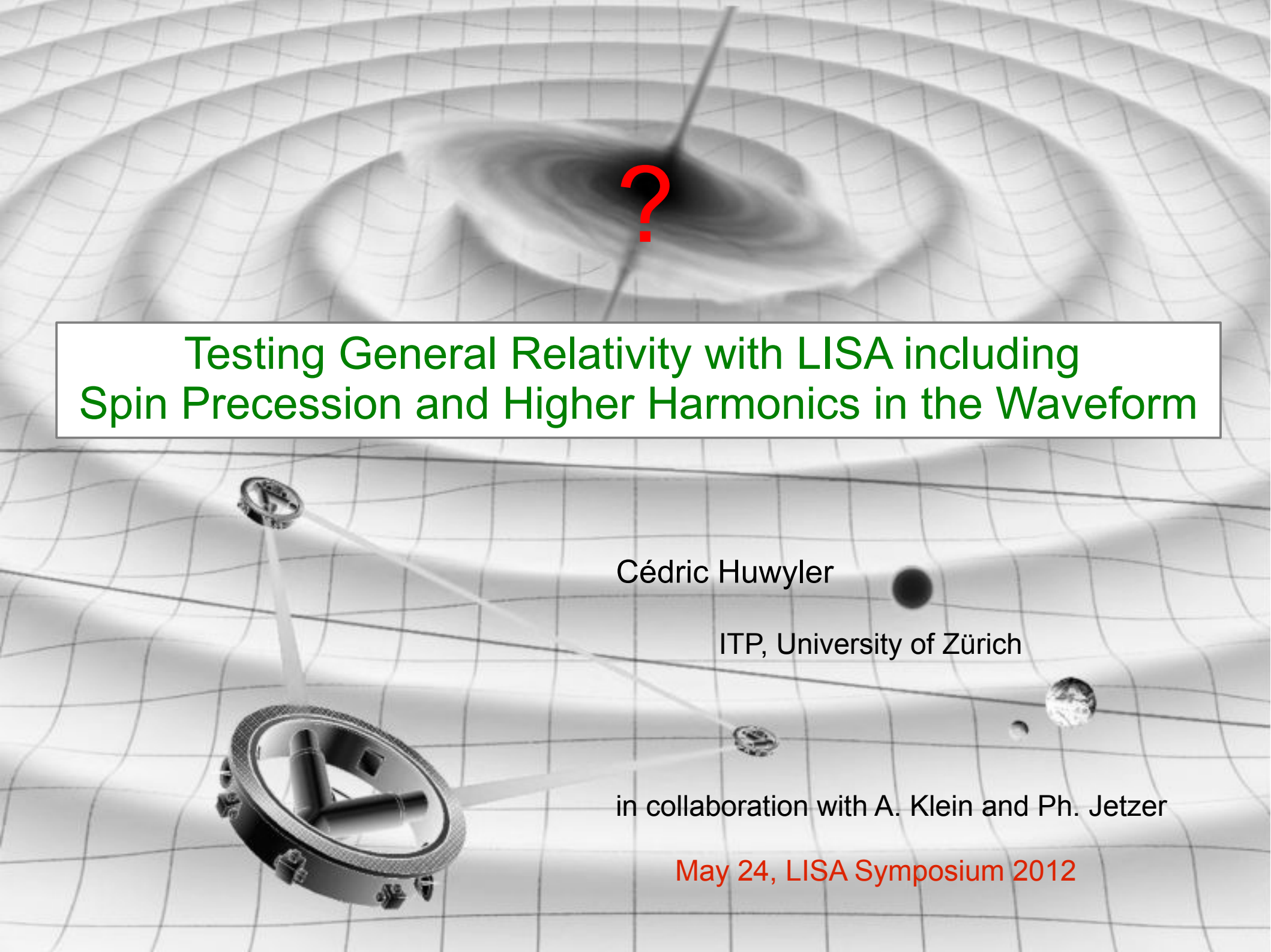

Testing General Relativity with LISA including Spin Precession and Higher Harmonics in the Waveform

Cédric Huwyler

ITP, University of Zürich

in collaboration with A. Klein and Ph. Jetzer

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data analysis and alternative theories

- **'Fundamental bias'**

  What if we used GR templates but GR would be the wrong theory?

  ➡ We would most certainly detect all the sources, but with slightly lower SNRs and wrong parameters

  ➡ It is necessary to check the underlying theory and how it affects gravitational wave sources and propagation

- **Supermassive black hole binaries are testbeds for alternative gravity theories**
  - strongest gravitational wave sources we know of
  - produce very clean signals

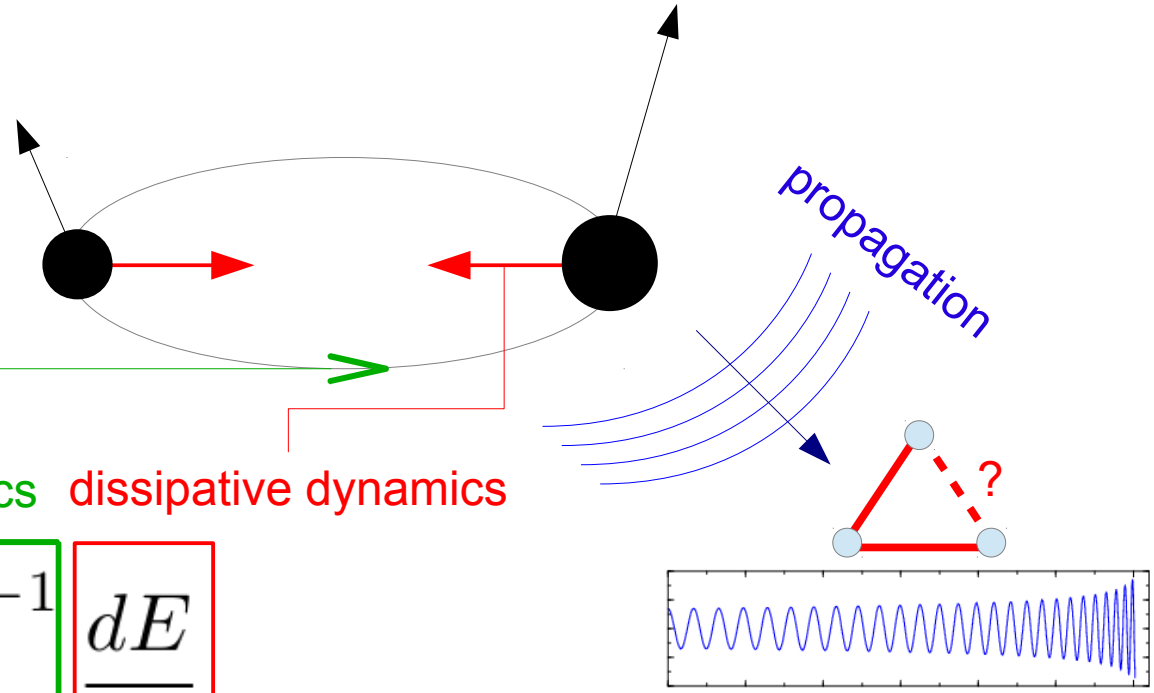
- **Spin precession**
  - Modulation of the waveform → more information in the signal

    → less degeneracy in the parameter space

- **Higher harmonics**
  - restricted waveform (**RWF**): leading order amplitude, 2PN expanded phase
  - Full waveform (**FWF**): 2PN amplitude, 2PN phase

    → additional structure, longer observation time

alternative theories and the dynamics of compact binaries



conservative dynamics dissipative dynamics

$$\frac{df}{dt} = \boxed{\left(\frac{d}{df}E\right)^{-1}}\,\boxed{\frac{dE}{dt}}$$

introduce modifications through alternative gravity theories

$$\begin{aligned} h(t) &= A(t)\, e^{i\Psi(t)} \\ \Psi(t) &= \Phi_c + 2\pi \int_{tc}^{t} f\, dt \end{aligned}$$

towards alternative theory parameters

Examples:

Brans-Dicke theory $$[\Psi(f)]_{\rm mod} = \frac{3}{256\nu} x^{-5/2} \left[\Psi_{\rm GR} + \beta_{\rm BD}\, x^{-1}\right]$$

massive gravitons $$[\Psi(f)]_{\rm mod} = \frac{3}{256\nu} x^{-5/2} \left[\Psi_{\rm GR} + \beta_{\rm MG}\, x\right]$$

$$x = \left(\frac{GM\omega}{c^3}\right)^{2/3}$$

2PN expanded phase term

'general' approach / toy model

$$[\Psi(f)]_{\rm mod} = \frac{3}{256\nu} x^{-5/2} \left[\Psi_{\rm GR} + \sum_i \Psi_i\, x^i\right] \qquad i = \{-1, 0, 1/2, 1, 3/2, 2\}$$

basic questions:

- What is the effect of adding a theory parameter to every existing PN phase term (plus two new ones) on the measurement accuracy?
- How much accuracy is gained by using the FWF instead of the RWF?

- **Fisher matrix** analysis of **quasi-circular** BBH systems in the **2PN** approximation with **fully precessing spins** using the **classic LISA** configuration.
- All parameter are randomized, except for masses, redshift and theory parameters.
- Since we are interested in deviations from GR, all the Ψ_i are set to 0.

relation to the ppE scheme

$$\tilde{h}(f) = \tilde{h}_{\rm GR}(f)\,[1+\alpha\,u^a]\,e^{i\beta\,u^b} \qquad u = \pi\mathcal{M}f$$

catches leading order deviations from GR

ppE scheme

Yunes / Pretorius 2009

$$[\Psi(f)]_{\rm mod} = \frac{3}{256\nu}x^{-5/2}\left[\Psi_{\rm GR} + \sum_i \Psi_i\,x^i\right] \qquad i = \{-1, 0, 1/2, 1, 3/2, 2\}$$

no amplitude corrections, correction for every PN phase term + new ones

our variant

in our variant..

→ ..not all classes of alternative theories are visible (at leading order)

→ ..effects of next-to-leading order terms can be investigated

→ ..every of the six theory parameters can be turned on and off, even after the simulations

parameter estimation: loss of accuracy in binary parameters

Which binary parameter errors are inflated due to correlations with theory parameters?

'low-mass' binaries $10^5\ M_{sun} \leq M \leq 10^7\ M_{sun}$

- Angular position in the sky nearly unaffected (10% loss)
- Luminosity distance, masses and spin parameters: worse by factor of less than one order of magnitude

quite nice!

'high-mass' binaries $3 \times 10^7\ M_{sun} \leq M \leq 10^8\ M_{sun}$

- Angular position in the sky: factor of ~2
- Luminosity distance worse by factor of ~10-20
- Mass and spin parameters worse by a factor of ~3

➡ Correlation coefficient of luminosity distance with theory parameters is increasing with mass from ~0.2 ($10^5\ M_{sun}$) to ~0.95 ($6 \times 10^7\ M_{sun}$)

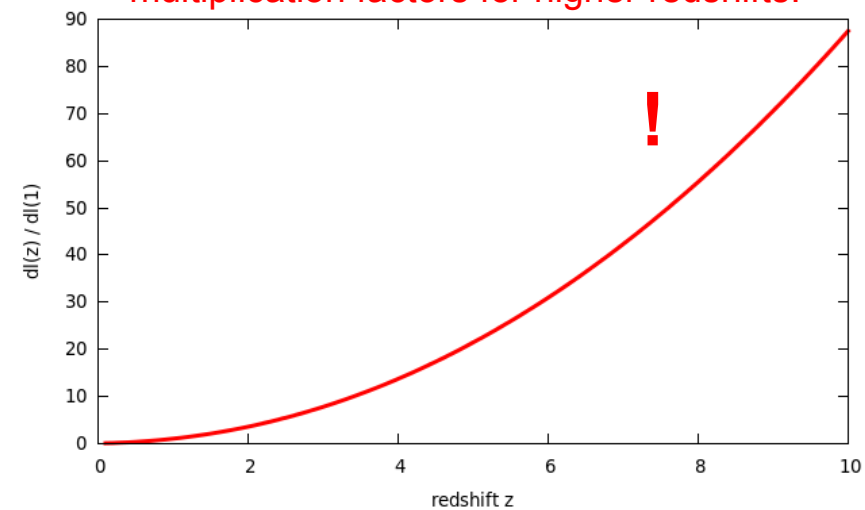
parameter estimation: error on theory parameters

Is the accuracy of the theory parameters reasonable?

- **Theory parameters with fiducial GR values (at z=1)**

	low-mass	high-mass
$\Delta\Psi_0/\psi_{\mathrm{GR},0}$	~10^{-3}-10^{-1}	~1-10^{2}
$\Delta\Psi_1/\psi_{\mathrm{GR},1}$	~10^{-2}-10^{-1}	~1-10^{2}
$\Delta\Psi_{3/2}/\psi_{\mathrm{GR},3/2}$	~10^{-2}-10^{-1}	~1-10^{2}
$\Delta\Psi_2/\psi_{\mathrm{GR},2}$	~10^{-1}-1	~10-10^{2}

multiplication factors for higher redshifts:



- **Theory parameters without fiducial GR values (at z=1)**

	low-mass	high-mass
$\Delta\Psi_{-1}$	~10^{-6}-10^{-4}	~10^{-2}-1
$\Delta\Psi_{1/2}$	~10^{-2}-10^{-1}	~1-10^{3}

?

desired accuracy depends on theory to test for and will most probably only serve as an upper limit!

parameter estimation: FWF vs RWF

Is it always necessary to use the FWF? What increase of accuracy can be gained?

low-mass binaries $10^5\ M_{sun} \leq M \leq 10^7\ M_{sun}$

- Factor of ~1.5-4 gained in accuracy of both binary and theory parameters

high-mass binaries $3 \times 10^7\ M_{sun} \leq M \leq 10^8\ M_{sun}$

- Angular position in the sky: 10-100
- Luminosity distance: 50-1400
- Mass and spin parameters: 30-200
- Theory parameters: Ψ_{-1}: ~100 - 1000, rest: ~10-100

→ FWF breaks correlations between highly correlated parameters, especially luminosity distance!

Use of the FWF is mandatory above total masses of ~10^7 M_{sun}

But there's not much to get in that region anyway!

parameter estimation: lower bound on graviton wavelength

The median error $\Delta\Psi_1$ can be used as an upper limit for a constraint on Ψ_1

This results in a lower bound on the graviton wavelength:

$$\lambda_g > \sqrt{\frac{256}{3}\frac{G}{c^2}\frac{\pi^2 D(z) M\nu}{(1+z)\Delta\Psi_1}}$$

Yagi / Tanaka 2010:
(RWF, simple precession, without eccentricity)

$$\lambda_g > 4.9 \times 10^{21}$$

At redshift z=1, this corresponds to the following lower bounds:

All six theory parameters included:

$$\lambda_g > 1.2 \times 10^{21}\ \mathrm{cm}\ (\mathrm{FWF}) \qquad \lambda_g > 7.8 \times 10^{20}\ \mathrm{cm}\ (\mathrm{RWF})$$

All theory parameters turned off except for Ψ_1:

$$\lambda_g > 7.6 \times 10^{21}\ \mathrm{cm}\ (\mathrm{FWF}) \qquad \lambda_g > 4.9 \times 10^{21}\ \mathrm{cm}\ (\mathrm{RWF})$$

summary

- We have to question the underlying model in order to avoid data analysis results that do not reflect the true situation.
- We included four theory corrections to the existing 2PN phase parameters and added two additional ones.
- For 'low-mass' binaries, the binary parameter errors are not inflated dramatically while for 'high-mass' binaries mainly the error in the luminosity distance is inflated by an order of magnitude.
- A simultaneous measurement of all the theory parameters is most probably only possible for 'low-mass' binaries.
- The FWF makes great improvements only for 'high-mass' binaries and there especially for the highly correlated parameters

Thank you for listening!