Testing General Relativity with LISA including Spin Precession and Higher Harmonics in the Waveform

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May 24, LISA Symposium 2012
data analysis and alternative theories

- 'Fundamental bias'
  
  What if we used GR templates but GR would be the wrong theory?
  
  We would most certainly detect all the sources, but with slightly lower SNRs and wrong parameters
  
  It is necessary to check the underlying theory and how it affects gravitational wave sources and propagation

- Supermassive black hole binaries are testbeds for alternative gravity theories
  
  - strongest gravitational wave sources we know of
  
  - produce very clean signals

- Spin precession
  
  - Modulation of the waveform → more information in the signal
    
    → less degeneracy in the parameter space

- Higher harmonics
  
  - restricted waveform (RWF): leading order amplitude, 2PN expanded phase
  
  - Full waveform (FWF): 2PN amplitude, 2PN phase
    
    → additional structure, longer observation time
alternative theories and the dynamics of compact binaries

\[ \frac{df}{dt} = \left( \frac{d}{df} \frac{E}{E} \right)^{-1} \frac{dE}{dt} \]

introduce modifications through alternative gravity theories

\[ h(t) = A(t) e^{i\Psi(t)} \]
\[ \Psi(t) = \Phi_c + 2\pi \int_{t_c}^t f \, dt \]
towards alternative theory parameters

Examples:

Brans-Dicke theory

$$[\Psi(f)]_{\text{mod}} = \frac{3}{256\nu} x^{-5/2} \left[ \Psi_{\text{GR}} + \beta_{\text{BD}} x^{-1} \right]$$

massive gravitons

$$[\Psi(f)]_{\text{mod}} = \frac{3}{256\nu} x^{-5/2} \left[ \Psi_{\text{GR}} + \beta_{\text{MG}} x \right]$$

2PN expanded phase term

'general' approach / toy model

$$[\Psi(f)]_{\text{mod}} = \frac{3}{256\nu} x^{-5/2} \left[ \Psi_{\text{GR}} + \sum_{i} \Psi_{i} x^{i} \right] \quad i = \{-1, 0, 1/2, 1, 3/2, 2\}$$

basic questions:

• What is the effect of adding a theory parameter to every existing PN phase term (plus two new ones) on the measurement accuracy?

• How much accuracy is gained by using the FWF instead of the RWF?

• **Fisher matrix** analysis of quasi-circular BBH systems in the 2PN approximation with fully precessing spins using the classic LISA configuration.

  • All parameter are randomized, except for masses, redshift and theory parameters.

  • Since we are interested in deviations from GR, all the $\Psi_{i}$ are set to 0.
relation to the ppE scheme

\[ \tilde{h}(f) = \tilde{h}_{\text{GR}}(f) \left[ 1 + \alpha u^a \right] e^{i \beta u^b} \quad u = \pi \mathcal{M} f \]

catches leading order deviations from GR

Yunes / Pretorius 2009

\[ [\Psi(f)]_{\text{mod}} = \frac{3}{256 \nu} x^{-5/2} \left[ \Psi_{\text{GR}} + \sum_i \Psi_i x^i \right] \quad i = \{-1, 0, 1/2, 1, 3/2, 2\} \]

no amplitude corrections, correction for every PN phase term + new ones

in our variant..

→..not all classes of alternative theories are visible (at leading order)
→..effects of next-to-leading order terms can be investigated
→..every of the six theory parameters can be turned on and off, even after the simulations
**Parameter estimation:** loss of accuracy in binary parameters

- Which binary parameter errors are inflated due to correlations with theory parameters?

'**Low-mass' binaries** $10^5 \ M_{\text{sun}} \leq M \leq 10^7 \ M_{\text{sun}}$

- Angular position in the sky nearly unaffected (10% loss)
- Luminosity distance, masses and spin parameters: worse by factor of less than one order of magnitude

'**High-mass' binaries** $3 \times 10^7 \ M_{\text{sun}} \leq M \leq 10^8 \ M_{\text{sun}}$

- Angular position in the sky: factor of ~2
- Luminosity distance worse by factor of ~10-20
- Mass and spin parameters worse by a factor of ~3

Correlation coefficient of luminosity distance with theory parameters is increasing with mass from ~0.2 ($10^5 \ M_{\text{sun}}$) to ~0.95 ($6 \times 10^7 \ M_{\text{sun}}$)
**parameter estimation: error on theory parameters**

Is the accuracy of the theory parameters reasonable?

- **Theory parameters with fiducial GR values (at z=1)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>low-mass</th>
<th>high-mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \Psi_0 / \psi_{GR,0}$</td>
<td>$\sim 10^{-3} - 10^{-1}$</td>
<td>$\sim 1 - 10^2$</td>
</tr>
<tr>
<td>$\Delta \Psi_1 / \psi_{GR,1}$</td>
<td>$\sim 10^{-2} - 10^{-1}$</td>
<td>$\sim 1 - 10^2$</td>
</tr>
<tr>
<td>$\Delta \Psi_{3/2} / \psi_{GR,3/2}$</td>
<td>$\sim 10^{-2} - 10^{-1}$</td>
<td>$\sim 1 - 10^2$</td>
</tr>
<tr>
<td>$\Delta \Psi_2 / \psi_{GR,2}$</td>
<td>$\sim 10^{-1} - 1$</td>
<td>$\sim 10 - 10^2$</td>
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- **Theory parameters without fiducial GR values (at z=1)**

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<tr>
<td>$\Delta \Psi_{-1}$</td>
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<td>$\sim 10^{-2} - 1$</td>
</tr>
<tr>
<td>$\Delta \Psi_{1/2}$</td>
<td>$\sim 10^{-2} - 10^{-1}$</td>
<td>$\sim 1 - 10^3$</td>
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desired accuracy depends on theory to test for and will most probably only serve as an upper limit!
parameter estimation: FWF vs RWF

Is it always necessary to use the FWF? What increase of accuracy can be gained?

low-mass binaries $10^5 \, M_{\text{sun}} \leq M \leq 10^7 \, M_{\text{sun}}$

- Factor of $\sim 1.5-4$ gained in accuracy of both binary and theory parameters

high-mass binaries $3 \times 10^7 \, M_{\text{sun}} \leq M \leq 10^8 \, M_{\text{sun}}$

- Angular position in the sky: 10-100
- Luminosity distance: 50-1400
- Mass and spin parameters: 30-200
- Theory parameters: $\Psi_{-1}$: $\sim 100 - 1000$, rest: $\sim 10-100$

$\rightarrow$ FWF breaks correlations between highly correlated parameters, especially luminosity distance!

Use of the FWF is mandatory above total masses of $\sim 10^7 \, M_{\text{sun}}$

But there's not much to get in that region anyway!
parameter estimation: lower bound on graviton wavelength

The median error $\Delta \Psi_1$ can be used as an upper limit for a constraint on $\Psi_1$

This results in a lower bound on the graviton wavelength:

$$\lambda_g > \sqrt{\frac{256}{3} \frac{G}{c^2} \frac{\pi^2 D(z) M \nu}{(1 + z) \Delta \Psi_1}}$$

Yagi / Tanaka 2010: (RWF, simple precession, without eccentricity)

$$\lambda_g > 4.9 \times 10^{21}$$

At redshift $z=1$, this corresponds to the following lower bounds:

All six theory parameters included:

$$\lambda_g > 1.2 \times 10^{21} \text{ cm (FWF)} \quad \lambda_g > 7.8 \times 10^{20} \text{ cm (RWF)}$$

All theory parameters turned off except for $\Psi_1$:

$$\lambda_g > 7.6 \times 10^{21} \text{ cm (FWF)} \quad \lambda_g > 4.9 \times 10^{21} \text{ cm (RWF)}$$
summary

- We have to question the underlying model in order to avoid data analysis results that do not reflect the true situation.

- We included four theory corrections to the existing 2PN phase parameters and added two additional ones.

- For 'low-mass' binaries, the binary parameter errors are not inflated dramatically while for 'high-mass' binaries mainly the error in the luminosity distance is inflated by an order of magnitude.

- A simultaneous measurement of all the theory parameters is most probably only possible for 'low-mass' binaries.

- The FWF makes great improvements only for 'high-mass' binaries and there especially for the highly correlated parameters

Thank you for listening!