9th *LISA Symposium, Paris*
ASP Conference Series, Vol. 467
G. Auger, P. Binétruy and E. Plagnol, eds.

Testing General Relativity with LISA Including Spin Precession and Higher Harmonics in the Waveform

Cédric Huwyler

Institute for Theoretical Physics, University of Zurich

Abstract.
I present the updated results of Huwyler et al. (2011), where we study the use of multiple alternative theory parameters to determine the accuracy at which a classic LISA configuration should be able to observe deviations from General Relativity through the inspiral of supermassive black holes in the low frequency approximation. In order to reach higher accuracy, we include the correlation breaking effects of full spin precession and higher harmonics. We use an implementation similar to the ppE formalism by Yunes and Pretorius. While many studies performing Monte Carlo simulations with the ppE formalism consider leading order deviations, we try to take another viewpoint and introduce six simultaneous phase correction parameters accounting for modified gravity in the second post-Newtonian gravitational wave phase. This approach will not be able to find all currently proposed types of alternative theories (some will fall through our 'sieve'), but will enable us to quantify the accuracy lost by introducing multiple corrections at the same time and to investigate next-to-leading order effects. In our study we performed simulations for binaries with total masses in the range of $10^5 M_\odot < M < 10^8 M_\odot$ on quasi-circular orbits. In order to find error distributions for the alternative theory parameters and to investigate correlations, we apply the Fisher information formalism for 10^3 randomly distributed points in the parameter space each, comparing the full (FWF) and restricted (RWF) version of the gravitational waveform. As an application, we compute an optimal lower bound on the graviton 'mass'.

1. Introduction

Several weak field tests of General Relativity (GR) have been conducted to this day (Will 2006), albeit only for weak fields such as by the dynamics of the Solar System ($v/c \sim 10^{-4}$) or Pulsars ($v/c \sim 10^{-3}$). Black hole mergers could approach $v/c \sim 0.4$ and open a new tool for tests of GR: gravitational waves (GWs). The proposed space-based eLISA/NGO detector will be sensitive to the strongest dynamical sources in the universe: supermassive black hole (SMBH) binaries with masses between $10^5 - 10^7 M_\odot$. Such SMBHs are thought to be brought together by various astrophysical mechanisms to form a binary that could, after a long inspiral phase, merge into a single Kerr black hole through the emission of GWs. The very clean, long-lasting and theoretically well-understood signal emitted by such SMBH binaries can be used to learn more about GR in the strong field regime through matched filtering techniques.

If GR is not the correct model for describing the dynamics of black holes, the underlying true theory will reveal itself to us through its imprints in the observed GW signal. However, we should expect that such modified dynamics would not differ significantly from GR, implicating only small deviations in the observed waveforms. Ignoring the previous considerations, we would most certainly detect almost all the signals, but with slightly lower signal-to-noise ratio (SNR) and incorrect parameter estimation. Such a possible *fundamental bias* (Yunes & Pretorius 2009) makes it necessary to create a signal template that hunts down deviations from GR. This can be done by introducing additional theory parameters that control such deviations.

Yunes & Pretorius (2009) have proposed their so called *parameterized post-Einsteinian* formalism to do that. We use a similar, albeit slightly differently parametrized version. Adding parameters to the model while getting the same amount of information has its price: the accuracy of all the recovered parameter values is decreased through degeneracies in the parameter space. The only way to get rid of degeneracies is to increase the amount of information contained in the signal model. This can be done by including effects which add further modulation to the waveform, such as spin precession and higher harmonics, which we will consider in this work. Many studies in the field of GR tests did not include higher harmonics so far, considering the GW phase up to second post-Newtonian (PN) order, but only taking the amplitude to leading order (restricted waveform, RWF). We use the full waveform (FWF) that uses a 2PN expanded amplitude. For simplicity, we do not consider the effects of eccentric orbits.

A variety of alternatives to GR has been proposed (see e.g Yunes & Pretorius (2009) or Cornish et al. (2011)), either through attempts to account for unsatisfactorily explained theoretical, astrophysical or cosmological phenomena or as toy models to investigate effects which are inexistent in GR but can be modeled theoretically. The ppE formalism and our implementation allow to map most (resp. some) of such extensions to a set of alternative theory parameters. A post-detection analysis is then able to evaluate the odds for the different proposed alternatives using a Bayesian framework (see e.g. Li et al. (2012)) which depend on the accuracy with which the alternative theory parameters can be measured by a LISA-like mission.

In this study we simultaneously introduce six alternative theory parameters to the post-Newtonian coefficients of the orbital phase of a quasi-circular black binary hole (BHB) inspiral, including the full 2PN precession of spins and angular momentum. We add higher harmonics to the waveform by considering the full 2PN amplitude. Since matched filtering is far more sensitive to the GW phase than to the amplitude, we do not consider corrections to the amplitude of the wave. We evaluate the measurement accuracy with which a classic LISA-like mission will be able to detect such corrections for BHBs. To estimate the errors on the parameters, we make use of the Fisher information formalism which is legitimate in the limit of high SNR which LISA will provide. As an application of our general study, we compute an optimal lower bound to the graviton Compton wavelength, in order to compare our work with previous massive graviton studies considering different models and effects (Will 1998; Berti et al. 2005; Stavridis & Will 2009; Arun & Will 2009; Yagi & Tanaka 2010; Keppel & Ajith 2010; Berti et al. 2011).

2. Theory

If emission of GWs is put aside, the conservative orbital motion of a BHB is governed by one quantity: its Hamiltonian or total energy. However, we know from the observation of several binary pulsar systems that during the orbital evolution energy is radiated away, presumably in the form of GWs. Such dissipative dynamics lead to a shrinking of the orbit, ending up in a possible merger of the two black holes. Theoretical models for only two quantities, the total energy and the GW luminosity, should therefore provide us with complete knowledge about the GW frequency evolution of the system, since

$$\frac{dx}{dt} = \left(\frac{d}{dx}E\right)^{-1}\frac{dE}{dt}, \qquad \text{for } x = \left(\frac{GM\pi f_{GW}}{c^3}\right)^{2/3}. \tag{1}$$

For GR, the PN expansion of eq. 1 is known up to 2PN, including time-dependent spin-orbit and spin-spin terms which have to be evolved separately. If, however, GR is not the true underlying theory, this will result in a modified Hamiltonian and a different GW luminosity. In the PN framework, such modifications result in a change to the PN coefficients of eq. 1. Scalar field theories, such as Brans-Dicke theory (Brans & Dicke 1961), cause for example dipole radiation proportional to x^{-1}, which is not there for GR. Also, alternatives to GR are proposed which change the propagation of GWs and make their speed frequency dependent. Such so called *massive graviton* effects (Will 1998) lead to a change in the 1PN coefficient of the GW phase at leading order. We account for such deviations by introducing corrections to eq. 1, resulting in a modified GW phase of the form $[\Psi(f/n)]_{mod} = [\Psi(f/n)]_{GR} + \Delta\Psi$ where

$$\Delta\Psi = \frac{3}{256\nu} x^{-5/2} \left(\Psi_{-1} x^{-1} + \Psi_0 + \Psi_{1/2} x^{1/2} + \Psi_1 x + \Psi_{3/2} x^{3/2} + \Psi_2 x^2 \right). \tag{2}$$

This modified phase can be incorporated into the n-th harmonic of the gravitational waveform $\tilde{h}_n(f) \sim A_n(f/n)\, e^{i n \Psi(f/n)}$ in frequency space (using the stationary phase approximation). Because of the lack of theoretical predictions for the dependency of the alternative theory parameters Ψ_i on the rest of the binary parameters, we treat the Ψ_i as constants. Our implementation is a subset of the ppE formalism (Yunes & Pretorius 2009; Yunes & Hughes 2010; Cornish et al. 2011) with the amplitude parameters $\alpha_k = a_k = A_k = 0$ and the phase parameter $b_k = \frac{2}{3}\left(i_k - \frac{5}{2}\right)$, where $i_k = \{-1, 0, 1/2, 1, 3/2, 2\}$ are our summation indices. Including these additional 6 theory parameters to the waveform which for BHBs on quasi-circular orbits with spins is parametrized by 15 parameters, we end up with a model for combined binary parameter estimation and GR tests with 21 parameters in total. In the next subsection, we investigate the accuracy reached with this model.

3. Results

We performed Monte Carlo simulations for 17 different mass configurations, with total masses between $10^5 - 10^8 M_\odot$, mass ratios varying between 1:1 and 1:10, and using 10^3 points in the parameter space for each configuration. In order to compare our results with other studies and also since it is currently unknown with what technical specifications eLISA/NGO will fly, we work with a three arm classic LISA configuration. This leads us to the resulting error distributions by inspection of the elements of the inverted Fisher matrix. We use the following set of parameters: masses $\log_{10} m_1$, $\log_{10} m_2$, spins with orientation $\mu_{1,2} = \cos\theta_{1,2}$, $\phi_{1,2}$ and spin parameter $\chi_{1,2}$, orientation of the angular momentum unit vector $\mu_l = \cos\theta_l$ and ϕ_l, orientation in the sky $\mu_n = \cos\theta_n$ and ϕ_n, time and phase at coalescence $\log t_c$ and $\log d_L$, and alternative theory parameters Ψ_i, $i \in \{-1, 0, 1/2, 1, 3/2, 2\}$. Since we look for deviations from GR, we set $\Psi_i = 0$. The redshift has been kept fixed to $z = 1$, since it is not possible to disentangle redshift, mass and distance. The integration of the Fisher matrix elements is usually started from the well-known innermost stable circular orbit $r_{ISCO} = 6\,GM/c^2$ for test masses around a black hole; although the definition of such an ISCO is non-trivial in the case of spinning black holes with comparable masses, our choice serves as a good cutoff point (Huwyler et al. 2011).

We find that the binaries can roughly be divided into two classes: low-mass (LM) binaries ($M \lesssim 10^7 M_\odot$) and high-mass (HM) binaries ($M \gtrsim 3 \times 10^7 M_\odot$). For LM binaries we find that in general, using the FWF instead of the RWF improves the measurement errors $\Delta\Psi_i$ on the alternative theory parameters by a factor of $\sim 1.5 - 3$. The correlation with the new parameters causes a decrease in the accuracy of the 15 binary parameters. For both the FWF and the RWF, the errors on the mass and spin parameters are typically worse by a factor of $2 - 5$ while the luminosity distance is approximately half as accurate. The angular orientation errors increase only by $\sim 10\%$. In figs. 1 and 2 we show the error distribution (left) of the parameters m_1 and Ψ_1 for a LM binary with $m_1 = 10^6 M_\odot$, $m_2 = 3 \times 10^5 M_\odot$.

By using the FWF instead of the RWF for HM binaries with total masses $\gtrsim 3 \times 10^7 M_\odot$, we find significant improvements for the measurement errors of the alternative theory parameters by factors of $\sim 100 - 1000$ for $\Delta\Psi_{-1}$, $\sim 30 - 60$ for $\Delta\Psi_0$ and $\Delta\Psi_{1/2}$, and $\sim 10 - 100$ for $\Delta\Psi_1$, $\Delta\Psi_{3/2}$ and $\Delta\Psi_2$. This makes it clear that it is inevitable to use the FWF in the HM regime to perform precision tests of GR. In any case, since the second harmonic spends only a few orbits in the LISA band, the use of the RWF is not trustworthy. Moreover, for BHBs with total masses higher than $10^8 M_\odot$, LISA will not be able to see the second harmonic at all and so the RWF cannot be used. For both the FWF and the RWF, the errors on the mass and spin parameters are typically worse by a factor of $\sim 1.2 - 4$ when accounting for alternative gravity parameters. The luminosity distance is about $50 - 1000$ times less accurate for the RWF while for the FWF it is only $\sim 10 - 100$ times worse. For the FWF, the angular orientation error is at maximum 5 times worse while the RWF loses up to a factor of ~ 10 in accuracy. In figs. 1 and 2 we show the error distribution (right) of two selected parameters for a BHB with $m_1 = 3 \times 10^7 M_\odot$, $m_2 = 10^7 M_\odot$.

The above considerations are all at redshift $z = 1$: for higher redshifts, the errors increase. If we desire to measure the deviations Ψ_0, Ψ_1, $\Psi_{3/2}$, Ψ_2 with a relative error of 10%, we have to find sources at redshifts of $z \lesssim 10$ ($\lesssim 100$ for Ψ_0) for low masses and $z \lesssim 0.1$ for high masses, however we suspect deviations to be smaller.

The combination of higher harmonics and spin precession enables us to increase the optimal lower bound on the graviton Compton wavelength by about a factor of 1.6 compared to the one reached by the RWF. We achieve an optimal lower bound of $\lambda_g > 7.6 \times 10^{21}$ cm for the classic LISA detector design if all the alternative theory parameters except for Ψ_1 are turned off. Using the RWF, we find $\lambda_g > 4.9 \times 10^{21}$ which is (by coincidence, we use different models) equal to the one Yagi & Tanaka (2010) found using the RWF and simple precession at $d_L = 3$ Gpc.

4. Conclusion

We conclude that it is not necessary to use the FWF instead of the RWF for the sole purpose of measuring alternative gravity parameters in the low-mass regime, however the use of the FWF is indispensable for high masses. Spin precession and higher harmonics achieve a lower bound for the Compton wavelength of $\lambda_g > 7.6 \times 10^{21}$, about a factor of 1.6 higher than the one recovered with spin precession and the RWF only. In future work, the effects of eccentric orbits, amplitude corrections and alternative GW polarization modes should be considered.

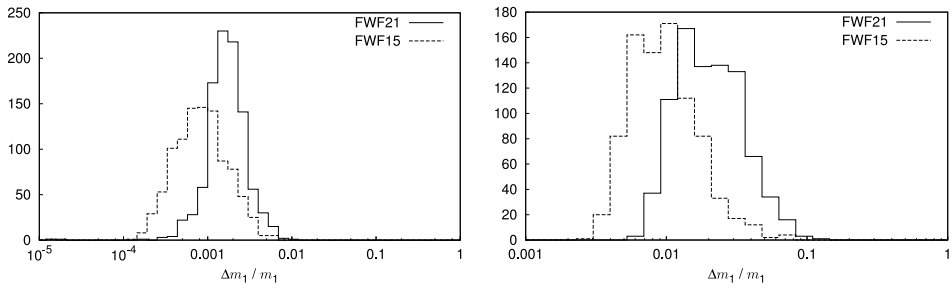

Figure 1. Expected classic LISA full waveform measurement error distributions for the mass m_1, computed with (FWF21) and without (FWF15) alternative theory parameters for a low-mass binary (left; $m_1 = 10^6 M_\odot$, $m_2 = 3 \times 10^5 M_\odot$) and a high-mass binary (right; $m_1 = 3 \times 10^7 M_\odot$, $m_2 = 10^7 M_\odot$).

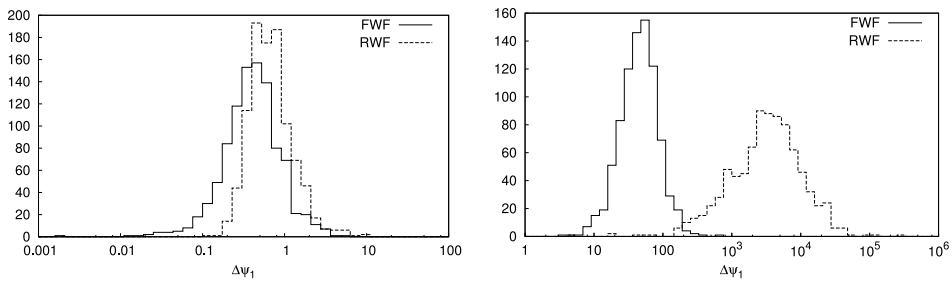

Figure 2. Expected classic LISA measurement error distributions for the alternative theory parameter Ψ_1, computed with the full (FWF) and restricted waveform (RWF) for a low-mass binary (left) and a high-mass binary (right) with the same mass configurations as in fig. 1.

References

Arun, K. G., & Will, C. M. 2009, Classical and Quantum Gravity, 26, 155002
Berti, E., Buonanno, A., & Will, C. M. 2005, Phys. Rev. D, 71, 084025
Berti, E., Gair, J., & Sesana, A. 2011, Phys Rev D, 84, 101501
Brans, C., & Dicke, R. H. 1961, Phys. Rev., 124, 925
Cornish, N., Sampson, L., Yunes, N., & Pretorius, F. 2011, Phys Rev D, 84, 062003
Huwyler, C., Klein, A., & Jetzer, P. 2011, ArXiv e-prints. 1108.1826
Keppel, D., & Ajith, P. 2010, Phys Rev D, 82, 122001
Li, T. G. F., Del Pozzo, W., Vitale, S., Van Den Broeck, C., Agathos, M., Veitch, J., Grover, K., Sidery, T., Sturani, R., & Vecchio, A. 2012, Phys Rev D, 85, 082003
Stavridis, A., & Will, C. M. 2009, Phys Rev D, 80, 044002
Will, C. M. 1998, Phys Rev D, 57, 2061
— 2006, Living Reviews in Relativity, 9. arXiv:gr-qc/0510072
Yagi, K., & Tanaka, T. 2010, Phys Rev D, 81, 064008
Yunes, N., & Hughes, S. A. 2010, Phys Rev D, 82, 082002
Yunes, N., & Pretorius, F. 2009, Phys Rev D, 80, 122003