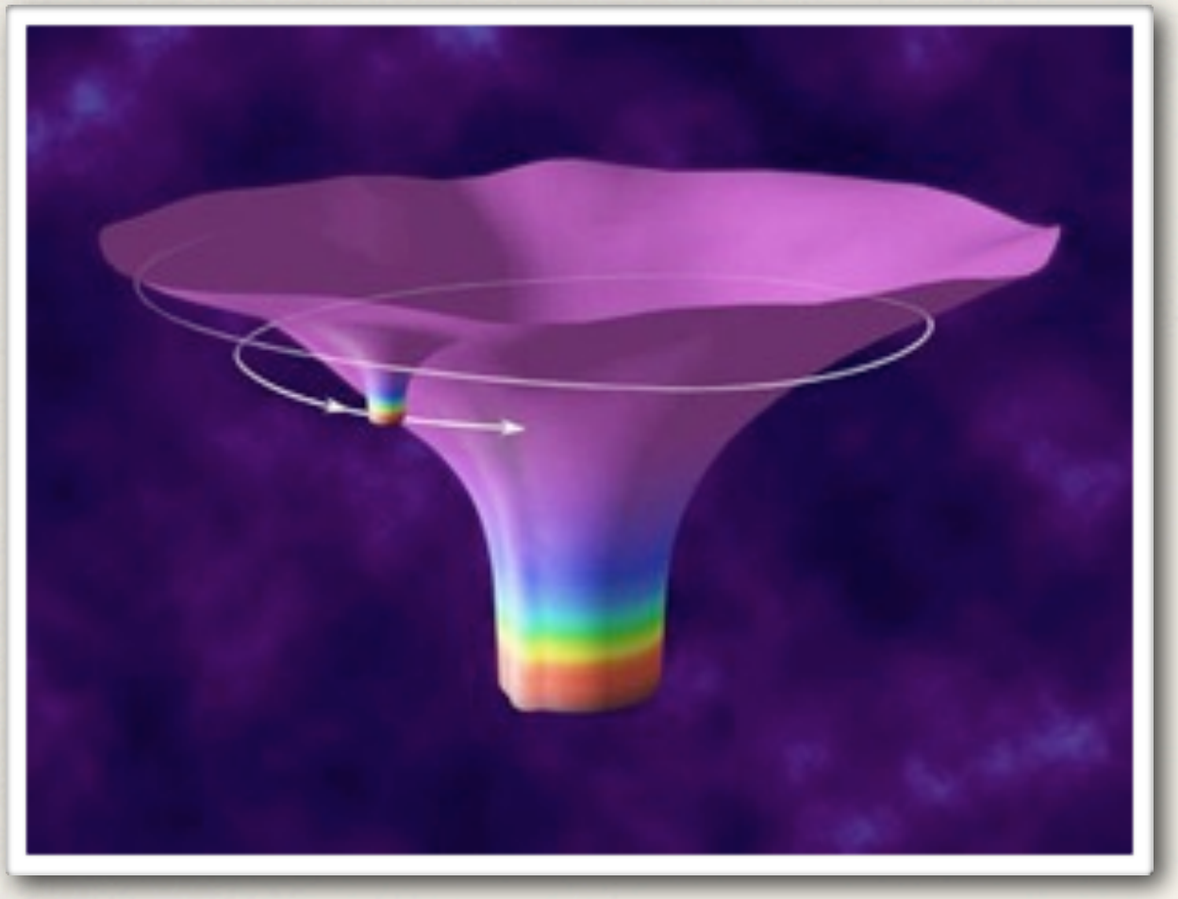

Gravitational self-force approach to extreme mass-ratio inspirals

Eric Poisson, University of Guelph

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UNIVERSITY of GUELPH

CHANGING LIVES IMPROVING LIFE

Motivation

- The inspiral of a solar-mass compact object onto a massive black hole is a promising source of low-frequency gravitational waves for a space-borne detector.

- Such systems involve very small mass ratios, strong gravitational fields, large orbital velocities, high eccentricities, and large black-hole spins.

- Measurement of these waves is rich in scientific payoffs, such as mapping out the Kerr spacetime around a rapidly rotating black hole, testing general relativity, and probing the astrophysics of supermassive black holes.

Motivation

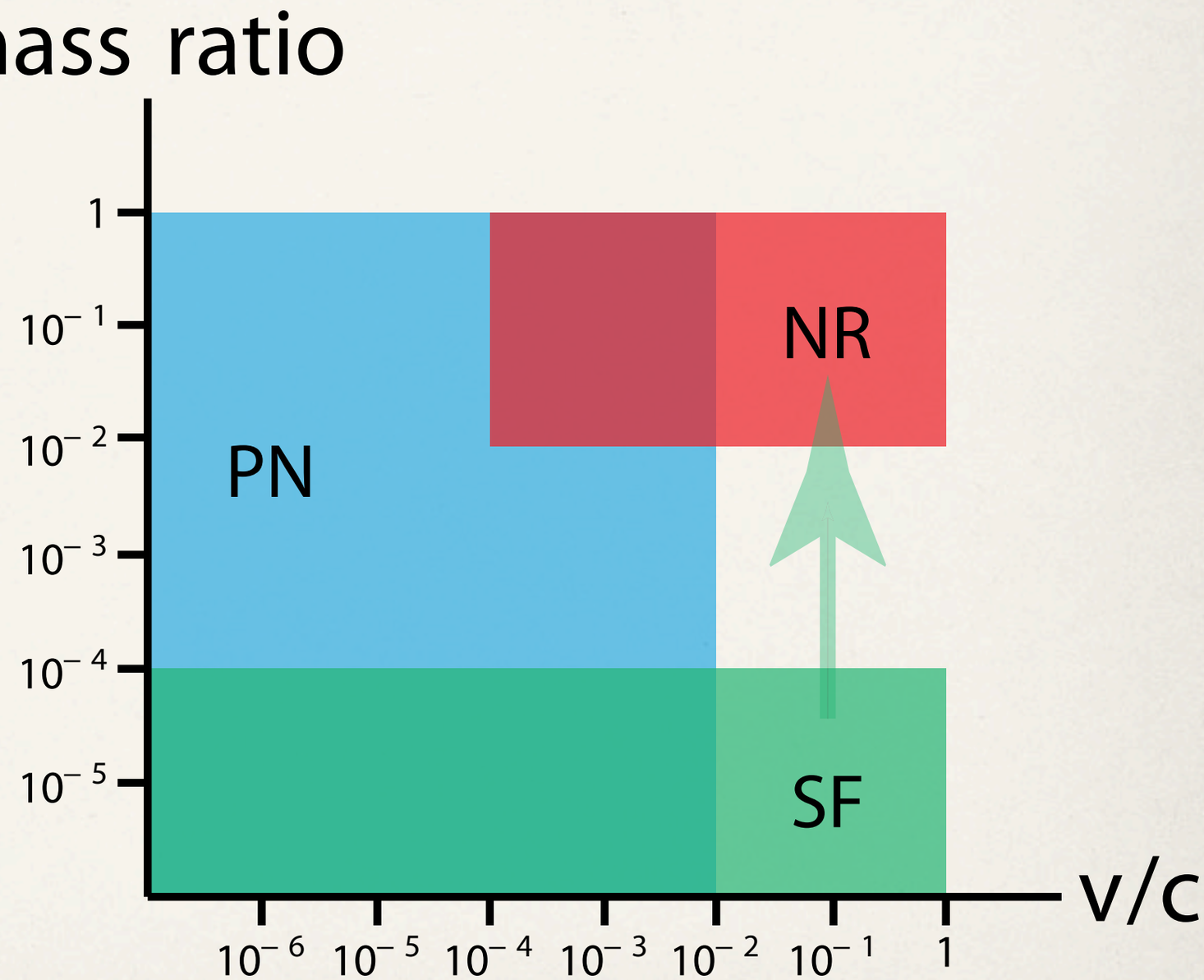
- Signal detection and source determination require the accurate modeling of the orbital motion and the emission of gravitational waves.
- The description of the motion must take into account the gravitational effects of the small compact object; in the test-mass limit the motion is geodesic and there is no inspiral.
- The computations must be done in a context of strong gravitational fields, large orbital velocities, and high eccentricities.

Motivation

Post-Newtonian theory and numerical relativity are not the best tools.

A fresh approach is required.

In the self-force approach, the small mass ratio is used as an expansion parameter. There is no restriction on the orbital velocity.



Self-force

- The small body creates a gravitational imprint in the spacetime of the large black hole.
- With a small mass ratio, the change in the gravitational field is small and can be calculated as a perturbative expansion.
- The perturbation acts on the compact object and alters its motion, which is no longer geodesic in the spacetime of the large black hole.
- The motion is accelerated, due to the action of the **gravitational self-force**.
- The perturbation propagates outward as a gravitational wave.

Foundations

- The standard formulation of the self-force was first given in 1997, derived from point-particle models. [**Mino, Tanaka, Sasaki; Quinn and Wald**]

- In 2003, Detweiler and Whiting introduced a meaningful decomposition of the perturbation into **singular and regular pieces**; the regular piece is entirely responsible for the self-force.

$$h^{\mathsf{R}}_{\alpha\beta} = h_{\alpha\beta} - h^{\mathsf{S}}_{\alpha\beta}$$

- The singular piece is constructed locally, and the motion is geodesic in a perturbed spacetime with metric

$$g^{\mathrm{Kerr}}_{\alpha\beta} + h^{\mathsf{R}}_{\alpha\beta}$$

- Equivalently, the motion is accelerated in the Kerr metric.

Foundations

- The foundations were revisited in the last few years. Two main issues were overcome:
  - The point-particle model were justified on the basis of an extended-body approach, in an approximation in which the details of internal structure are unimportant. [**Gralla and Wald (2008); Pound (2010); Harte (2012)**]
  - First-order perturbation theory requires geodesic motion as a source, but produces accelerated motion as an outcome; the contradiction can be avoided by generalizing the formulation of perturbation theory. [**Gralla and Wald (2008); Pound (2010)**]
- The second-order self-force is now available [**Pound (2012); Gralla (2012)**]

Implementations

- Mode-sum approaches:

  Symmetries of the Schwarzschild and Kerr spacetimes permit a mode decomposition of the perturbation; the singular field can be computed mode-by-mode; and the self-force can be evaluated as a mode sum. [**Barack and Ori (2002); Mino, Nakano, Sasaki (2003)**]

- Effective-source methods:

  The perturbation equations can be formulated for the regular field, resulting in a nonsingular effective source. [**Barack and Golbourn (2007); Vega and Detweiler (2008)**]

- Adiabatic approximations:

  The self-force can be approximated by its dissipative piece, which can be obtained by energy and angular-momentum balance. [**Hugues, Drasco, Flanagan, Franklin (2005)**]

Implementations

- Gravitational self-force on fixed orbits in Schwarzschild spacetime

  Barack and Sago (2010): eccentric orbits, time-domain code, mode sum

- Scalar self-force on fixed orbits in Kerr spacetime

  Warburton and Barack (2011): eccentric, equatorial orbits, frequency-domain code, mode sum

- Inspirals in Schwarzschild spacetime

  Warburton, Akcay, Barack, Gair, Sago (2012): gravitational self-force, hybrid frequency-domain/time-domain code, mode sum

  Diener, Vega, Wardell, Detweiler (2012): scalar self-force, time-domain, fully self-consistent, effective source

Results

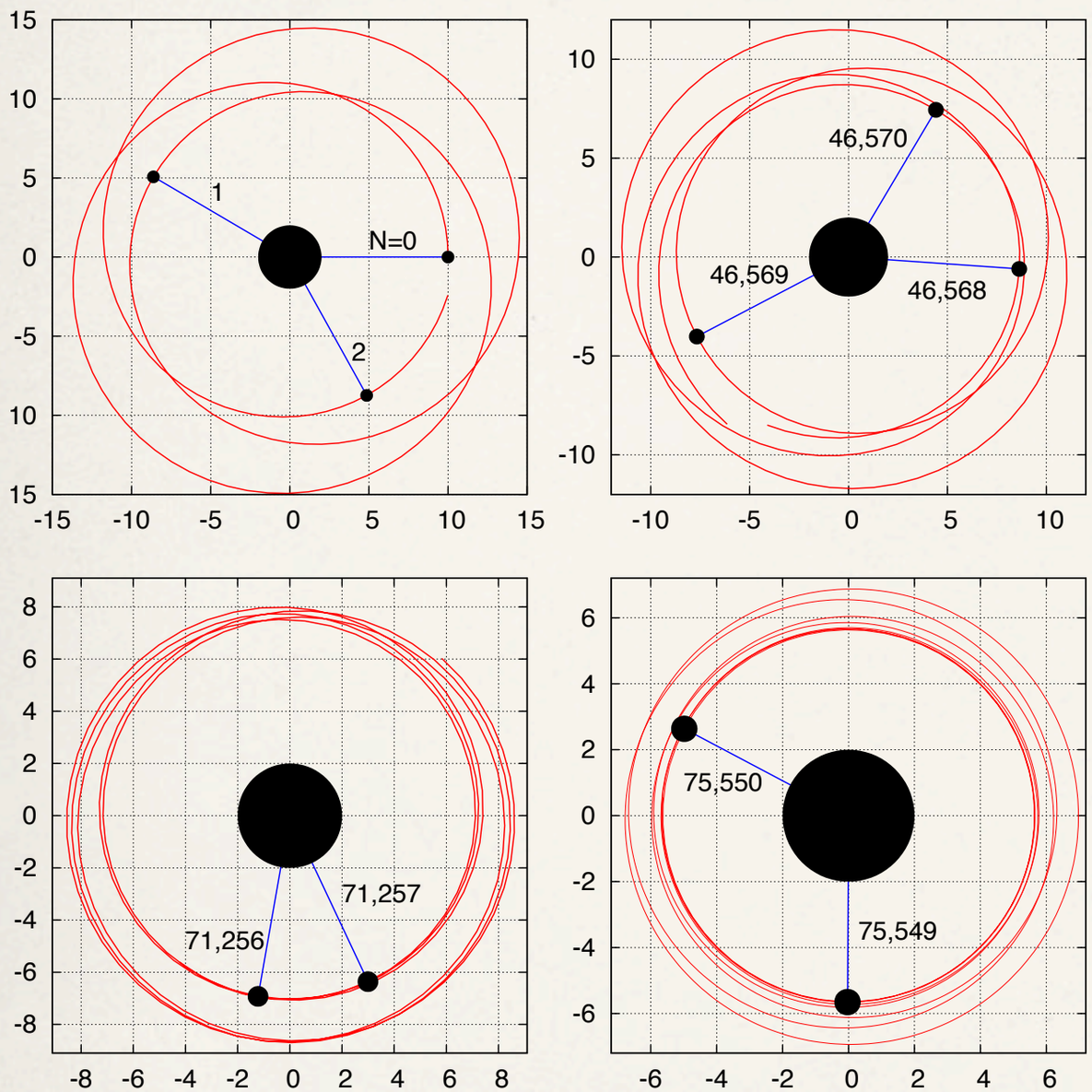
- Shift in the innermost stable circular orbit (ISCO) of a Schwarzschild black hole caused by the conservative piece of the gravitational self-force [**Barack and Sago (2012)**]:

$$\frac{\Delta\Omega}{\Omega} = 0.4867\frac{m}{M}$$

- Shift in the innermost stable equatorial circular orbit (ISCEO) of a Kerr black hole caused by the conservative piece of the scalar self-force [**Warburton (2012)**]:

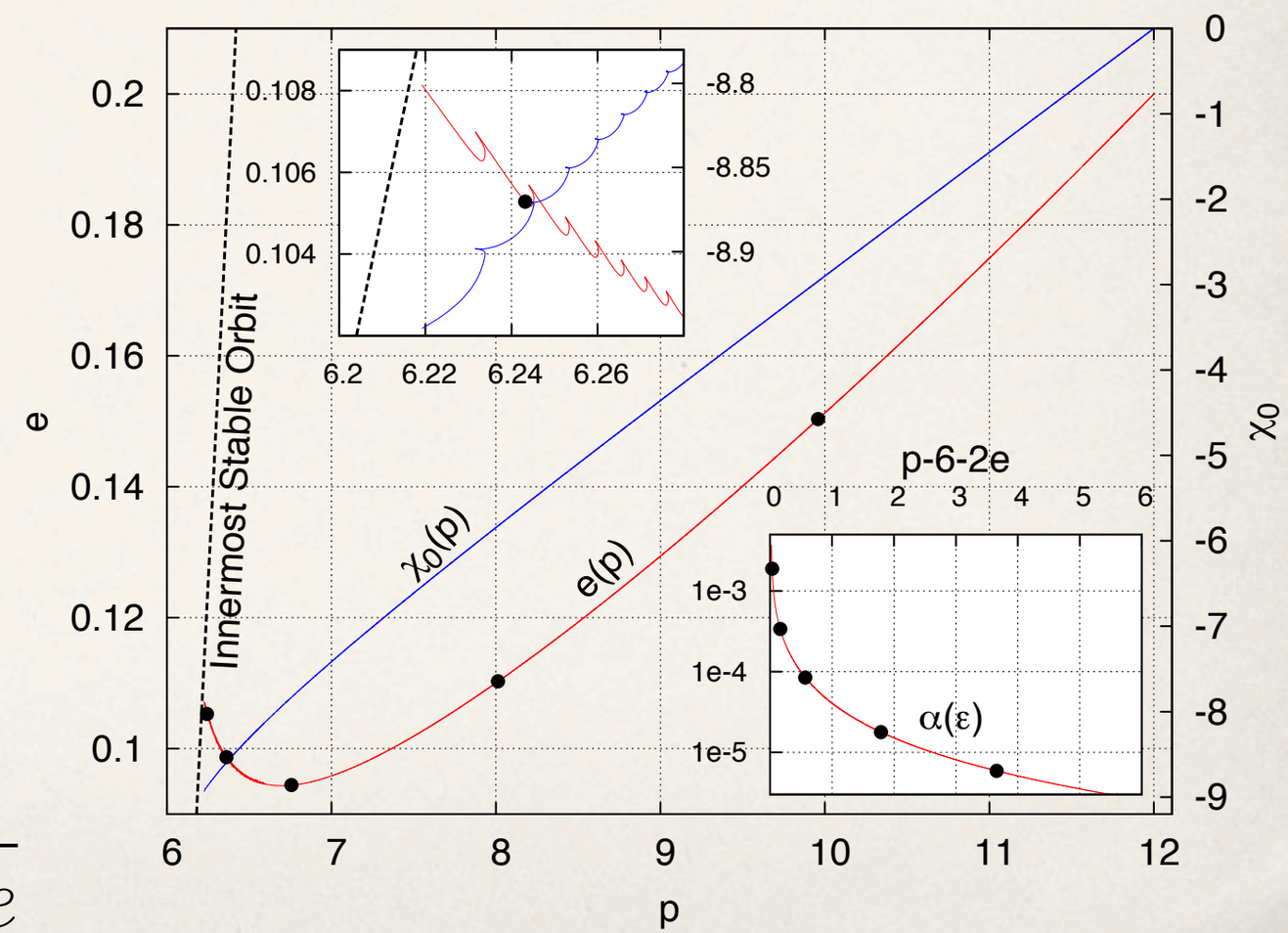
$$\frac{\Delta\Omega}{\Omega} > 0 \quad \text{for} \quad \frac{a}{M} < 0.8$$
$$\frac{\Delta\Omega}{\Omega} < 0 \quad \text{for} \quad \frac{a}{M} > 0.8$$

Results



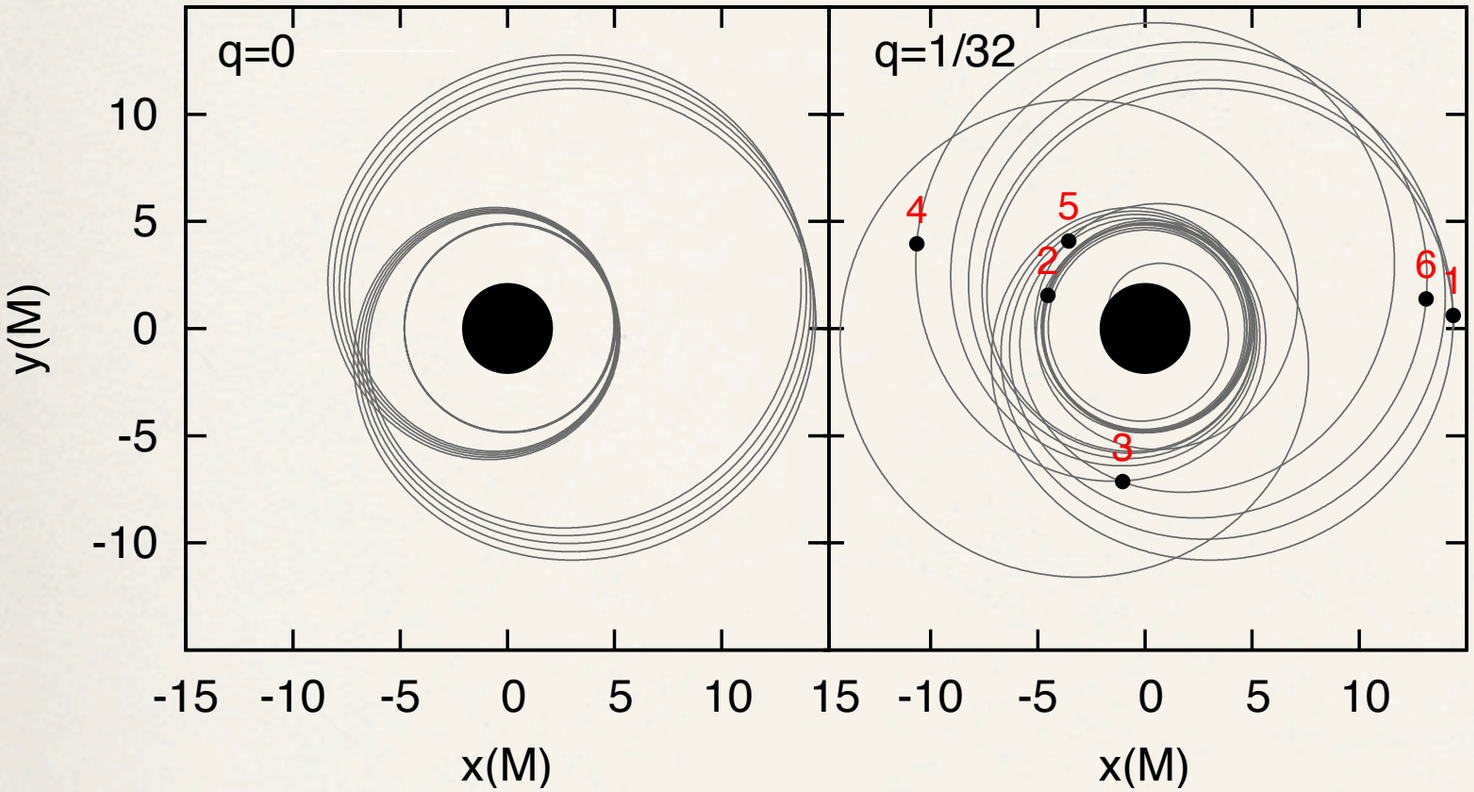
Warburton, Akcay, Barack, Gair, Sago (2012):

Gravitational self-force inspiral

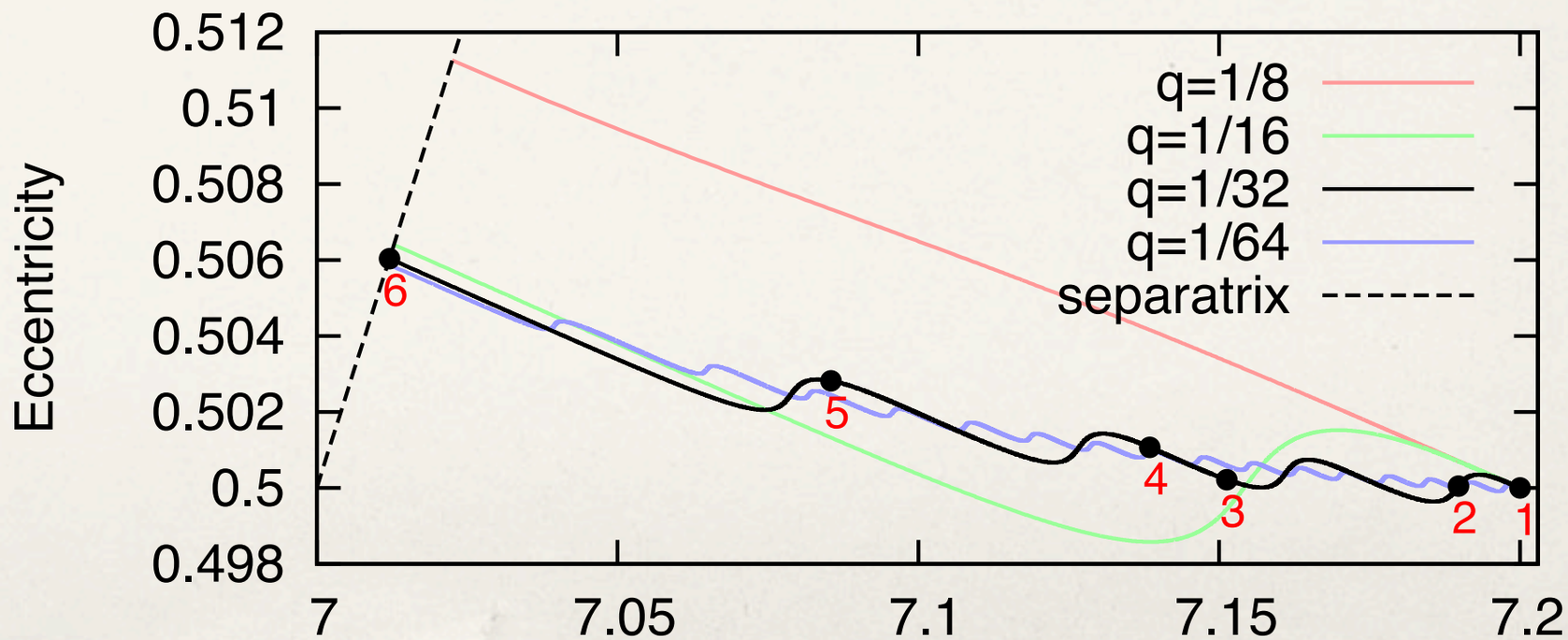


$$r_{\min} = \frac{pM}{1+e} \qquad r_{\max} = \frac{pM}{1-e}$$

Results



Diener, Vega, Wardell, Detweiler (2012): Scalar self-force inspiral



Current status

- Foundational aspects of the self-force are now well understood.
- Second-order self-force is now available, at least formally.
- Implementations are maturing (inspirals in Schwarzschild spacetime; meaningful comparisons with post-Newtonian theory, and possible extrapolations to large mass ratios).
- Rate of progress is good.

Pressing issues

- Methods to compute the gravitational self-force in Kerr spacetime are still incomplete; there is no clear path to completion.
- Practical methods to compute the second-order self-force have yet to be devised.
- The self-force must still be incorporated in a consistent wave-generation formalism; this requires a proper foundation, now within reach.
- Modeling of inspirals is made more difficult by the existence of transient resonances, which occur when two of the fundamental orbital frequencies become commensurate. [**Hinderer and Flanagan (2010)**]