Gravitational self-force approach to extreme mass-ratio inspirals

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Motivation

- The inspiral of a solar-mass compact object onto a massive black hole is a promising source of low-frequency gravitational waves for a space-borne detector.

- Such systems involve very small mass ratios, strong gravitational fields, large orbital velocities, high eccentricities, and large black-hole spins.

- Measurement of these waves is rich in scientific payoffs, such as mapping out the Kerr spacetime around a rapidly rotating black hole, testing general relativity, and probing the astrophysics of supermassive black holes.
Motivation

- Signal detection and source determination require the accurate modeling of the orbital motion and the emission of gravitational waves.

- The description of the motion must take into account the gravitational effects of the small compact object; in the test-mass limit the motion is geodesic and there is no inspiral.

- The computations must be done in a context of strong gravitational fields, large orbital velocities, and high eccentricities.
Motivation

Post-Newtonian theory and numerical relativity are not the best tools.

A fresh approach is required.

In the self-force approach, the small mass ratio is used as an expansion parameter. There is no restriction on the orbital velocity.
Self-force

- The small body creates a gravitational imprint in the spacetime of the large black hole.

- With a small mass ratio, the change in the gravitational field is small and can be calculated as a perturbative expansion.

- The perturbation acts on the compact object and alters its motion, which is no longer geodesic in the spacetime of the large black hole.

- The motion is accelerated, due to the action of the gravitational self-force.

- The perturbation propagates outward as a gravitational wave.
Foundations

* The standard formulation of the self-force was first given in 1997, derived from point-particle models. [Mino, Tanaka, Sasaki; Quinn and Wald]

* In 2003, Detweiler and Whiting introduced a meaningful decomposition of the perturbation into singular and regular pieces; the regular piece is entirely responsible for the self-force.

\[
h^R_{\alpha\beta} = h_{\alpha\beta} - h^S_{\alpha\beta}
\]

* The singular piece is constructed locally, and the motion is geodesic in a perturbed spacetime with metric

\[
g^{\text{Kerr}}_{\alpha\beta} + h^R_{\alpha\beta}
\]

* Equivalently, the motion is accelerated in the Kerr metric.
Foundations

- The foundations were revisited in the last few years. Two main issues were overcome:
  - The point-particle model were justified on the basis of an extended-body approach, in an approximation in which the details of internal structure are unimportant. [Gralla and Wald (2008); Pound (2010); Harte (2012)]
  - First-order perturbation theory requires geodesic motion as a source, but produces accelerated motion as an outcome; the contradiction can be avoided by generalizing the formulation of perturbation theory. [Gralla and Wald (2008); Pound (2010)]

- The second-order self-force is now available [Pound (2012); Gralla (2012)]
Implementations

* Mode-sum approaches:

Symmetries of the Schwarzschild and Kerr spacetimes permit a mode decomposition of the perturbation; the singular field can be computed mode-by-mode; and the self-force can be evaluated as a mode sum. [Barack and Ori (2002); Mino, Nakano, Sasaki (2003)]

* Effective-source methods:

The perturbation equations can be formulated for the regular field, resulting in a nonsingular effective source. [Barack and Golbourn (2007); Vega and Detweiler (2008)]

* Adiabatic approximations:

The self-force can be approximated by its dissipative piece, which can be obtained by energy and angular-momentum balance. [Hugues, Drasco, Flanagan, Franklin (2005)]
Implementations

* Gravitational self-force on fixed orbits in Schwarzschild spacetime
  
  **Barack and Sago (2010):** eccentric orbits, time-domain code, mode sum

* Scalar self-force on fixed orbits in Kerr spacetime
  
  **Warburton and Barack (2011):** eccentric, equatorial orbits, frequency-domain code, mode sum

* Inspirals in Schwarzschild spacetime
  
  **Warburton, Akcay, Barack, Gair, Sago (2012):** gravitational self-force, hybrid frequency-domain/time-domain code, mode sum

  **Diener, Vega, Wardell, Detweiler (2012):** scalar self-force, time-domain, fully self-consistent, effective source
Results

Shift in the innermost stable circular orbit (ISCO) of a Schwarzschild black hole caused by the conservative piece of the gravitational self-force [Barack and Sago (2012)]:

\[ \frac{\Delta \Omega}{\Omega} = 0.4867 \frac{m}{M} \]

Shift in the innermost stable equatorial circular orbit (ISCEO) of a Kerr black hole caused by the conservative piece of the scalar self-force [Warburton (2012)]:

\[ \frac{\Delta \Omega}{\Omega} > 0 \quad \text{for} \quad \frac{a}{M} < 0.8 \]
\[ \frac{\Delta \Omega}{\Omega} < 0 \quad \text{for} \quad \frac{a}{M} > 0.8 \]
Results


\[ r_{\text{min}} = \frac{pM}{1 + e} \quad r_{\text{max}} = \frac{pM}{1 - e} \]
Results

Current status

- Foundational aspects of the self-force are now well understood.

- Second-order self-force is now available, at least formally.

- Implementations are maturing (inspirals in Schwarzschild spacetime; meaningful comparisons with post-Newtonian theory, and possible extrapolations to large mass ratios).

- Rate of progress is good.
Pressing issues

- Methods to compute the gravitational self-force in Kerr spacetime are still incomplete; there is no clear path to completion.

- Practical methods to compute the second-order self-force have yet to be devised.

- The self-force must still be incorporated in a consistent wave-generation formalism; this requires a proper foundation, now within reach.

- Modeling of inspirals is made more difficult by the existence of transient resonances, which occur when two of the fundamental orbital frequencies become commensurate. [Hinderer and Flanagan (2010)]