Self Force Orbit-Integrated gravitational waveforms for EMRI}s

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Extreme Mass Ratio Inspirals

Credits: NASA
Sources for eLISA/NGO

- Galactic binaries
- Astrophysical black holes
- **EMRIs**
- GR tests
- Cosmology

We choose mass ratio of $10^{-2}$

<table>
<thead>
<tr>
<th></th>
<th>NGO</th>
<th>LISA</th>
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<tbody>
<tr>
<td>expected rate</td>
<td>1 yr$^{-1}$ to 100 yr$^{-1}$</td>
<td>10 yr$^{-1}$ to 1000 yr$^{-1}$</td>
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<tr>
<td>expected number</td>
<td>10 to 20</td>
<td>a few tens</td>
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<tr>
<td>(2 year mission duration)</td>
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**NGO**: Expected rate: 1 yr$^{-1}$ to 100 yr$^{-1}$

**LISA**: Expected rate: 10 yr$^{-1}$ to 1000 yr$^{-1}$

**NGO**
- **Capability to detect gravitational waves emitted during the last two years of inspiral for a stellar-mass compact object**
  - $(m_2 \sim 5 M_\odot - 20 M_\odot)$ orbiting a massive black hole
  - $(m_1 \sim 10^3 M_\odot - 10^6 M_\odot)$ up to $z = 0.7$ with an SNR > 20.
  - Determine the mass with an relative error smaller than 0.1%, the spin of the MBH with an error smaller than $10^{-3}$, and the mass of the compact object with a relative error smaller than 0.1%, as well as the orbital eccentricity before the plunge with an error smaller than $10^{-3}$.

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**LISA**
- **Capability to detect gravitational waves emitted during the last year of inspiral for a stellar-mass compact object**
  - $(m_2 \sim 5 M_\odot \ldots 100 M_\odot)$ orbiting a massive black hole (with $m_1 \sim 10^5 M_\odot \ldots$ few $\times 10^6 M_\odot$) at $z = 1$ with SNR > 30
  - (averaged over source locations and orientations)

**NGO**
- **Capability to detect gravitational waves emitted by a $10^5 M_\odot - 10^4 M_\odot$ IMBH spiralling into an MBH with mass**
  - $3 \times 10^5 M_\odot - 10^7 M_\odot$ out to $z \sim 2 - 4$ (for a mass ratio around $10^{-2}$ to $10^{-3}$)

**LISA**
- **Capability to detect gravitational waves emitted by a $10^3 M_\odot - 10^4 M_\odot$ intermediate-mass black hole (IMBH) spiralling into an MBH with mass in the range $3 \times 10^5 M_\odot - 10^6 M_\odot$ out to $z = 3$**
  - (with SNR ~ 30).

Credits: eLISA/NGO
Code development: convergence
The Orbit
The Orbit

![Graph of the orbit showing the energy balance, osculating, and direct methods.](image-url)
Testing the waveforms: Conservative effects turned off

\[ f_{\mu}^{\text{SF}} = f_t \delta_{\mu}^t + f_\phi \delta_{\mu}^\phi + f_r \delta_{\mu}^r \]

\[ \text{dissipative} \quad \text{conservative} \]
The waveforms: Effect of the conservative self force

\[ \Delta \phi \approx 8.4 \pm 0.4 \text{ rad} \]
The waveforms: Effect of the conservative self force

\[ C_{\text{max}} := \max_{\tau} \frac{\langle \psi_1(t) | \psi_2(t - \tau) \rangle}{\sqrt{\langle \psi_1(t) | \psi_1(t) \rangle \langle \psi_2(t) | \psi_2(t) \rangle}} \]

\[ C_{\text{max}} = 0.9899 \]

Overlap Integral

EB / OS

Window Size = 350 M
The waveforms: Effect of the conservative self force

\[ C(t) \]

-1 \[ \rightarrow \] 1

L = 820 M
L = 350 M

t / M

\[ L = 820 \text{ M} \]
\[ L = 350 \text{ M} \]
The waveforms: Effect of the conservative self force

\[ R^2 = 0.9983 \]
The waveforms: Effect of the conservative self force
Future Directions

• Eccentric orbits in Schwarzschild (straightforward – Warburten et al.)
• Kerr orbits
• (True) self consistent waveforms