Neutrinos and Particle Physics Models

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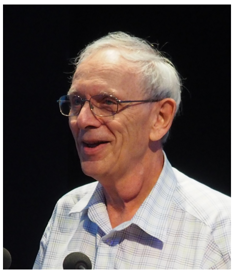

As in Greek mythology, the neutrino was born in the mind of Wolfgang Pauli to salvage a fundamental principle. Its existence met with universal skepticism by a scientific community used to infer particles from experiment. Its detection in 1956 brought particle physics acceptance; its chirality explained maximal parity violation in β decay; its (apparent) masslessness led theorists to imagine new symmetries. Neutrinos are pioneers of mutli-messenger astronomy, from the Sun, from SNA1987, and now through IceCube's blazar. The discovery of neutrino masses opened a new era in particle physics aswell as unexplored windows on the Universe. -Tiny neutrino masses suggest new physics at very short distances through the Seesaw. - Neutrinos and quarks, unified by gauge structure, display different mass and mixing patterns: small quark mixing angles and two large neutrino mixing angles. This difference in mass and mixings in the midst of gauge unification may be an important clue towards Yukawa unification. - Neutrino mixings provide a new source of CP-violation, and may solve the riddle of matter-antimatter asymmetry. We present a historical journey of these “enfants terribles” of particle physics and their importance in understanding our Universe.

1 Preamble

When asked my occupation in life, I often answer that I study neutrinos. My attempts at elaboration motivated an artist acquaintance to produce these visual portaits of neutrinos,

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On my home office wall they remind me of the evocative powers of neutrinos on our imagination.

This talk consists of four parts:

- Editorial
- Early History
- Neutrino Masses
- Neutrinos & Yukawa Unification

2 Editorial

The idea of a neutrino was revealed to Wolfgang Pauli, not through direct experimental evidence but as a "desperate" attempt to rescue what he believed to be a fundamental principle: the conservation of energy. He was right, of course, but Pauli's neutron (neutrino) was difficult if not impossible to detect, and for a while he lamented on his fate, having invented a particle impossible to detect [b]. In his days, inventing a new particle seemed like an admission of failure, to be contrasted with the present sociology where a mere glitch in the data generates a whole Kaluza-Kein tower of particles!

For experimentalists (and most theorists) his hypothesis was not taken seriously at first, even though his proposal added a spin one half particle in the nucleus, thereby explaining in addition the intensity of Raman lines from the Nitrogen nucleus.

This disrespect of the neutrino concept was surely misplaced as neutrinos are the misfits of the particle world; they never fit current dogma. Retrospectively,

- Neutrinos are left-handed in an ambidextruous world, generating parity violation in β decay.

- Neutrinos appeared to be massless, motivating theorists to seek a general principle for their lack of mass; witness Volkov and Akulov's non-linear representation of supersymmetry with the neutrino as Nambu-Goldstone fermion, and Fayet's proposal of a supersymmetric Standard Model.

- Neutrinos may be Majorana particles, leading to leptogenesis and possibly explaining matter-antimatter asymmetry.

- Absurdly light neutrinos require a new scale of physics?

- Neutrinos as keys to Yukawa Unification: they display the same gauge structure as quarks, yet their Yukawa patterns are strikingly different. This outstanding problem begs explanation.

- Neutrinos are messengers from the Universe, from the center of the Sun, from Supernovae, and recently detected by IceCube from a four billion years old blazar!

Except for dark matter, Neutrino masses and mixings provide the only "Physics Beyond the Standard Model". Today a small proportion of particle physicists work on neutrino, even though over the years a number of neutrino prospectors found their study very rewarding:

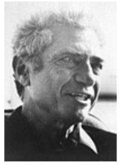
[b] not unlike the axion?

E. Fermi
1938

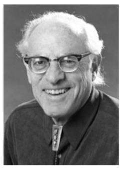
W. Pauli
1945

L. Lederman
1988

M. Schwartz
1988



J. Steinberger
1988



F. Reines
1995

R. Davis
2002

M. Koshiba
2002

T. Kajita
2015

A. McDonald
2015

Not to mention those notables who belong to the Neutrino Hall of Fame:

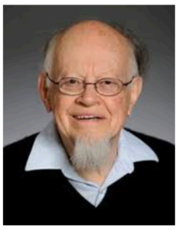
E. Majorana

B. Pontecorvo

M. Goldhaber

S. Sakata

J. Bahcall



L. Wolfenstein

Their past achievements suggest that it may not be a bad idea to study everything possible about neutrinos[c].

Enough editorializing, and let us look at the neutrino's early history.

[c]In the absence of direct evidence, theorists should put wax in their ears and chain themselves to the mast to resist the lure of light sterile neutrinos, while of course urging experimentalists to look for them.

3 Early History

It is customary to begin with Pauli's famous letter to Lise Meitner and friends of 4$^{\text{th}}$ December 1930,

> I have hit upon a desperate remedy to save the "exchange theorem" of statistics
> and the energy theorem. ... there could exist in the nuclei electrically
> neutral particles... which have spin 1/2, and ... do not travel with the velocity of light.
> The continuous beta spectrum would then become understandable.
> I do not feel secure enough to publish anything about this idea ...
> only those who wager can win.
>
> Unfortunately, I cannot personally appear in Tübingen, since I am indispensable
> here on account of a ball...
>
> Pauli

which is noteworthy in many different ways. Pauli postulates the existence of a neutral particle in the nucleus. Its existence would then solve two experimental facts. Raman scattering of the Nitrogen nucleus implies it is a boson. In Pauli's world, the Nitrogen nucleus is made up of protons and electrons and to account for its atomic weight and chemistry it must contain $7+7$ protons and 7 electrons, thus making it a fermion. This is the "exchange theorem" part as a new spin one-half fermion in the nucleus solves that problem. It is only later in the letter that he mentions the continuous spectrum of the β electron, and in order to account for his particle to be in the nucleus, he endows it with a magnetic moment, and therefore a mass!

Chadwick's discovery of neutron two years later solves the Nitrogen problem, and does not require Pauli's light neutron to be inside the nucleus. However it is still needed, although in a new world rocked by quantum mechanics, even the great Bohr entertained the idea that nuclear processes might violate energy conservation.

The sociological context of the letter is revealing. Pauli is clearly nervous at the idea of introducing a new particle! So much so that he does not publish the idea. Six months later, at the APS June 1931 meeting in Pasadena, Pauli gave a talk where he is said to have discussed his new particle and believed it lived in the nucleus. I have not been able to find a copy of his talk.

One might wonder if the Neutron had been discovered earlier (as it could have been) would Pauli have suggested a new light neutral particle? Did the founding fathers think that they should solve every puzzle without introducing new degrees of feedom? Contrast with today's practice where any experimental anomaly is interpreted by new particles, even towers thereof. "O Tempora O Mores".

Another aspect of the letter is that he foregoes a physics meeting to go on a date! Pauli was in the midst of a divorce from actress Kate Depner who left him for a chemist! Within a year Pauli was under analysis with Carl Jung.

It was of course E. Fermi who in 1933 and 1934 papers identifies Pauli's particle as being created by the decay process. Being Italian he named it neutrino, the little neutron, after the discovery of the neutron by Chadwick in February 1932.

A revealing testimony of the place the neutrino idea occupied in particle physics is Hans Bethe and Robert Bacher's 1936 Review of Modern Physics [1]:

> There is thus considerable evidence for the neutrino hypothesis. Unfortunately, all this evidence is indirect; and more unfortunately, there seems at present to be no way of getting any direct evidence. At least, it seems practically impossible to detect neutrinos in the *free state*, i.e., *after* they have been emitted by the radioactive atom. There is only *one* process which neutrinos can *certainly* cause. That is the inverse β-process, consisting of the capture of a neutrino by a nucleus together with the emission of an electron (or positron). This process is, however, so extremely rare (§42) that a neutrino has to go, in the average, through 10^{16} km of solid matter before it causes such a process. The present methods of detection must be improved at least by a factor 10^{13} in sensitivity before such a process could be detected.

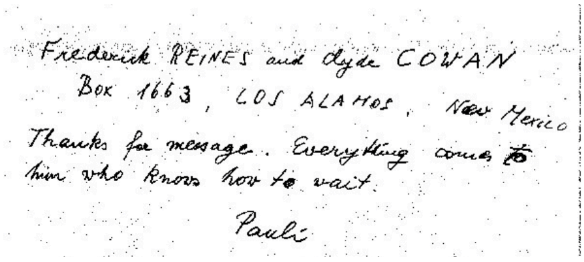
Interesting as it may be, the neutrino idea offers no proof of its existence. Still they identify the process by which the (anti)neutrino was detected twenty years later: inverse β decay. Its detection required an improvement of 10^{13} in sensitivity, making it all but insurmountable!

Bethe and Bacher still denote the neutrino by n' to distinguish it from the neutron n. L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad [2] seem to be the first to use the greek letter ν in 1937 (E. M. Lyman a year later [3]). I am not aware of any earlier attribution. It is universally used from then on.

Ten years later, the 1948 Reviews of Modern Physics article by H. R. Crane [4] summarizes the community's attitude on the neutrino, as a useful idea but still not universally accepted:

> NOT everyone would be willing to say that he believes in the existence of the neutrino, but it is safe to say there is hardly one of us who is not served by the neutrino hypothesis as an aid in thinking about the beta-decay process.

This attitude is about to change when Clyde Cowan and Frederick Reines use inverse β decay to finally detect antineutrinos coming from the Savannah River reactor at the Georgia-South Carolina border. The neutrino is the only elementary particle discovered south of the Mason-Dixon line. At first their discovery met with skepticism, as the titles of their papers suggest: 1953 "Detection of the Free Neutrino" [5] announce the experiment, the 1954 talk "Status of an Experiment to detect the free neutrino" at the January APS Meeting, and finally their 1956 article "Detection of a Free Neutrino: a Confirmation", published in Nature [?]. Earlier, Cowan and Reines had sent Pauli news of their discovery who responded thus:


Frederick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico
Thanks for message. Everything comes to him who knows how to wait.
Pauli

A comment very much applicable to the present state of particle physics!

In 1937 E. Majorana [7] noticed that as a neutral particle the neutrino could, without violating Lorentz invariance, be its own antiparticle, in constrast with electrons and positrons easily distinguishable by their electrical charge.

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di Ettore Majorana

This brilliant theoretical remark will assume more importance in later years. Neutrinos and antineutrinos can be distinguished by their lepton number since Majorana particles necessarily break lepton number. Further progress along these lines was cut short by his tragic disappearance.

Starting from Maria Goeppert-Mayer's 1935 study of double β decay [8], Wendell Furry [9] applied the Majorana idea to a similar decay neutrinoless double β decay ($\beta\beta 0\nu$) with the difference that the two electrons are expelled without their usual antineutrinos. Furry's process gave reality to the Majorana or non Majorana nature of neutrinos.

In 1946 Bruno Pontecorvo proposes a way to look for neutrinos [10]:

$$\nu_e + {}^{37}Cl \quad \longrightarrow \quad {}^{37}Ar + e^-$$

Whenever a neutrino hits a vat of cleaning fluid C_2Cl_4, an Argon isotope and an electron are produced. The beauty of the reaction is that Argon is chemically inert and is radioactive with a half life of the order of one month which provides a beautiful signature. Pontecorvo approached his teacher Fermi who said that although it was a nice idea, it will never be seen because the rates are so low. So it remained a preprint from Chalk River, the Canadian reactor laboratory where Pontecorvo was working. Being classified, it was not published; even when declassified a few years later Pontecorvo did not submit it for publication [d].

Pontecorvo's elegant reaction had not escaped Ray Davis' attention, whose skills as a radio chemist were taylor-made for this experiment. He proposes a pilot experiment near the same Savannah river nuclear plant, which generates plenty of antineutrinos but no neutrinos.

A rumor soon appears according to which Davis had detected one neutrino event. Rumors propagate faster than the speed of light since they contain no information. Sure enough the rumor was just that but it had the unintended effect to motivate Pontecorvo with another

[d] When I met Pontecorvo (once) at Erice, he gave me a reprint of his paper.

beautiful idea [11]: could it be that a reactor antineutrino oscillates into Davis' neutrino? He reasoned by analogy with the analysis of the neutral kaon anti-kaon system the year before.

GELL-MANN and Pais[1] were the first to point out the interesting consequences which follow from the fact that K^0 and $\tilde{K}^0$ are not identical particles.[2] The possible $K^0 \rightarrow \tilde{K}^0$ transition, which is due to the weak interactions, leads to the necessity of considering neutral K-mesons as a superposition of particles K_1^0 and K_2^0 having a different combined parity.[3] In the present note the question is treated whether there exist other "mixed" neutral particles (not necessarily "elementary") besides the K^0-meson, which differ from their anti-particles and for which the particle → antiparticle transitions are not strictly forbidden.

It was assumed above that there exists a conservation law for the neutrino charge, according to which a neutrino cannot change into an antineutrino in any approximation. This law has not yet been established; evidently it has been merely shown that the neutrino and the antineutrino are not identical particles.[9] If the two-component neutrino theory[10] should turn out to be incorrect (which at present seems to be rather improbable) and if the conservation law of neutrino charge would not apply, then in principle neutrino → antineutrino transitions could take place in vacuo. Even in this case, as well as in the case where one assumes that to every world there exists an antiworld, the number of neutrinos and antineutrinos in the universe would have to be the same.

Thus was born the idea of vacuum neutrino-antineutrino oscillations ("transmutations").

After the Cowan-Reines experiments it was soon realized on harmonious grounds that there must be a different neutrino associated with the muon. Shoichi Sakata, Ziro Maki and Masami Nakagawa [12] applied the flavor mixing ideas of Gell-Mann and Levy to neutrinos [e]

Finally, we would like to add remarks on some characteristic properties of leptons in our scheme.

a) The weak neutrinos must be re-defined by a relation

$$\left.\begin{aligned} \nu_e &= \nu_1 \cos\delta - \nu_2 \sin\delta, \\ \nu_\mu &= \nu_1 \sin\delta + \nu_2 \cos\delta. \end{aligned}\right\} \qquad (2\cdot18)$$

The leptonic weak current (2·9) turns out to be of the same form with (2·1). In the present case, however, weak neutrinos are *not stable* due to the occurrence of a virtual transmutation $\nu_e \rightleftarrows \nu_\mu$ induced by the interaction (2·10). If the mass difference between ν_2 and ν_1, i.e. $|m_{\nu_2} - m_{\nu_1}| = m_{\nu_2}$ *) is assumed to be a few Mev, the transmutation time $T(\nu_e \rightleftarrows \nu_\mu)$ becomes $\sim 10^{-18}$ sec for fast neutrinos with a momentum of $\sim$Bev/c. Therefore, a chain of reactions such as[10)]

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \qquad (2\cdot19a)$$

$$\nu_\mu + Z(\text{nucleus}) \rightarrow Z' + (\mu^- \text{ and/or } e^-) \qquad (2\cdot19b)$$

is useful to check the two-neutrino hypothesis only when $|m_{\nu_2} - m_{\nu_1}| \lesssim 10^{-6}$ Mev under a conventional geometry of experiments. Conversely, the absence of e^- in the reaction (2·19b) will be able not only to verify the two-neutrino hypothesis but also to provide an upper limit of the mass of the second neutrino (ν_2) if the present scheme should be accepted.

They refer to the transmutation between the two flavors of neutrinos ν_e and ν_μ. Thus was born the idea of vacuum flavor oscillation.

This concludes my short and selective description of neutrino prehistory.

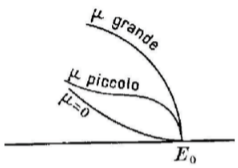
4 Neutrino Masses

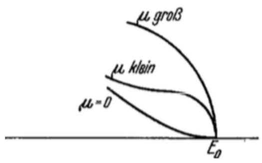
It was Fermi who first attempted to determine the neutrino mass from the continuous spectrum of the β electron. He proposed to look at the electron's spectrum at the end of its kinematically allowed range [f].

[e] Kobayashi and Maskawa who discovered CP violation in quark mixing were students at Nagoya University where Sakata had extended his egalitarian ideas to particle mixings.

[f] An early example of extreme kinematics used today to distinguish different topologies of LHC events.

Fermi had first published his findings in Italian [13] and few weeks later almost simultaneously in an Italian and in a German journal [14], which explains the two erroneous but suggestive figures:





In fact neutrinos are absurdly light, to the point that it was widely believed that they were massless [g], in which case the mixing would be irrelevant.

There are many ways to incorporate neutrino masses in the Standard Model. All require new degrees of freedom, either bosons and/or fermions. They are distinguished by their couplings to the three weak doublets and three weak singlets of the Standard Model leptons,

$$L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}, \qquad \bar{e}_i,$$

where $i = 1, 2, 3 = e, \mu, \tau$ is the flavor index. There are three associated global lepton numbers ℓ_i, with $\ell_i = +1$ for L_i and $\ell_i = -1$ for $\bar{e}_i$.

Neutrino masses can be generated only if new degrees of freedom, bosons and/or fermions, are added to the Standard Model. We split the discussions into two cases:

- Leptonic Bosons Only

With no extra fermions, neutrino masses are of the Majorana type $\nu_i \nu_j \sim L_i L_j$, which break lepton numbers by two units. Lepton-number carrying scalars fields must be introduced. Their renormalizable couplings to the Standard Model leptons are of three types:

$$\bullet \text{ Flavor antisymmetric } \; L_{[i}L_{j]} \; \text{ weak singlets } \; \text{couple to } \; S^+,$$

where S^+ is a charged scalar field with hypercharge 2 and total lepton number $\ell = \ell_e + \ell_\mu + \ell_\tau = -2$.

$$\bullet \text{ Flavor symmetric } \; L_{(i}L_{j)} \; \text{ weak triplets } \; \text{couple to } \; T,$$

where T are isotriplet scalar fields with hypercharge 2 and total lepton number $\ell = -2$. Two of its three components T^{++}, T^+, T^0 carry electric charge. With two charged components, its signature makes it an experimental favorite.

$$\bullet \text{ Flavor symmetric } \; \bar{e}_i \bar{e}_j \; \text{ weak singlet } \; \text{couples to } \; S^{--},$$

where S^{--} is a doubly charged scalar field. In these generic couplings, possible flavor indices are not shown.

These models break lepton number explicitly [h] in the potential to enable Majorana masses. In all cases explicit breaking occurs through cubic couplings of dimension three:

$$m(H\,H)\,T \;\; \text{(Type II)}, \quad mS^{--}S^+S^+, \quad mS^{--}(T\,T), \quad m\bar{S}^+\bar{S}^+(T\,T),$$

[g] This "what else can it be" attitude on neutrino masses is reminiscent of the cosmological constant migrating from a "wecib" zero to a non-zero measured value.

[h] Spontaneous breaking generates experimentally ruled-out massless Majorons.

and combinations thereof. All break ℓ by two units. The arbitrary mass parameters are determined to generate a mass suppression through mixing light and heavy states (called seesaw by some).

There is a model (Zee) where the neutrino masses appear at one loop. It requires a second BEH scalar H' to enable the cubic coupling $(H\,H')\,S^+$, where S^+ couples to the flavor antisymmetric combination of two weak doublets.

- Leptonic Fermions Only

Extra fermions with lepton numbers couple renormalizably to the Standard Model in four ways using H the weak doublet BEH boson (again suppressing all flavor indices):

$$\bullet\ L_i\bar{H}\ \text{ weak singlets }\ \text{couple to }\ \bar{N},$$

where $\bar{N}$ are neutral leptons with zero hypercharge and $\ell = -1$. Here the BEH vacuum value generates Dirac mass terms of the form $\nu_i\bar{N}_j$ which does not violate total lepton number. But then why are they so small?

$$\bullet\ L_i\bar{H}\ \text{ weak triplets }\ \text{couple to }\ \bar{\Sigma},$$

where $\bar{\Sigma}$ are isotriplet fermions with zero hypercharge and $\ell = -1$.

$$\bullet\ L_iH\ \text{ weak singlets }\ \text{couple to }\ \bar{N}^+,$$

where $\bar{N}^+$ are charged leptons with two units of hypercharge and $\ell = -1$. The electroweak vacuum generates mass terms which mix $\bar{N}^+$ and $\bar{e}$.

$$\bullet\ L_iH\ \text{ weak triplets }\ \text{couple to }\ \vec{\Sigma},$$

where $\vec{\Sigma}$ are charged leptons with two units of hypercharge and $\ell = -1$

I consider only the first of the fermion-addition models generically described as "Seesaw Mechanisms". The extra fermions can have both Dirac and Majorana masses; the former preserves total lepton humber, and the latter violates it. Their combination leads to the iconic compound Majorana mass matrix:

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}, \qquad m_\nu \sim m\frac{m}{M}$$

where the natural suppression of the light neutrino masses stems from the ratio of two scales of physics.

The Dirac mass is generated in the electroweak vacuum, from $\Delta I_{\mathrm{w}} = 1/2$ physics at the electroweak scale $m \sim 240\,GeV$. The Majorana mass with $\Delta I_{\mathrm{w}} = 0$ unknown physics of unknown scale M. The three observed neutrino species have suitably suppressed masses, and the three right-handed neutrinos have masses of the order of the GUT scale.

The Seesaw Mechanism requires new particles with GUT scale masses: there is particle Physics Beyond the Standard Model.

5 Neutrino Masses and Mixings

The observable lepton mixing matrix results from an overlap between two types of mixings (PMNS for Pontecorvo, Maki, Nakagawa, Sakata – see [16]),

$$\mathcal{U}_{PMNS} = \mathcal{U}_{-1}^{\dagger}\,\mathcal{U}_{Seesaw}$$

where $\mathcal{U}_{-1}$ diagonalizes the charged lepton Yukawa Standard Model couplings, and $\mathcal{U}_{Seesaw}$ diagonalizes the Seesaw matrix, of unknown $\Delta I_w = 0$ origin[i].

Experimental neutrino mixing angles are a combination of two values,

$$\theta_{Expt} \sim \theta_{EW} \text{ ``}+\text{''} \theta_{Seesaw}$$

where θ_{EW} is expected to be like quark mixings, of the order of Cabibbo angle, a sort of "Cabibbo Haze" correction to the Seesaw mixing θ_{Seesaw}.

The neutrino masses are constrained by both oscillation experiments and the early Universe. Oscillations data (normal hierarchy, PDG values [15]) yield:

$$\begin{aligned}
\Delta^2_{12} &\equiv |m^2_{\nu_1} - m^2_{\nu_2}| = (8.68\ meV)^2,\\
\Delta^2_{13} &\equiv |m^2_{\nu_1} - m^2_{\nu_3}| = (50.10\ meV)^2.
\end{aligned}$$

They suggest either the "normal hierarchy" with $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$, or the "inverted hierarchy" $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$, although the former appears slightly favored.

The energy in neutrino masses in the very early Universe is limited to:

$$m_{\nu_1} + m_{\nu_2} + m_{\nu_3} \le 220\ meV.$$

(even smaller in the last PDG issue [15]). The measured three lepton mixing angles,

$$\begin{aligned}
\theta_{23} &= 40.2^{\circ}{}^{+1.4^{\circ}}_{-1.6^{\circ}} \quad \text{“atmospheric angle”}\\
\theta_{12} &= 33.6^{\circ} \pm .8^{\circ} \quad \text{“solar angle”}\\
\theta_{13} &= 8.37^{\circ} \pm .16^{\circ} < \theta_{Cabibbo} \quad \text{“reactor angle”}
\end{aligned}$$

display two large angles and a small angle less than Cabibbo's. The two large angles were unexpected while the reactor angle falls in line with naive expectations.

The present data tends towards a CP-violating phase in the PMNS matrix.

The Seesaw mechanism predicts two other phases linked with Majorana physics that violate total lepton number. There is no sign of total lepton number violation in the data.

[i] Although any neutrino mass model could generate this matrix, I consider only the Seesaw Mechanism where the scale is motivated by Grand-Unification.

6 Neutrinos & Yukawa Unification

In his famous (but forgotten) lecture for the James Scott Prize "The Relation between Mathematics and Physics" [17], Dirac discusses the principles of simplicity and mathematical beauty. Simplicity is Newton's equation while mathematical beauty is the symmetry of special relativity. He even goes as far as saying:

"It often happens that the requirements of simplicity and of beauty are the same, but where they clash the latter must take precedence".

We follow Dirac's path in search of an organizing principle for Yukawa couplings.

Beauty can be found in the quarks and leptons gauge couplings which suggest a unifying gauge symmetry at much shorter distances.

Neither simplicity nor beauty is easily discerned in the masses and mixings of quarks and leptons.

Quark masses and charged leptons are strongly hierarchical; neutrino masses are not. Quark mixings are small; neutrino mixings contain two large mixings.

Large angles suggest a crystal-like symmetry for a hypothetical Majorana crystal.

Can Dirac beauty emerge from a discrete symmetry? [j]

Three chiral families suggest finite subgroups of $SU(3)$. These were catalogued by mathematicians more than a century ago, and it is fair to say that each possibility can be found in the literature!

There is no compelling argument in favor of one group over another.

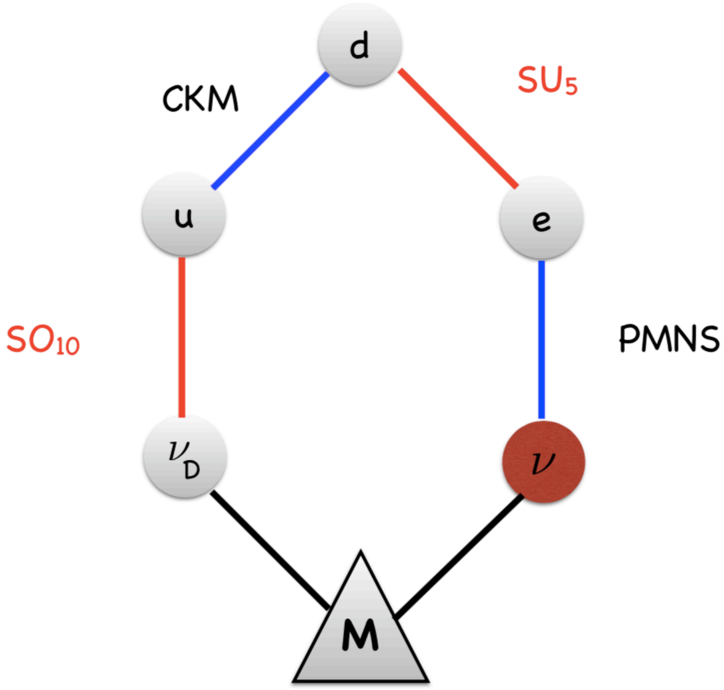
For the remainder of this talk I will be within this theorist's "Rabbit Hole" and single out the mathematically ubiquitous simple discrete $SU(3)$ subgroup with 168 elements, $PSL(2, 7)$.

It is useful to introduce a graphical rendition that shows how the different Yukawa couplings of the Standard Model are connected by Grand-Unification.
Let us represent the Standard Model Yukawa couplings by circles labelled by the particles whose masses they generate. The fourth circle is the electroweak Dirac mass Yukawa with one right-handed neutrino per chiral family.

To enable the Seesaw Mechanism we add the Majorana mass

[j] Discrete flavor symmetry, advocated long ago by Sugawara and Pakvasa, and also Ma, is now hugely popular

The $SU(5)$ and $SO(10)$ Grand-Unified groups connect these couplings through the "Flavor Ring" where the red links are GUT-inspired and the observable mixing matrices are the black links:



We invoke a principle of Seesaw Simplicity, which posits that

the two large (solar and atmospheric) angles come solely from the Seesaw side,

the small reactor angle is entirely due to the charged lepton mixing matrix.

An obvious choice for the Seesaw mixing matrix is the "Tri-Bi-Maximal Matrix" (TBM) [k] of Perkins et al [18]:

$$\begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ \sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Seesaw simplicity is most clearly enunciated when the Yukawa matrix of the charge $2/3$ quarks is diagonal: $Y^{2/3} \sim \mathrm{Diag}(\epsilon^4, \epsilon^2, 1)$ [l]

- $SO(10)$ link: charge $2/3$ and Dirac neutrino mass matrices are equal at GUT scale.
- $SU(5)$ link: $m_b = m_\tau$ determines the GUT scale M_{GUT} using the renormalization group [m].

Absence of dramatic hierarchy in neutrino masses $\rightarrow$ "correlated hierarchy" in the Majorana mass matrix

$$\mathcal{M} = \begin{pmatrix} \epsilon^4 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{M}' \begin{pmatrix} \epsilon^4 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathcal{M}'$ of order one with inverse eigenvalues proportional to neutrino masses. TBM diagonalization fixes relations among its elements:

[k] "The ugly matrix with a pretty name" (L. Everett)
[l] Natural when the family symmetry distinguishes diagonal from off-diagonal couplings
[m] Possible only because the b quark physical mass (half the Υ) is bigger than the τ lepton's

$$\mathcal{M}'_{12} = \mathcal{M}'_{13}; \quad \mathcal{M}'_{22} = \mathcal{M}'_{23}; \quad \mathcal{M}'_{11} + \mathcal{M}'_{12} + \mathcal{M}'_{23} = \mathcal{M}'_{22}$$

Choice of discrete group is predictive (G. Chen, J. M. Pérez):

$$PSL(2,7) \quad \rightarrow \quad \mathcal{M}'_{22} = \mathcal{M}'_{23} \quad \rightarrow \quad \left|\frac{m_{\nu_1}}{m_{\nu_2}}\right| = \frac{1}{2}$$

Folding this extra relation with the oscillation data yields

$$m_{\nu_3} \sim \ 50\ meV, \quad m_{\nu_2} \sim 11\ meV, \quad m_{\nu_1} \sim 5.5\ meV$$

The 2014 Florida flavor group (J. Kile, J. M. Pérez, J. Zhang) found that TBM mixing required flavor-asymmetric charged lepton Yukawa matrices [19].

Recently my students (M.H. Rahat and Bin Xu) and I presented a TBM texture that fits the GUT patterns and all mass and mixing angles data but only for a specific CP-violation.

• $SU(5)$ relate charge $-1/3$ and charge -1 Yukawa matrices with BEH along the $\bar{\mathbf{5}}$ and $\overline{\mathbf{45}}$ representations.

The Yukawa matrices are expressed in terms of the Wolfenstein parameters A, ρ, η, λ

$$\begin{aligned}
\bar{\mathbf{5}} \quad &: \quad \frac{1}{3}\begin{pmatrix} 2\sqrt{\rho^2+\eta^2}\lambda^4 & \lambda^3 & 3A\sqrt{\rho^2+\eta^2}\lambda^3 \\ \lambda^3 & 0 & 3A\lambda^2 \\ 3A\sqrt{\rho^2+\eta^2}\lambda^3 & 3A\lambda^2 & 3 \end{pmatrix} + \frac{2\lambda}{3A}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\
\overline{\mathbf{45}} \quad &: \quad \frac{\lambda^2}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

They reproduce the Wolfenstein CKM matrix, the Gatto relation and the GUT-scale Georgi-Jarlskog relations

$$\lambda \approx \sqrt{\frac{m_d}{m_s}}, \quad m_b = m_\tau, \quad m_\mu = 3m_s, \quad m_d = 3m_e.$$

The PMNS angles are also determined, but they differ from their PDG values,

$$\theta_{13}: \ 2.26^\circ \text{above pdg}, \quad \theta_{23}: \ 2.9^\circ \text{below}, \quad \theta_{12}: \ 6.16^\circ \text{above}.$$

These angles can be brought back to their PDG values by adding a CP-violating phase φ in the TBM matrix. This is possible because the reactor angle is above its experimental value.

Lowering the reactor angle to its PDG value demands $\cos\varphi \approx 0.2$, but leaves the sign of φ undetermined. The other two angles magically fall within PDG:

$$\cos\varphi \approx 0.2 \ \rightarrow \ \theta_{13} \text{ at pdg}, \quad \theta_{23}: \ 0.66^\circ \text{below}, \quad \theta_{12}: \ 0.51^\circ \text{above}.$$

The Jarlskog-Greenberg invariant is $J = |0.027|$. When folded into the PMNS matrix, we find $\delta_{CP} = 1.32\pi$ or $\delta_{CP} = 0.67\pi$, depending on the sign of the phase. Only the first value is consistent with the little we know from experiments.

A Neutrino Prediction

Two important measurements await neutrino physics, neutrinoless double β-decay which will determine if the total lepton number is broken, and the cosmic neutrino background. In the absence of technology which suggests these measurements in the near future, I turn whimsically to mathematics to offer a prediction for the year when lepton number violation will be detected:

$$\begin{array}{rcr} \text{Revelation} & : & 1930 \\ \text{Detection} & : & 26 = 2 \cdot 13 \text{ years later } \; 1956 \\ \text{Oscillations} & : & 68 = 2^2 \cdot 17 \text{ years later } 1998 \\ \beta\beta_{0\nu} \text{ Decay} & : & 152 = 2^3 \cdot 19 \text{ years later } 2052 \end{array}$$

The Sun Never Sets

On Neutrino Detectors

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