9^{th} *LISA Symposium, Paris*
ASP Conference Series, Vol. 467
G. Auger, P. Binétruy and E. Plagnol, eds.

Bayesian Model Selection for LISA Pathfinder

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Abstract. The LISA Pathfinder mission (LPF) aims at validating the displacement and acceleration noise models and to test key technologies for the future LISA mission. The LISA Technology Package (LTP) Data Analysis team has developed complex models of the LTP for simulations and data analysis during the mission. These models contain a large number of parameters to be estimated and for that reason, we need to recover only the essential ones that describe the observations. Being able to distinguish between competing models that describe the data introduces many possible applications in LTP Data Analysis. In our analysis we use two main different approximations to compute the Bayes Factor: the Reversible Jump Markov Chain Monte Carlo (RJMCMC) and the Laplace approximations. They are applied first to toy models and then verified with full LTP models. This work is part of the LTPDA Matlab toolbox.

4. Introduction

The motivation for developing tools to compare models of the LTP (Antonucci et al. (2011)) comes from the nature of the instrument itself. It is a complicated system with numerous parameters to estimate and with unknown effects to be determined during flight operations. Given a data-set $\mathbf{x}$ we can test our hypothesis for the dynamics of the LTP and investigate the importance of various contributions to the overall noise performance of the system. Working in a Bayesian framework, we can discriminate between different models by computing the so-called *Bayes factor* (MacKay (2003); Godsill (2001)).

5. Estimating the Bayes Factor.

Given two models X and Y, the Bayes Factor can be expressed as: $B_{\mathrm{XY}} = \pi_{\mathrm{X}}(\mathbf{x})/\pi_{\mathrm{Y}}(\mathbf{x})$, where π_{X} is the evidence of model X. The evidence (or *marginal likelihood*) of the model is difficult to to calculate, especially when the dimensionality of the models is high. A solution for this set of problems is the RJMCMC method which is a special Markov Chain Monte Carlo (MCMC) method that is capable of sampling parameter spaces of different dimensions. The MCMC implemented in the LTPDA toolbox described in Nofrarias et al. (2010) is based on the Metropolis algorithm and we can say that the RJMCMC developed is a generalization of the same algorithm (Lopes & West (2004)). The main idea is that the algorithm will spend more time iterating inside the most probable model and we can estimate the Bayes Factor as: B_{XY} = (# of iterations in model X) / (# of iterations in model Y). We can verify our findings with the Laplace approximation of the evidence of a model:

$$\pi_{\mathrm{X}}(\mathbf{x}) \simeq (2\pi)^{\mathbf{D}/2}\,|\mathbf{\Sigma}|^{-1/2}\,\pi(\theta_{\mathrm{MAP}}, \mathrm{X}\,|\mathbf{x}) \tag{23}$$

where θ_{MAP} is the set of parameters that maximize the likelihood $\pi(\theta_{\mathrm{MAP}}, \mathrm{X}\,|\mathbf{x})$ of model X and D is its dimensionality. The covariance matrix of the parameters $\mathbf{\Sigma}$, can be estimated via the calculation of the Fisher Information Matrix (we can call this version as the Laplace-Fisher approximation), or it can be extracted from previous MCMC runs (we can call this version as the Laplace-Metropolis approximation).

6. Applying to the LTP models.

The methods developed were tested and verified with simple toy harmonic oscillator models and then used to perform model selection between different LTP models. As expected, there are some aspects of the LTP that can not be tested on ground. In our case, this translates in creating different versions of the LTP models that include or exclude a set of parameters to estimate. An example used is the *guidance delays* of the model. They are time delays that are caused from functions of the on-board computer but cannot be verified on ground tests. Preliminary results obtained with synthetic data investigations showed that we can successfully perform the RJMCMC and decide if these parameters are significant for the description of the data collected. In Figure 2, we investigate the evolution of Bayes Factor as a function of the signal-to-noise ratio (SNR) using the methods that were described above. The results obtained by comparing different LTP models seem to be in good agreement. The output of all methods, as expected, is clearer when we are working in higher SNR areas. On the contrary, when the SNR is low (below 24 in the particular example) we cannot distinguish between the competing models.

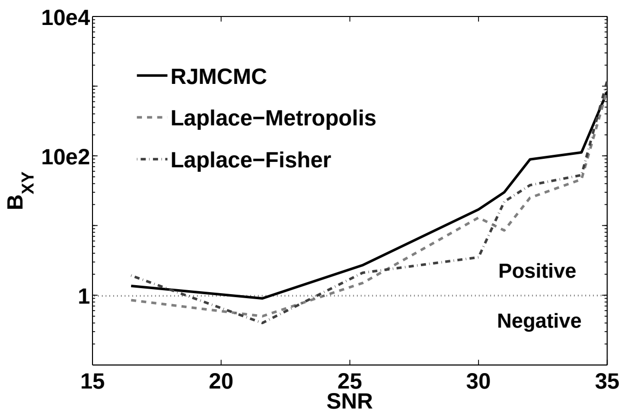

Figure 2. Comparison of the Bayes Factor as a function of SNR for all the approximations.

Acknowledgments. We acknowledge support from contracts AYA-2010-15709 (MICINN) and 2009-SGR-935 (AGAUR). NK acknowledges support from the FI PhD program of the AGAUR (Generalitat de Catalunya). CFS acknowledges support from the Ramón y Cajal Programme of the MICINN.

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