) \mu s$ $\chi^2 = 1.1 (34 \text{ DOF})$</td>
<td>$\delta_{ES} = 3.10(0.09)e^{-7}$ $\tau = -0.8(2.0) \mu s$ $\chi^2 = 1.1 (40 \text{ DOF})$</td>
<td>$\delta_{ES} = 2.58(0.09)e^{-7}$ $\tau = 6.7(1.9) \mu s$ $\chi^2 = 1.6 (46 \text{ DOF})$</td>
</tr>
</tbody>
</table>

1. $\delta_{ES}$ from 2f/4f/6f components consistent in $2\sigma$
2. $\tau$ from 2f/4f/6f components consistent in $2\sigma$ and close to zero, as expected
3. These features also found on 1N2S pair (the other sensitive diagonal pair)