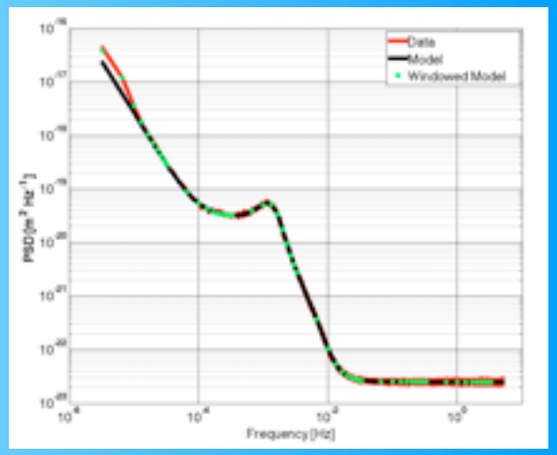
Statistical analysis of LISA Pathfinder noise

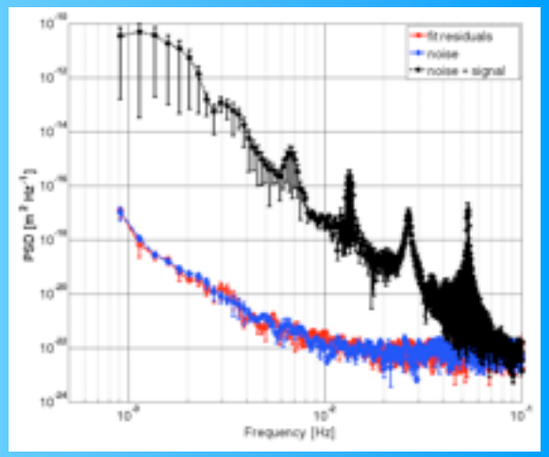
Luigi Ferraioli

Why we need a statistical tool for noise analysis?

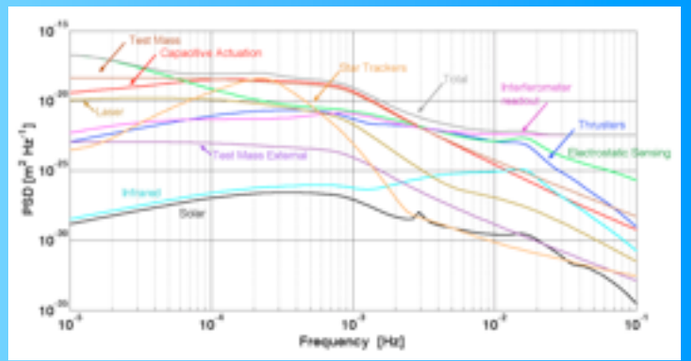
Daily noise investigation



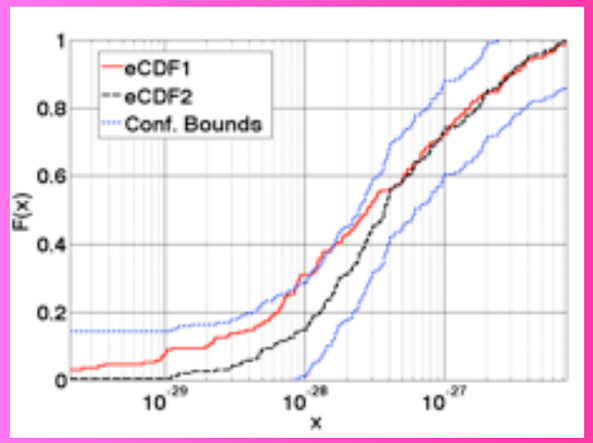
Analysis of fit residuals



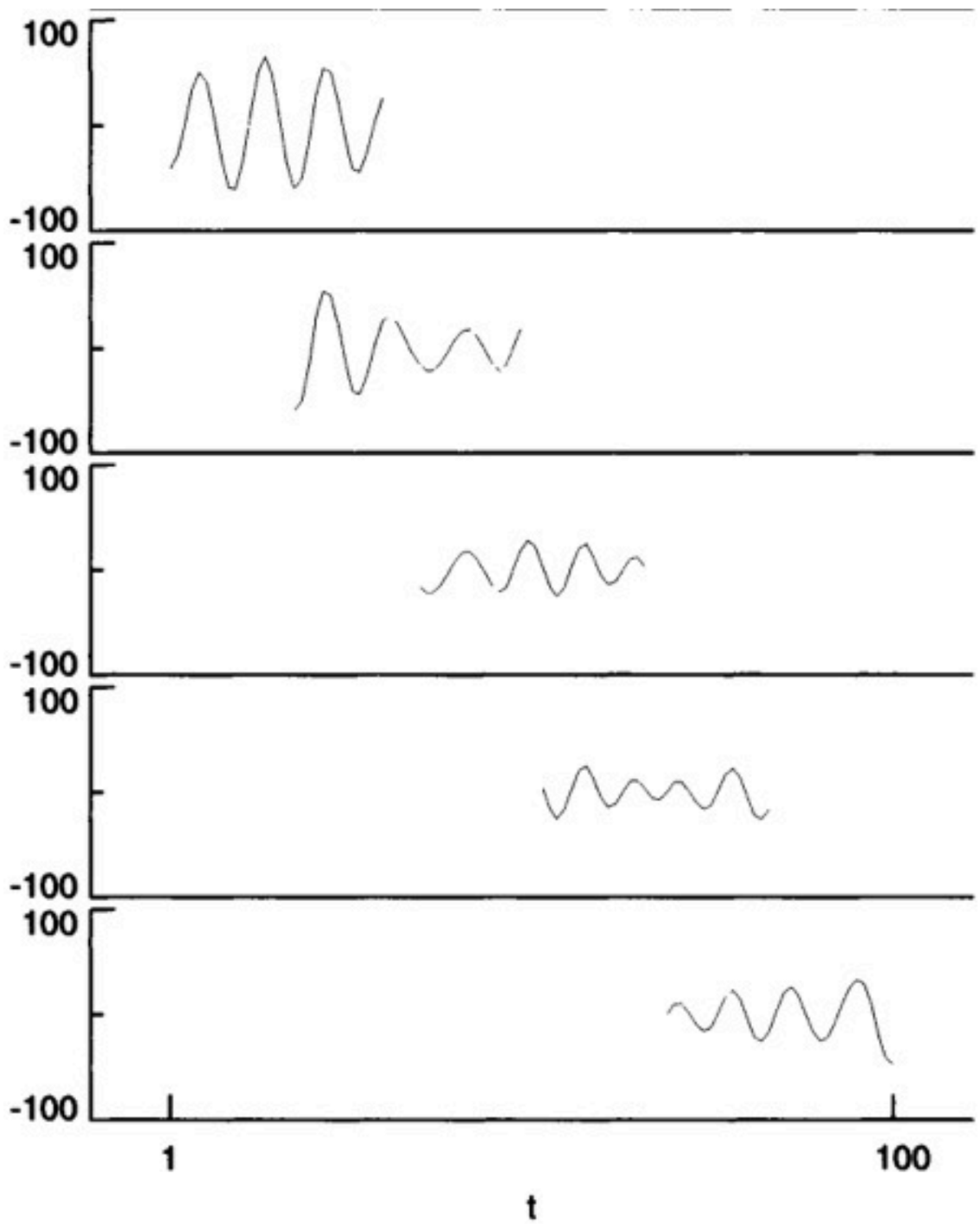
Quantitative noise analysis



Statistical analysis of noise in spectral domain



Starting from the beginning
Welch's Overlapped Segment Averaging



Breaking up a time series into windowed overlapping segments.

Averaging the spectra for the different segments.

Allows to lower spectrum variance by a factor N^2_{ave}

Donald B. Percival and Andrew T. Walden,
SPECTRAL ANALYSIS FOR PHYSICAL APPLICATIONS,
Cambridge University Press.

The Statistic of the sample spectrum is a Gamma

Welch's overlapped segment averaging (WOSA) spectrum $P_{\mathrm{WOSA}}(f_k)$

$$f_{\mathrm{WOSA}}\left(z; h, \delta\right) = \frac{z^{(h-1)} e^{-\frac{z}{\delta}}}{\delta^h \Gamma\left(h\right)}$$ Gamma distribution

$$\delta = S\left(f_k\right)/N_{\mathrm{ave}} \qquad S(f_k) = E[P_{\mathrm{WOSA}}(f_k)]$$

$$h = N_{\mathrm{ave}}$$

Normalized WOSA spectrum $R_{\mathrm{WOSA}}\left(f_k\right) = \dfrac{P_{\mathrm{WOSA}}\left(f_k\right)}{S\left(f_k\right)}$

Still a Gamma distribution with

$$\delta = 1/N_{\mathrm{ave}}$$

$$h = N_{\mathrm{ave}}$$

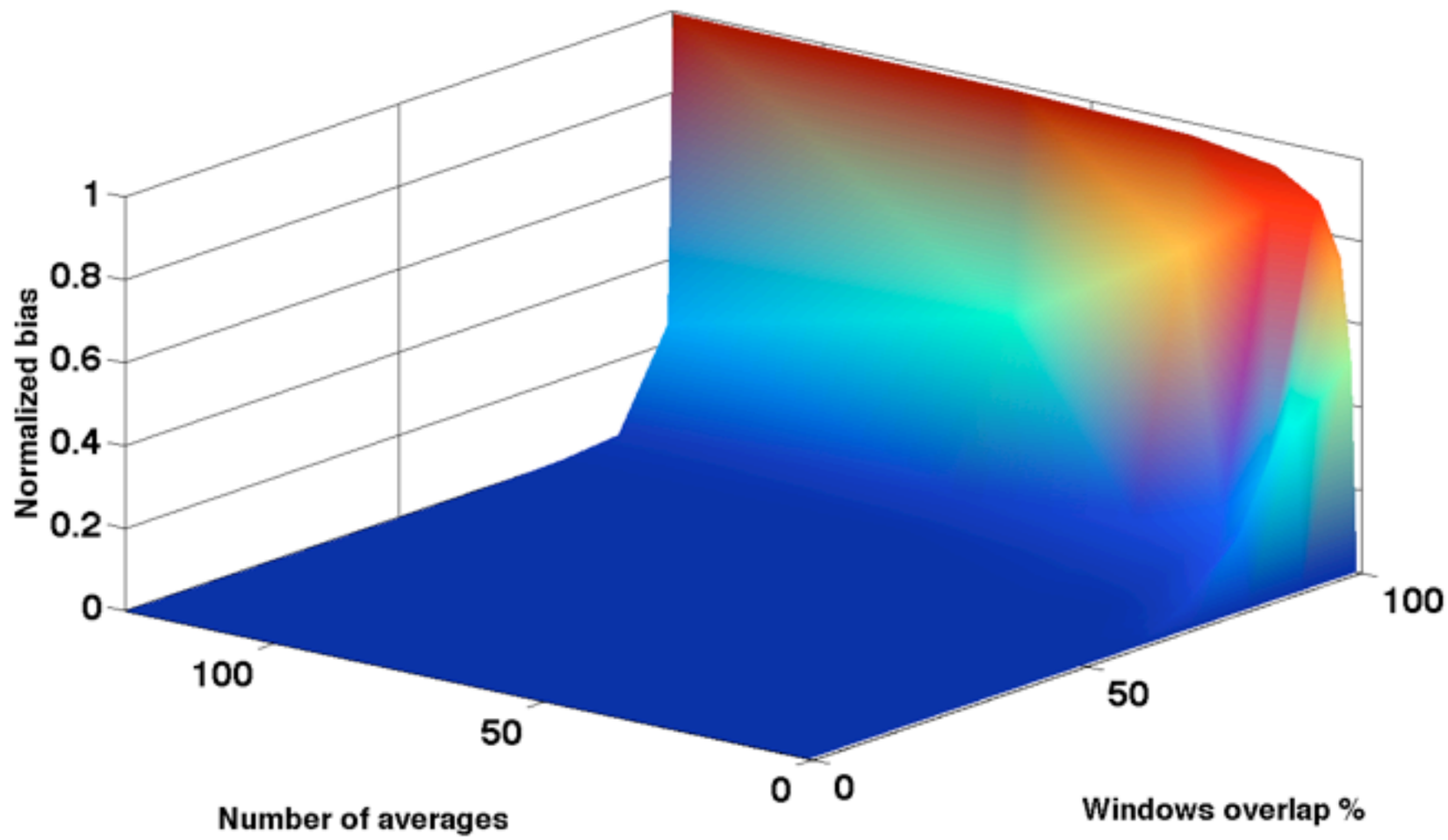
Phys. Rev. D. 84, 122003 (2011)

Overlapping segments are correlated

Averaging introduces a bias in the variance of the estimator

Normalized correction to the variance introduced by segment overlapping

50 % overlap is a good tradeoff at each average value



Blackman-Harris window

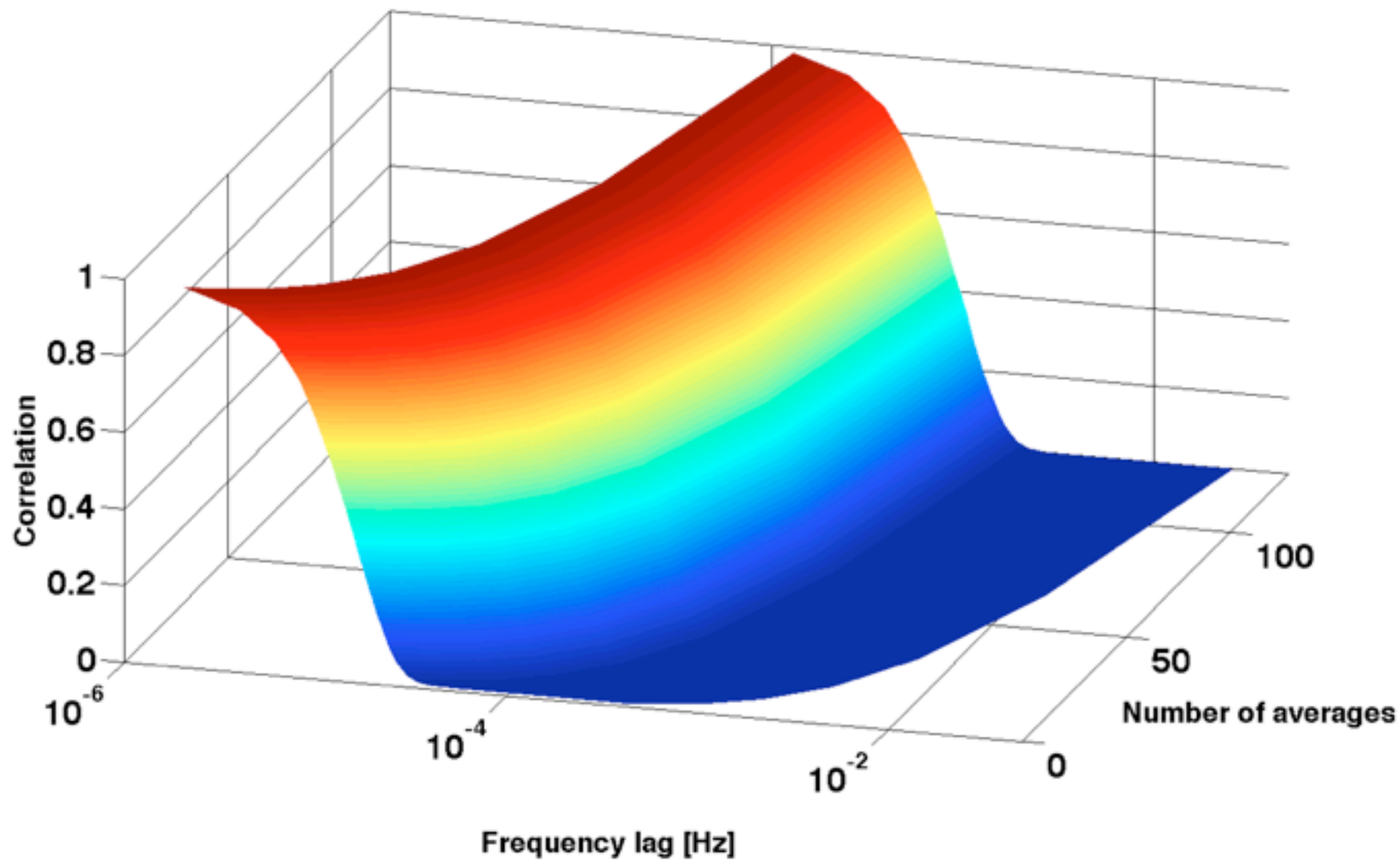
Frequency bins are correlated

Correlation introduced by windowing

Blackman-Harris window

10^5 data points

50% overlap



Kolmogorov-Smirnov noise excess estimator

$$d_K = max\left|F_N\left(x\right) - F\left(x\right)\right|$$

Test cumulative function

Reference cumulative function

Phys. Rev. D. 84, 122003 (2011)

Kolmogorov-Smirnov noise excess estimator

$$d_K = max\left|F_N\left(x\right) - F\left(x\right)\right|$$

Test cumulative function

Reference cumulative function

If $d_K > d_K\left(\alpha\right)$ the null hypothesis is rejected

$\alpha = P_{\mathrm{FA}}$

the null hypothesis is that the two CDFs are asymptotically equal

Phys. Rev. D. 84, 122003 (2011)

Kolmogorov-Smirnov noise excess estimator

$$d_K = max\left|F_N\left(x\right) - F\left(x\right)\right|$$

Test cumulative function

Reference cumulative function

If $d_K > d_K\left(\alpha\right)$ the null hypothesis is rejected

$\alpha = P_{\mathrm{FA}}$

the null hypothesis is that the two CDFs are asymptotically equal

The test works for uncorrelated data!!!

Phys. Rev. D. 84, 122003 (2011)

Kolmogorov-Smirnov noise excess estimator

$$d_K = max\,|F_N(x) - F(x)|$$

Test cumulative function

Reference cumulative function

If $d_K > d_K(\alpha)$ the null hypothesis is rejected

$\alpha = P_{\mathrm{FA}}$

the null hypothesis is that the two CDFs are asymptotically equal

The test works for uncorrelated data!!!

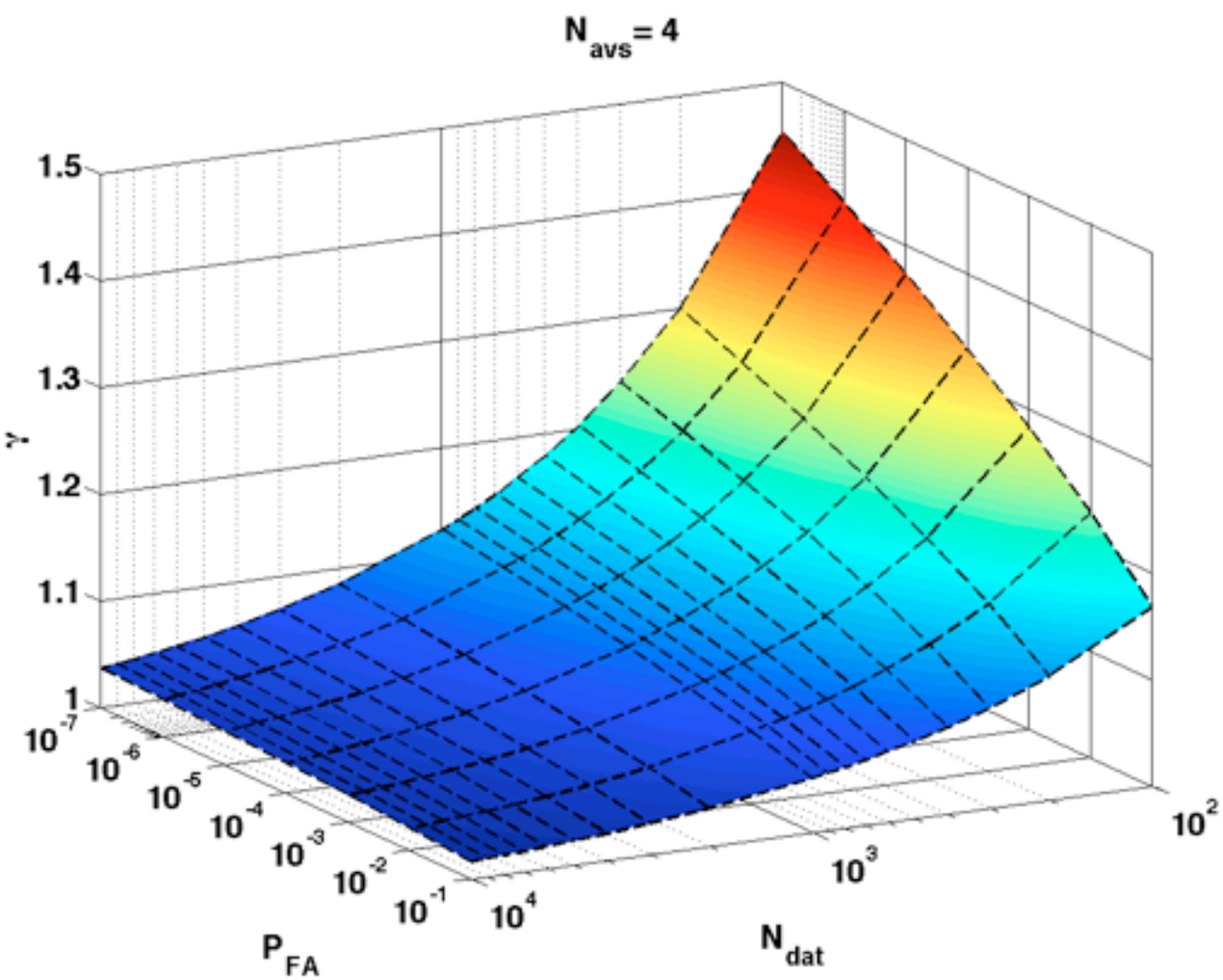
overlap bias → choice of the proper overlap % ☺

window correlations → calculate a correction factor by Monte Carlo on white noise ☺

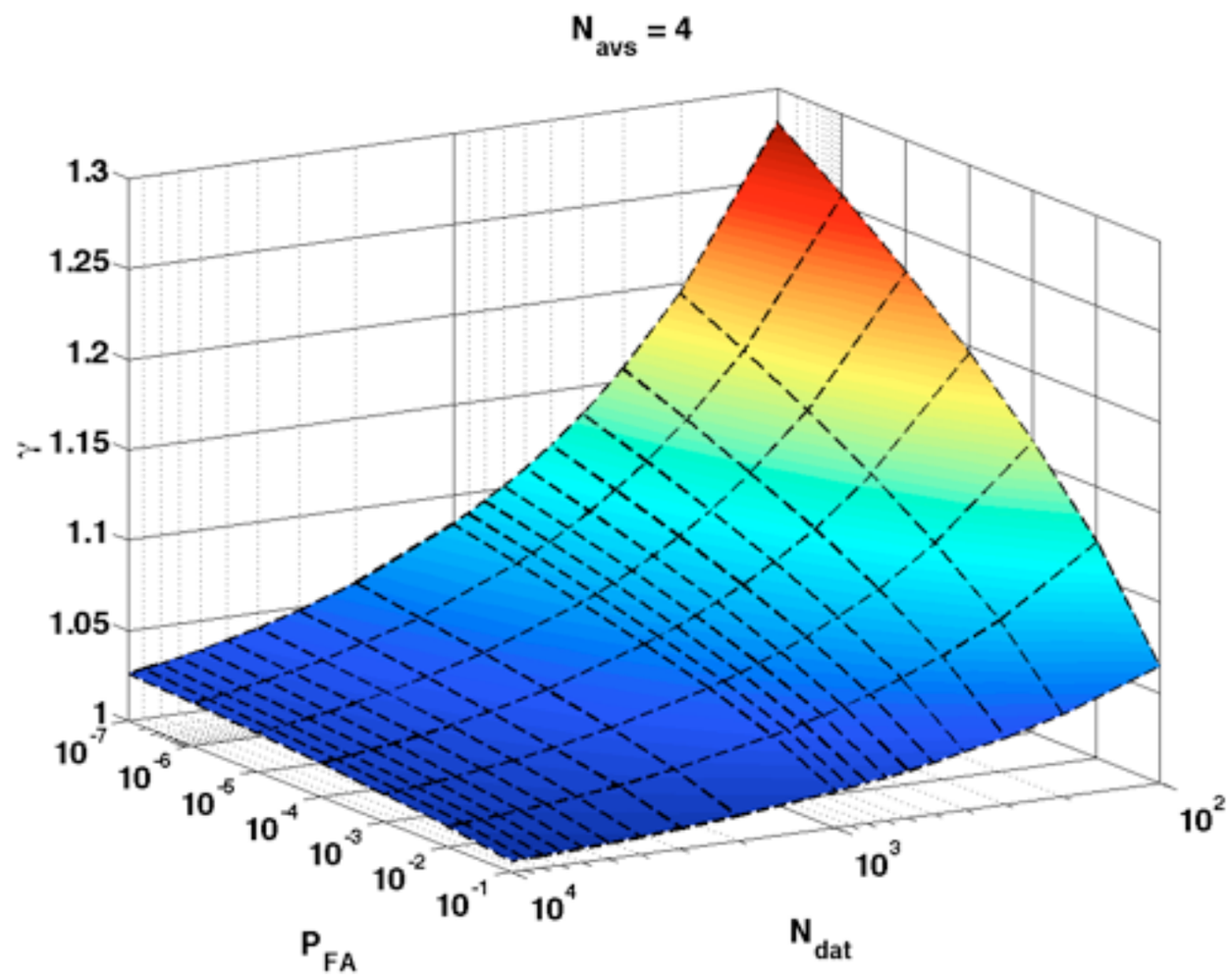
Phys. Rev. D. 84, 122003 (2011)

Test sensitivity - Data vs Model

Kolmogorov-Smirnov test



Likelihood Ratio test



Υ is the minimum detectable noise energy excess

Pros and Cons

KS Test Pros

- Independent from the data distribution
- Easy to use
- Compare data vs. data and data vs. model

Likelihood Ratio Test Pros

- Optimal, highest sensitivity

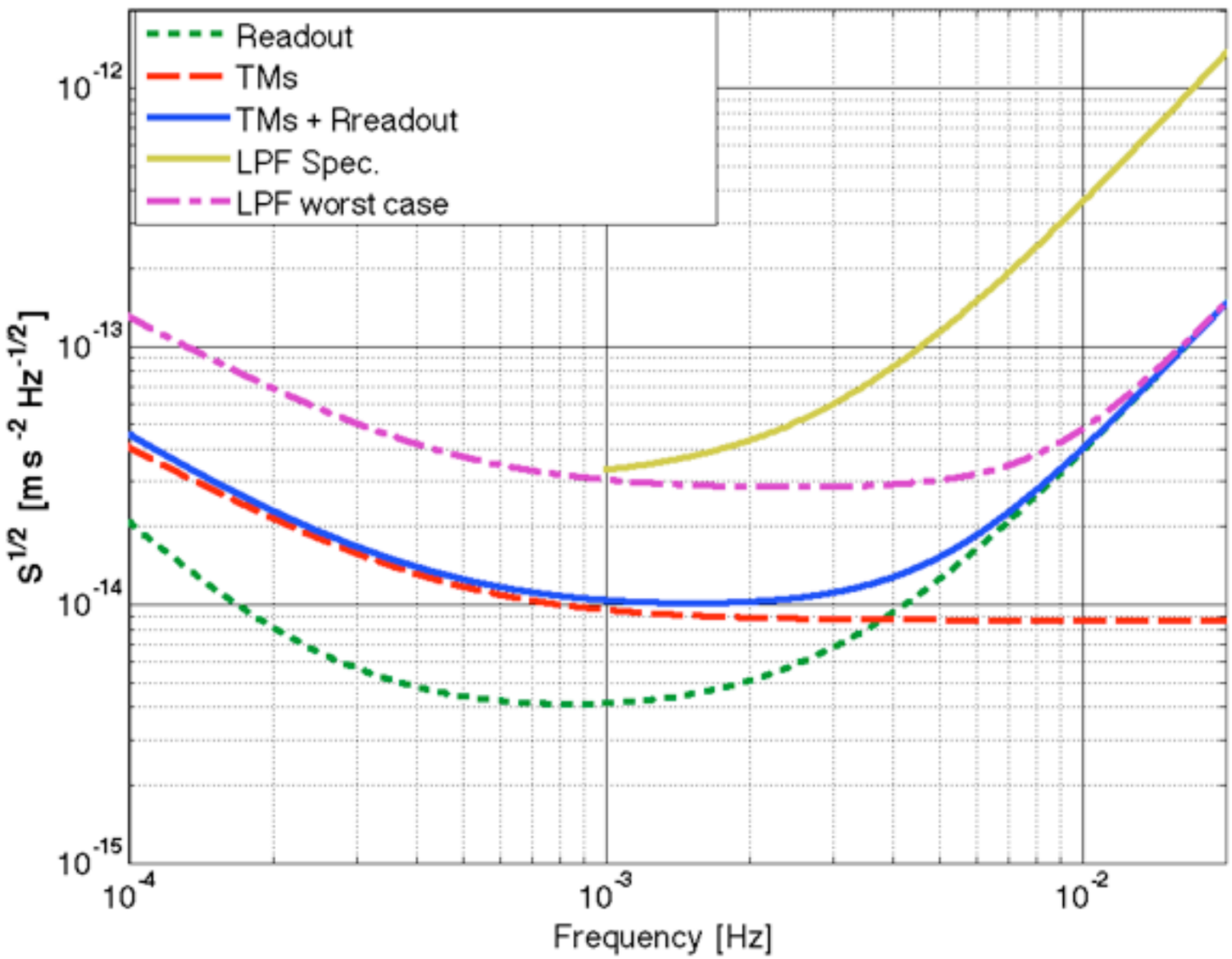
KS Test Cons

- Sub-optimal, less sensitive than the likelihood ratio

Likelihood Ratio Test Cons

- Unpractical for data vs. data comparison
- Once we depart from the simple case of constant excess over the frequency band, the distribution of data is unknown and the test is hardly implemented

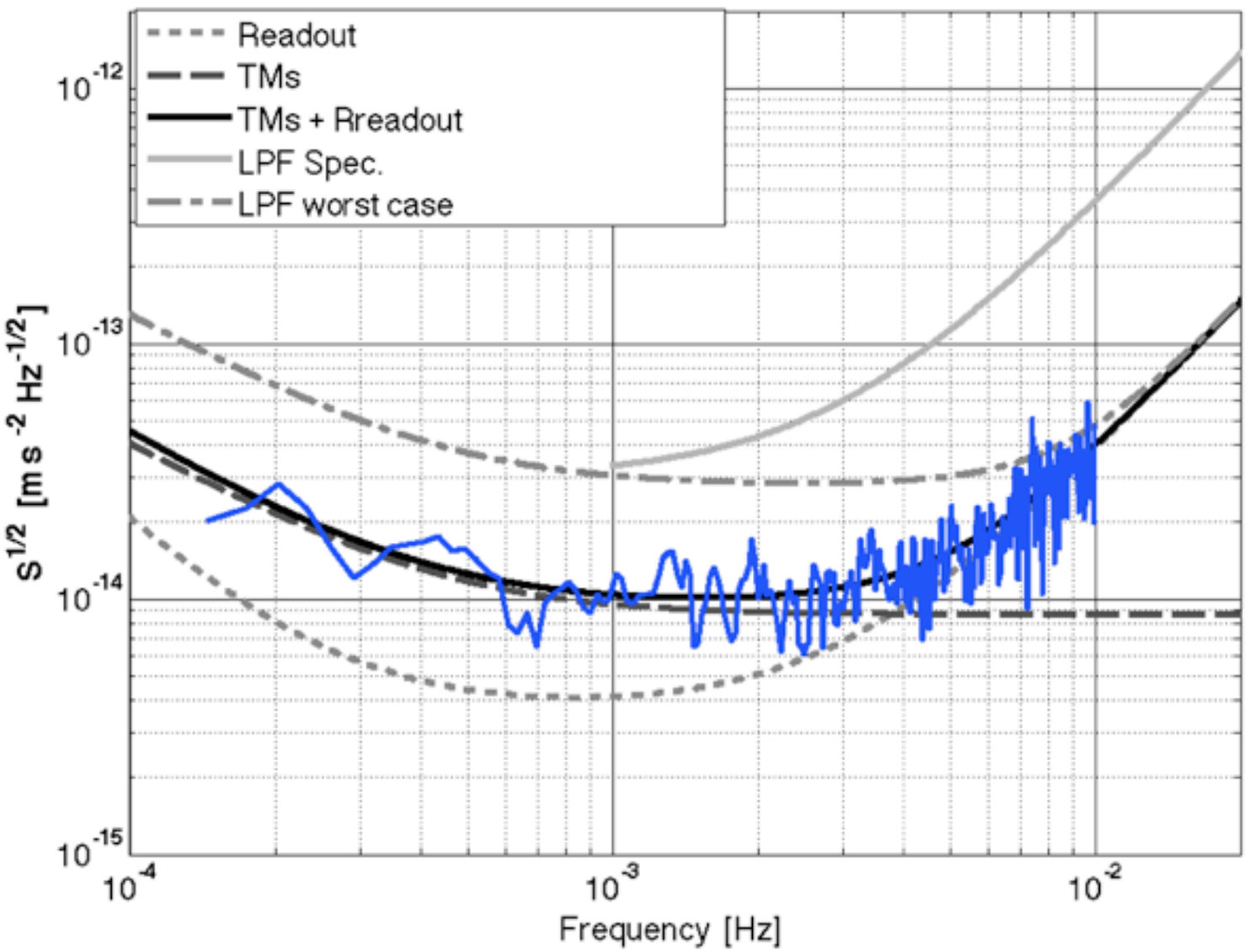
Application to LTP



Phys. Rev. D. 84, 122003 (2011)

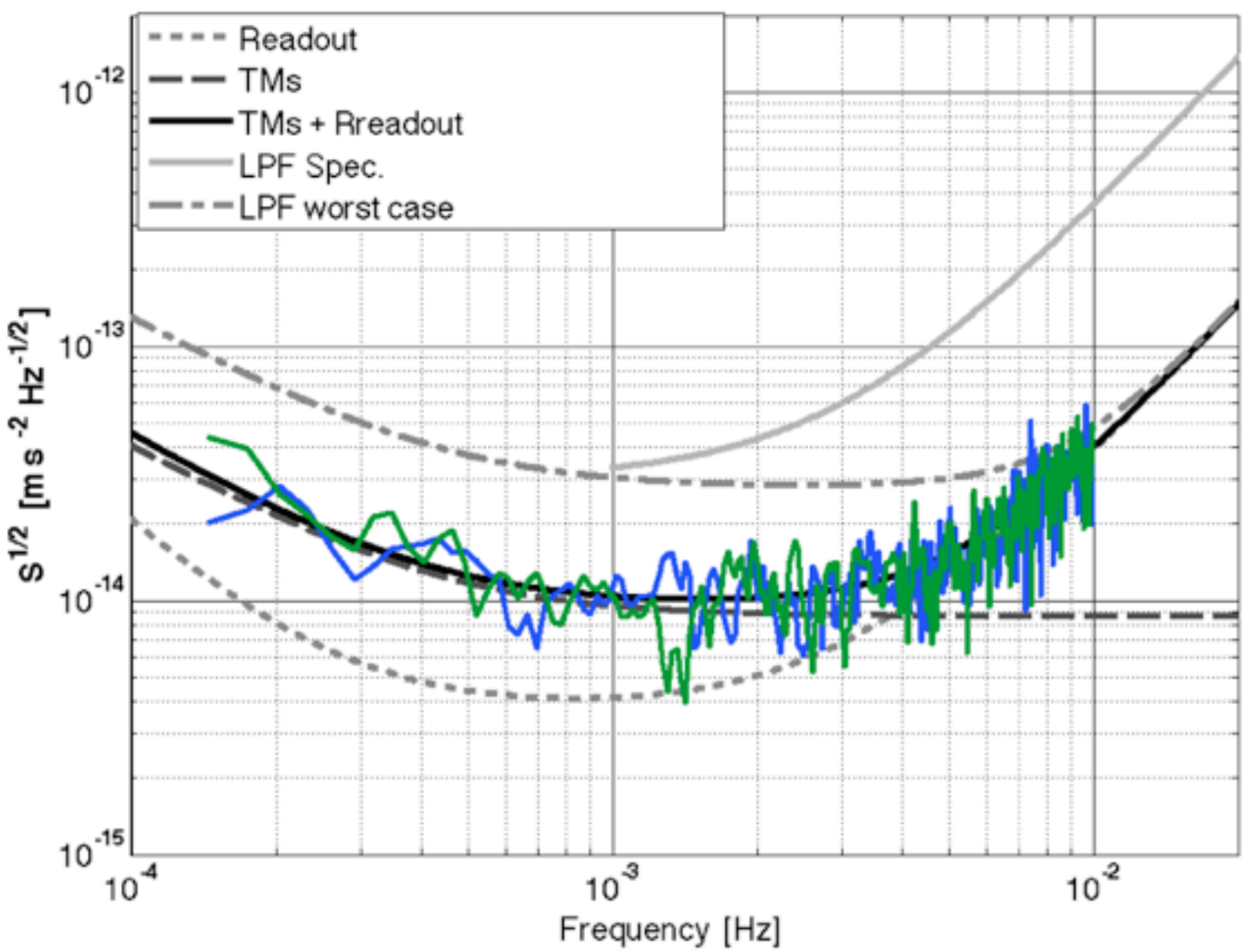
Application to LTP

Reference Spectrum



Application to LTP

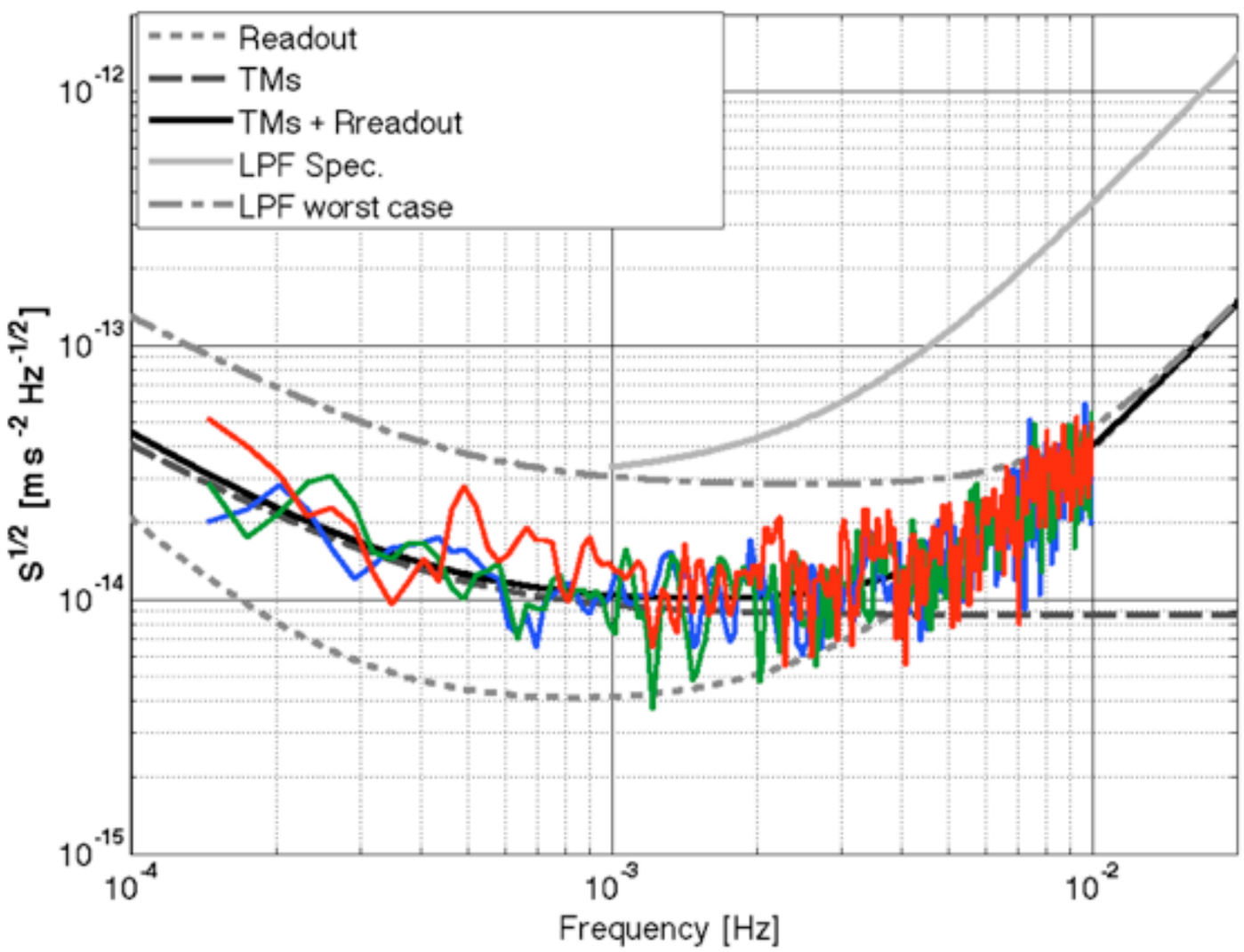
Energy Excess = 1.041



The test does not reject the hypothesis that the two spectra are compatible

Application to LTP

Energy Excess = 1.157



The test rejects the hypothesis that the two spectra are compatible

Noise parameter estimation

Write a parametrized WOSA estimator for the normalized sample spectrum $R_{\mathrm{WOSA}}(f_k;\theta)$

The closer θ is to θ_{true}

The better the distribution is described by a Gamma

$$d_K(\theta) = \max\left|F_R(x;\theta) - \mathcal{P}(N_s, xN_s)\right|$$

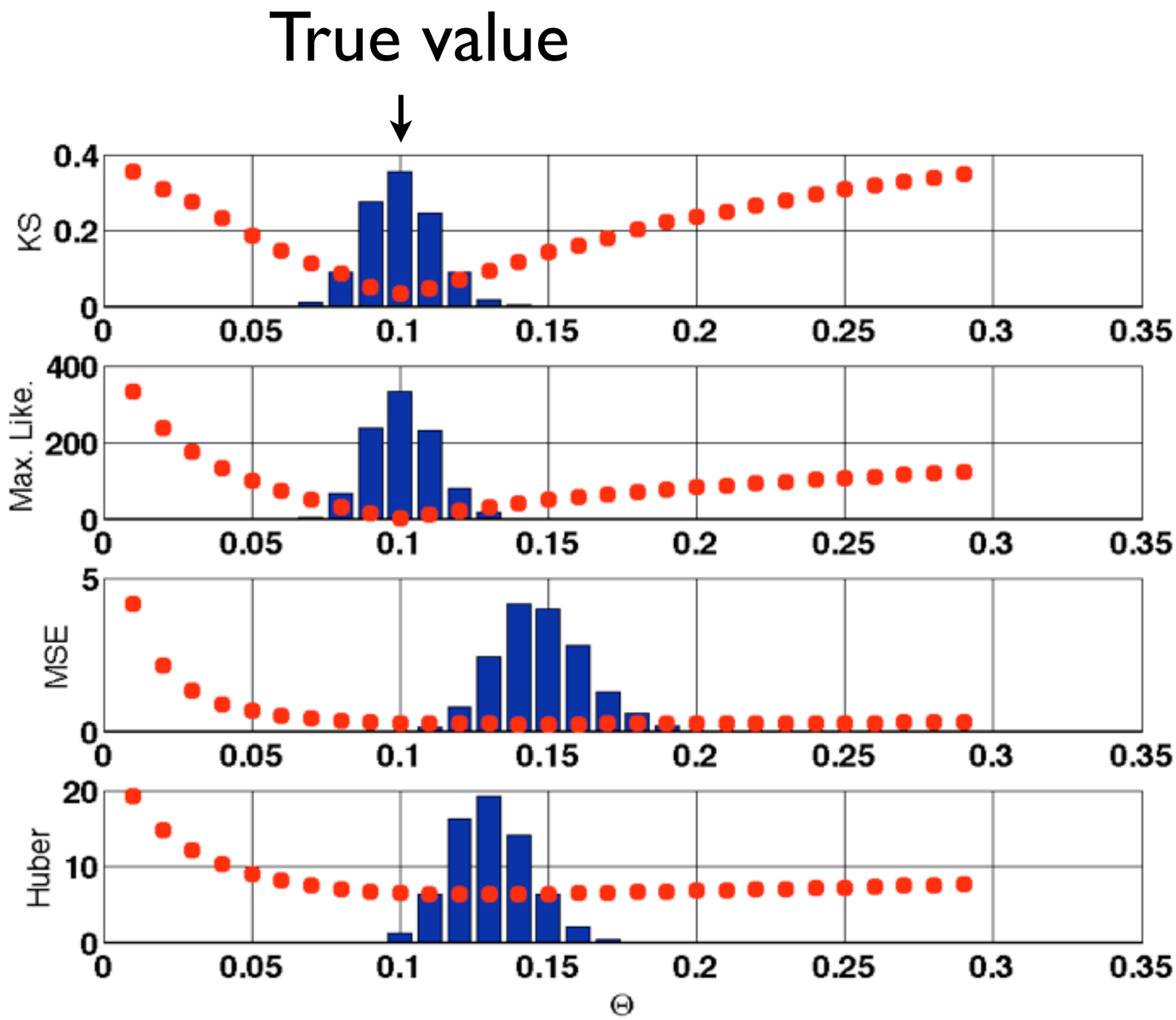
$F_R(x;\theta)$ is the ECDF for the current parameter estimate

$\mathcal{P}(N_s, xN_s)$ is the limiting CDF

The minimization of $d_K(\theta)$ vs. θ provides the KS estimation for the parameter

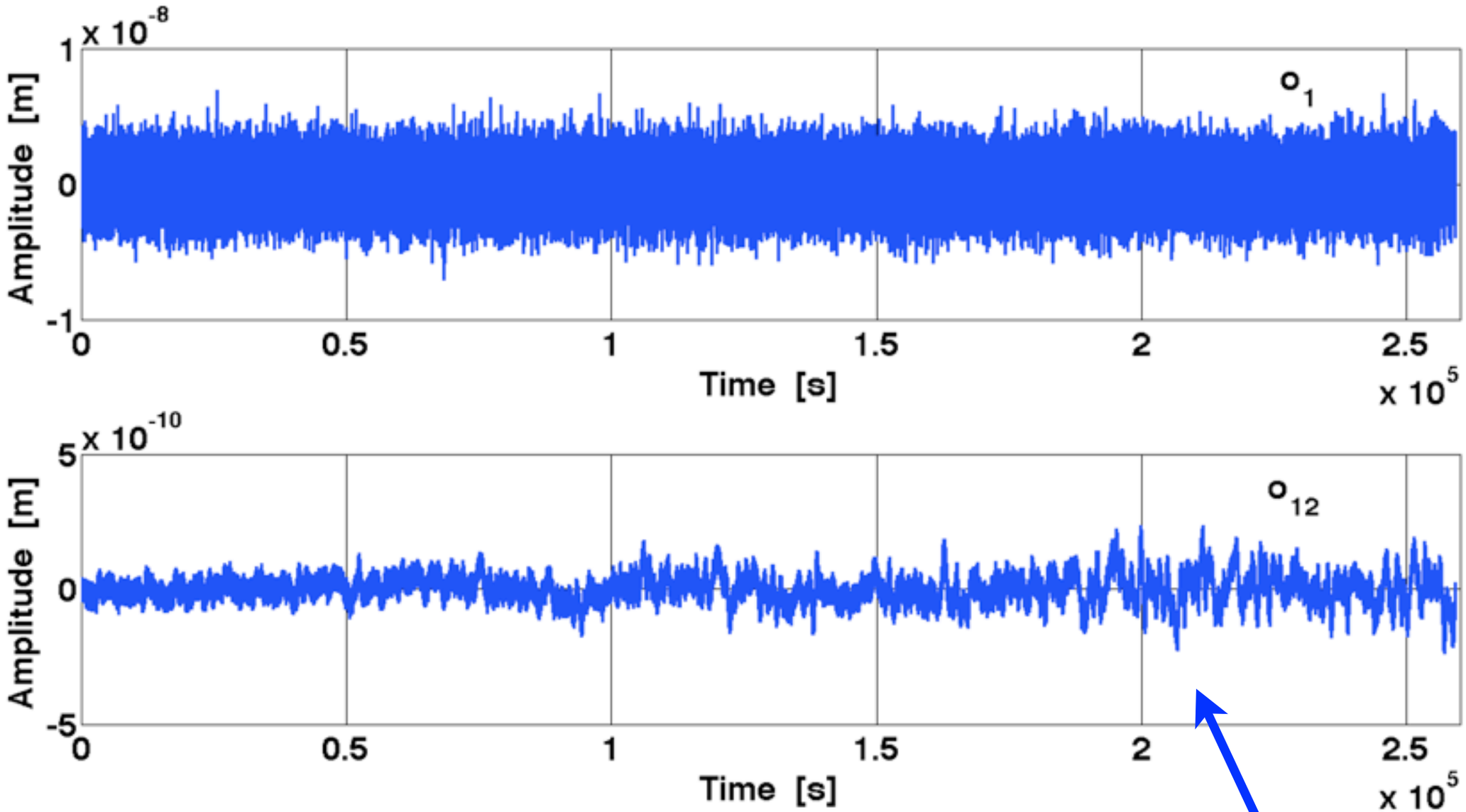
Phys. Rev. D. 84, 122003 (2011)

LTP Example - Grid search



Phys. Rev. D. 84, 122003 (2011)

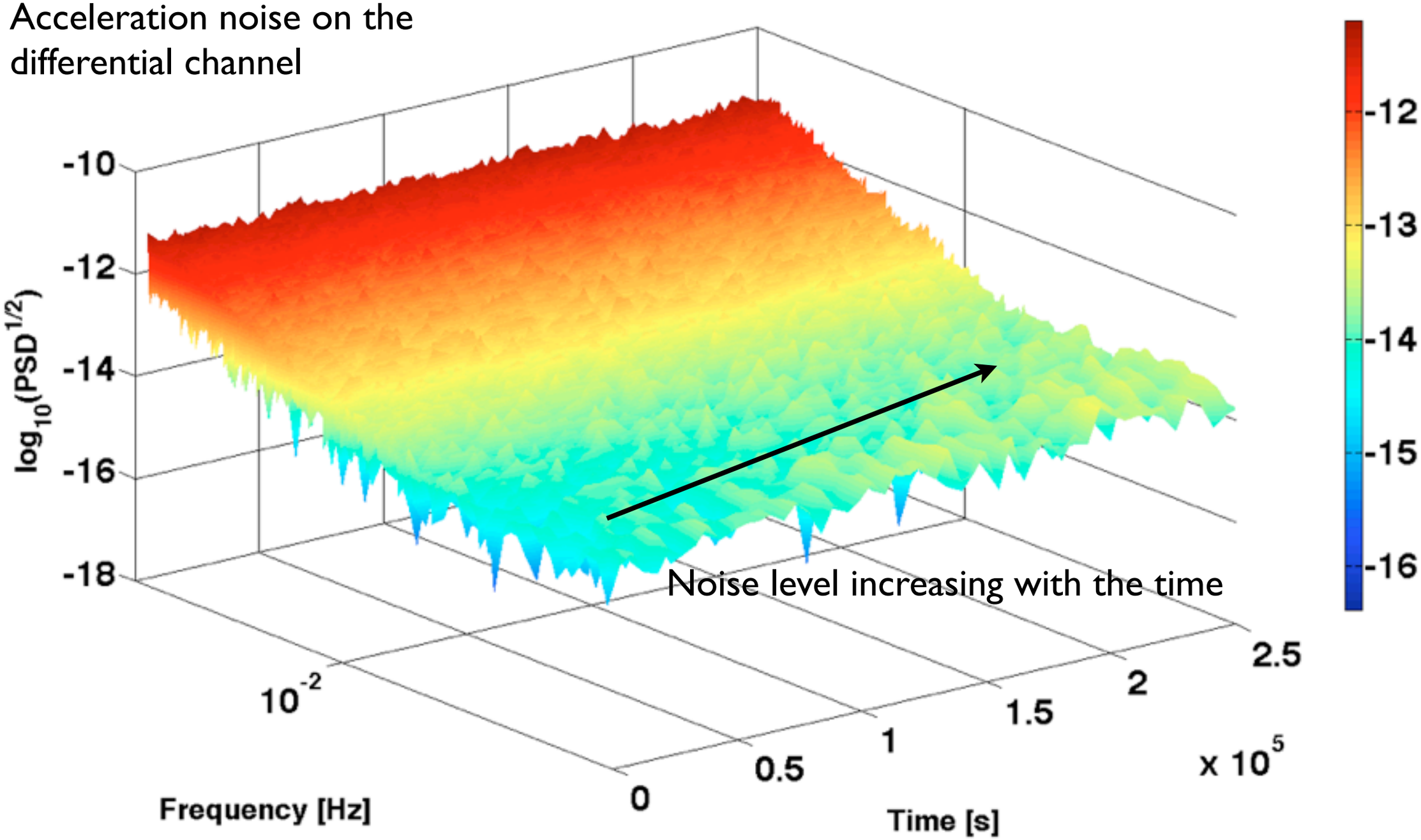
Non-stationary noise



3 days interferometer data @ 10 Hz
Non-stationary noise on capacitive actuators linearly increasing with the time

Non-stationary noise

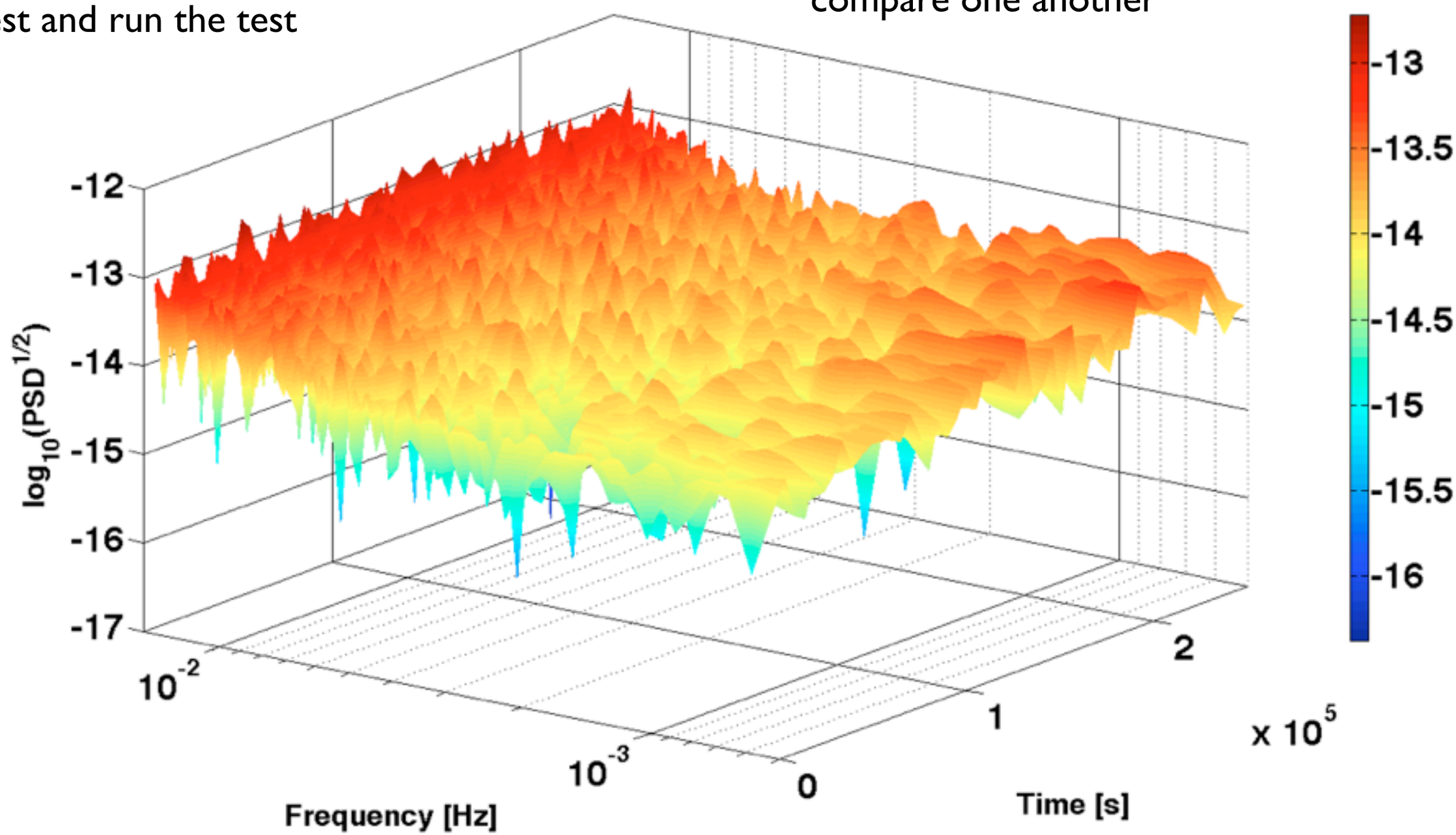
Acceleration noise on the differential channel



Non-stationary noise - KS test

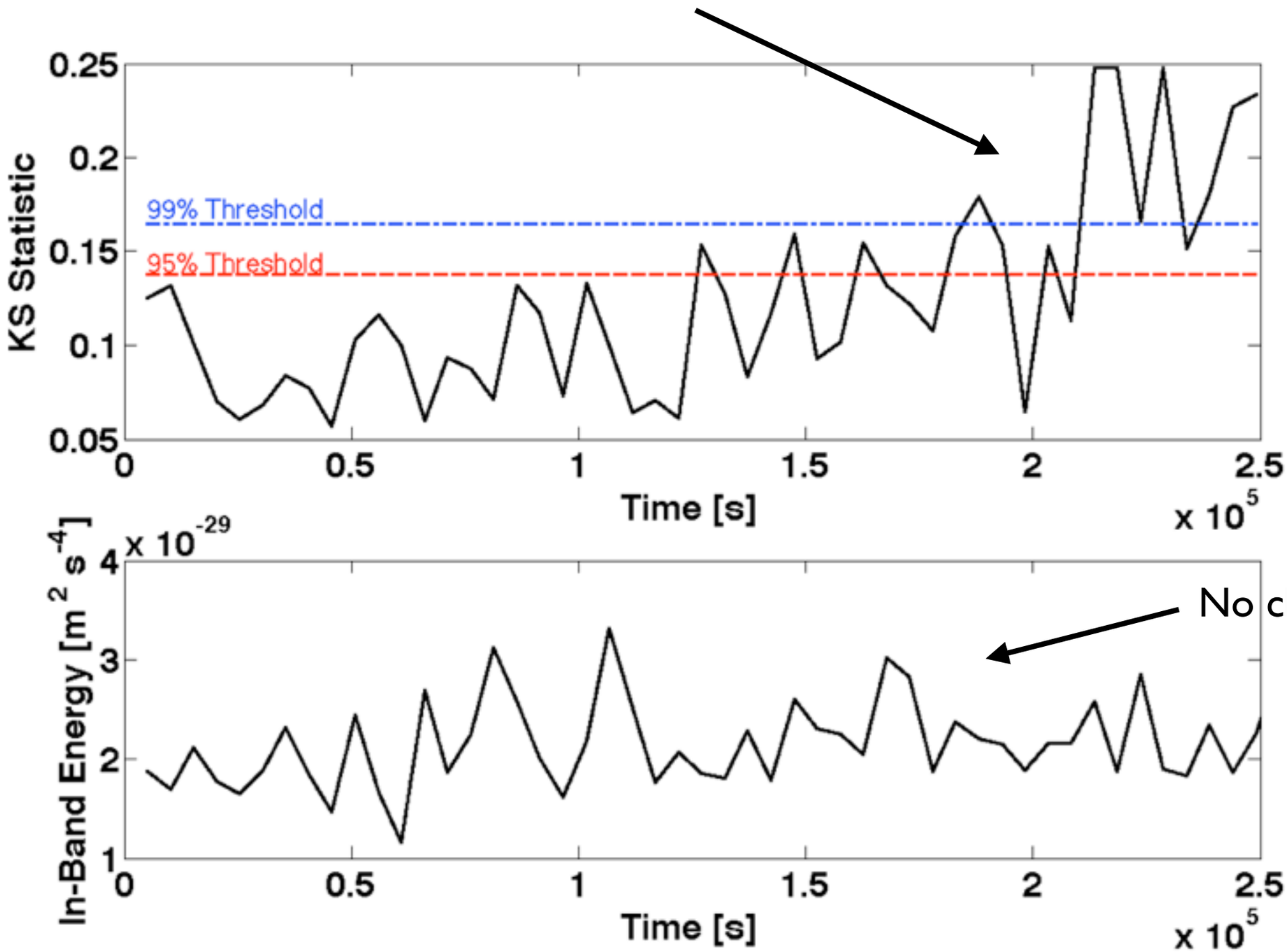
Restrict to a frequency range of interest and run the test

Cut slices at given times and compare one another



Non-stationary noise - KS test

Rising above the thresholds



No clear variation

Thank you for your attention