Journée des Doctorants

Félix-Louis Julié

Advisors : N.Deruelle, E.Porter

November 10th 2016

Research projects

Main topic : Modified gravities and how to test them with gravitational waves

Can we detect deviations to GR with compact binary systems ?

Study the example of Scalar-Tensor theories

Eintein-Hilbert action modified with a scalar field

introduced by Brans-Dicke (60's)

Scalar-Tensor theories can be expressed in two frames

Jordan Frame

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-\tilde{g}}\left(\phi\tilde{R} - \frac{\omega(\phi)}{\phi}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) + S_m[\Psi, \tilde{g}_{\mu\nu}]$$

A conformal transformation $\tilde{g}_{\mu\nu} = \frac{1}{\phi}g_{\mu\nu}$ and a scalar field redefinition $\phi(\varphi)$ leads to

Einstein Frame

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) + S_m[\Psi, A(\varphi)g_{\mu\nu}]$$

Matter is non-minimally coupled to the metric.

How to deal with compact stars ?

$\rightarrow$ replace them by point particles

In GR, "effacement" :

$S_m = -\int m_A d\tau = -\int m_A d\lambda \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$ so that a test body follows a geodesic of the metric.

In Scalar-Tensor theories :

$S_m = -\int m_A(\varphi) d\tau = -\int m_A(\varphi) d\lambda \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$

Trajectories depend on the internal structure of bodies through $m_A(\varphi)$: SEP violation.

But what are these masses ?

Katz Superpotentials

there are many ways to define masses in GR

How to define a mass ?
Noether charge associated to the gauge symmetry of GR (diffeomorphisms)

$\rightarrow$ Katz action, in GR :

$$I = \int d^4x \, (\hat{R} - \overline{\hat{R}} + \partial_\mu \hat{k}^\mu_{\mathrm{K}})$$

where we introduce the Katz vector and a background metric $\bar{g}_{\mu\nu}$

$$k^\mu_{\mathrm{K}} \equiv -(g^{\nu\rho}\Delta^\mu_{\nu\rho} - g^{\mu\nu}\Delta^\rho_{\nu\rho}) \quad \text{where} \quad \Delta^\mu_{\nu\rho} \equiv \Gamma^\mu_{\nu\rho} - \overline{\Gamma}^\mu_{\nu\rho}\,,$$

- Still GR (we only added a divergence)
- The boundary conditions are simply Dirichlet
- Invariant under diffeomorphisms

Katz Superpotentials

(bonus frame)

→ Katz action, in GR :

$$I = \int d^4x \, (\hat{R} - \overline{\hat{R}} + \partial_\mu \hat{k}^\mu_{\mathrm{K}})$$

Remark 1

Katz' action is the covariant version of Einstein's action :

$$I = \int d^4x \, [\hat{g}^{\mu\rho}(\Delta^\lambda_{\mu\sigma}\Delta^\sigma_{\rho\lambda} - \Delta^\sigma_{\mu\rho}\Delta^\lambda_{\sigma\lambda}) + (\hat{g}^{\mu\nu} - \overline{\hat{g}^{\mu\nu}})\bar{R}_{\mu\nu}]$$

Remark 2

Link with Gibbons-Hawking-York term :
$\int d\Sigma_\mu k^\mu = \int d^3x \sqrt{|\, h \,|} 2\epsilon \left(K - \frac{1}{2}\bar{K}_{ij}(\bar{h}^{ij} + h^{ij})\right)$

Katz Superpotentials

$\rightarrow$ Katz action, in GR :

$$I = \int d^4x \, (\hat{R} - \overline{\hat{R}} + \partial_\mu \hat{k}^\mu_{\rm K})$$

This action is a Scalar (diffeomorphism invariant)

We perform a translation of the system : $x^\mu \rightarrow x^\mu + \xi^\mu$

$$\partial_\mu \partial_\rho \hat{J}^{[\mu\rho]} \equiv 0 \quad \text{where} \quad \hat{J}^{[\mu\rho]} = D^{[\mu}\hat{\xi}^{\rho]} - \overline{D^{[\mu}\hat{\xi}^{\rho]}} + \xi^{[\mu}\hat{k}^{\rho]}$$

So we have defined a conserved current
$\nabla_\mu \hat{j}^\mu = 0$ that derives from the "superpotential" $\hat{j}^\mu = \nabla_\nu \hat{J}^{[\mu\nu]}$

Mass is associated to stationnary metrics : $\xi^\mu = \delta^\mu_t$ (Killing)

$$\int_{S^2, r\rightarrow\infty} dS_{\mu\nu} J^{[\mu\nu]} = \int_{r\rightarrow\infty} d\theta d\phi \hat{J}^{[0r]} \equiv M$$

Can we generalize this to Scalar-Tensor theories ?

A specific example : asymptotically AdS Black Holes in Scalar-Tensor theories arXiv:1606.05849 (Anabalón, Deruelle, Julié)

$$2\kappa I = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{U(\phi)}{\ell^2}\right)$$
$$-\int d^4x\sqrt{-\bar{g}}\left(\bar{R} + \frac{6}{\ell^2}\right) + \int d^4x\,\partial_\mu(\hat{k}^\mu_{\rm K} + \hat{k}^\mu_{\rm S})\,.$$

- Effective "negative Λ" from $U = -6 - \phi^2 + \mathcal{O}(\phi^3)$
- The background metric is AdS ($I = 0$ when no BH)
- Proposal : add a Katz vector associated to the scalar field (it describes gravity too)

$$\boxed{k^\mu_S = f(\phi)\partial^\mu\phi}$$

What vector should we choose ?

- Look for static, spherically symmetric case

$$ds^2 = -h(r)\, dt^2 + \frac{dr^2}{\tilde{h}(r)} + r^2 d\Omega^2,$$

$$\phi = \phi(r)\,,$$

- The extremalization gives field equations (for the bulk) and boundary terms

- Field equations from the bulk term

We only need the asymptotic behaviour to compute the mass ($M = \int_{r\to\infty} d\theta d\phi \hat{J}^{[0r]}$)

$$\phi(r) = \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \mathcal{O}(1/r^3)$$

$$h(r) = \ell^{-2}r^2 + 1 - \frac{2m_g}{r} + \mathcal{O}(1/r^2), \quad \tilde{h}(r) = \ell^{-2}r^2 + 1 + \alpha^2 - \frac{2m_i}{r} + \mathcal{O}(1/r^2)$$

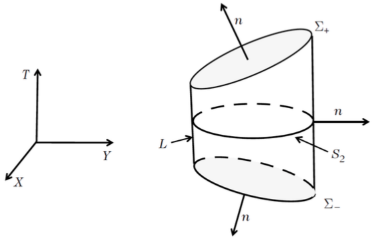
with

$$\alpha = \frac{\phi_1}{2\ell} \quad \text{and} \quad m_i = m_g - \frac{\phi_1\phi_2}{3\ell^2}\,.$$

3 integration constants : ϕ_1, ϕ_2, m_g.

Katz Superpotentials in Scalar-Tensor Theories

- Boundary terms : our prescription is to build k_S^μ so that I is extremal **for the broadest possible family of on-shell solutions**.



Specific boundary condition

On shell, this gives $k_S^\mu = f(\phi)\partial^\mu\phi$
with

$$f(\phi) = \frac{1}{2}\phi(1 + C\phi) \quad \textit{and} \quad \phi_2 = -3C\phi_1^2 + D\ell\phi_1$$

Results :

We have built a Katz vector associated to the Scalar field

$$k_S^\mu = \frac{1}{2}\phi(1 + C\phi)\partial^\mu\phi$$

And found back results obtained by other techniques (GHY+Counterterms)

Mass :

$$M = \int_{r\to\infty} d\theta d\phi \hat{J}^{[0r]} = m_g + D\frac{\phi_1^2}{24\ell}\,.$$

Indeed $f(\phi) = \frac{1}{2}\phi(1 + C\phi)$ is the only way to get a finite mass !

Gibbs relation

Our black hole is compatible with thermodynamics if its onshell action verifies Gibbs' relation :

$$I_{\rm onshell} = S - \beta M \,,$$

Again, $f(\phi) = \frac{1}{2}\phi(1 + C\phi)$ and $\phi_2 = -3C\phi_1^2 + D\ell\phi_1$ are necessary conditions (finite action + compatible with Gibbs).

The background AdS metric and the new Katz vector play a role similar to counterterms.

In Conclusion :

• We have managed to generalize Katz's definition of the mass of a spacetime to Scalar-Tensor theories in the example of BH-AdS.

• We also built an action that is compatible with Gibbs' relation.