Neutron Star Mergers Chirp about Vacuum Energy

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with
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The Evolution of vacuum energy

- The cosmological constant is very small today

$$\Lambda \sim (10^{-3}\ \mathrm{eV})^4$$

- Expectation is that microscopic origin of cc is vacuum energy of quantum field theory

- Why is it so small vs. $(TeV)^4, M_{Pl}^4$

- If it is so small why is it not zero?

- Is it always very small (ie. is there an adjustment mechanism)?

The Evolution of vacuum energy

- If CC result of microphysics, in traditional picture cc should undergo a series of jumps at every phase transition

- Expectation $\Delta\Lambda_i \propto T_{c,i}^4$

- Want CC to NOT dominate AFTER phase transition (otherwise Universe accelerates too early)

- CC AFTER PT should be of order of T_c of NEXT phase transition

- eg. before EWPT $\Lambda \sim M_W^4$

The Evolution of vacuum energy

- $\Delta\Lambda \sim M_W^4$ so tuning $\Lambda + \Delta\Lambda \sim \mathcal{O}(\Lambda_{QCD}^4)$

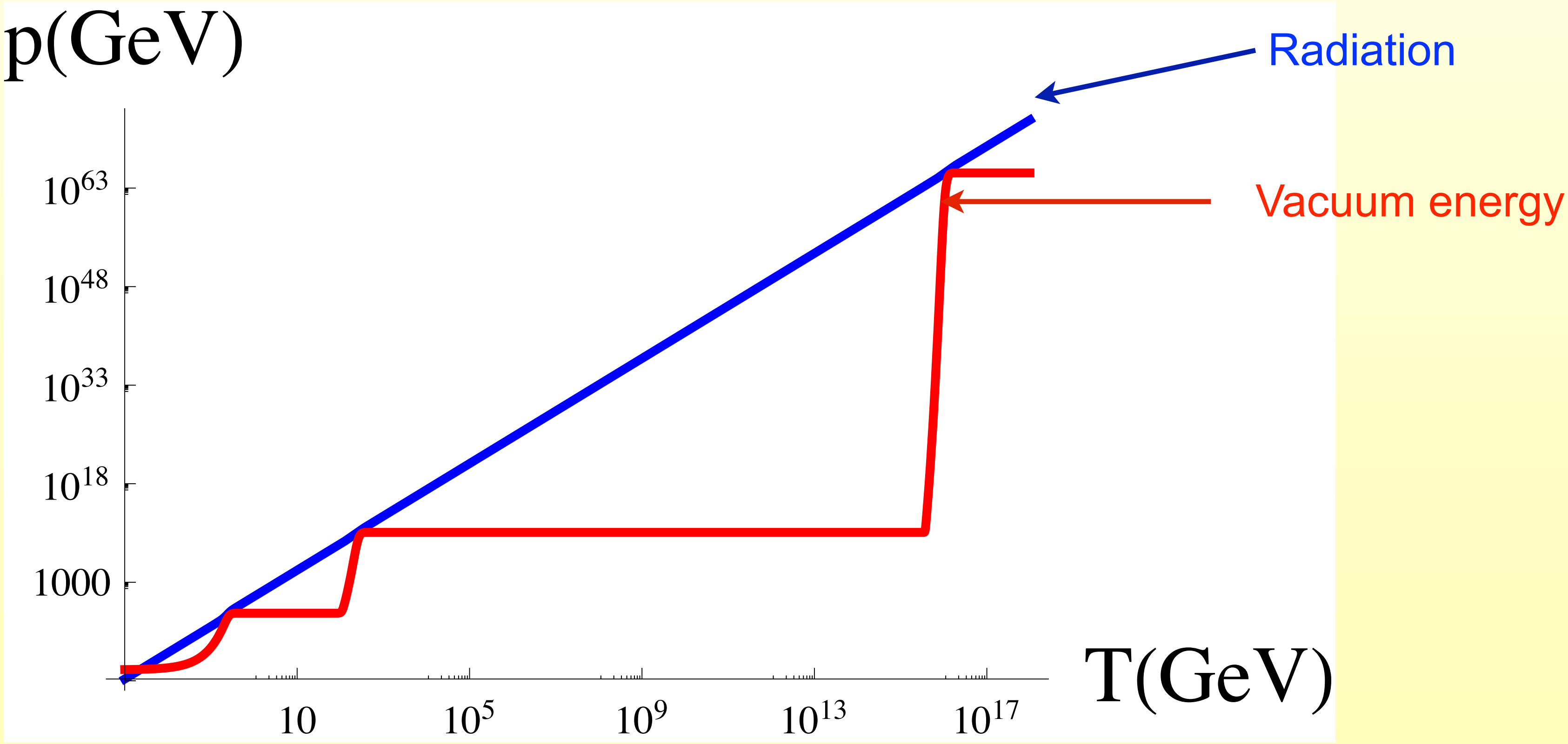
- At one phase transition Universe already ``knows" where the next phase transition will be

- At least QCD, EW PT, potentially also SUSY and/or GUT phase transition (if SUSY changes GUT expectations)

- In previous history Λ was much larger than now, but never dominated previously!

A simple sketch of the evolution of Λ

(Bellazzini, C.C., Hubisz, Serra, Terning)



The Evolution of vacuum energy

- Λ goes through steps during phase transitions

- Whenever Λ would start to dominate a new phase transition happens

- Λ is always subleading even though it was much bigger than it currently is - challenging to find experimental tests of this picture

- Size of step of order $(T_c^{(i)})^4$

- Amount of tuning given by $(T_c^{(i+1)})^4$

The Evolution of vacuum energy

- Expression of fact that tuning of CC

$$\Lambda_{\mathrm{eff}} = \Lambda_{\mathrm{bare}} + \sum_i \alpha_i T_{c,i}^4$$

- Final CC tuned against many different terms

- Each of very different order of magnitude

- Each set by very different dynamics (and sensitive to a different set of parameters)

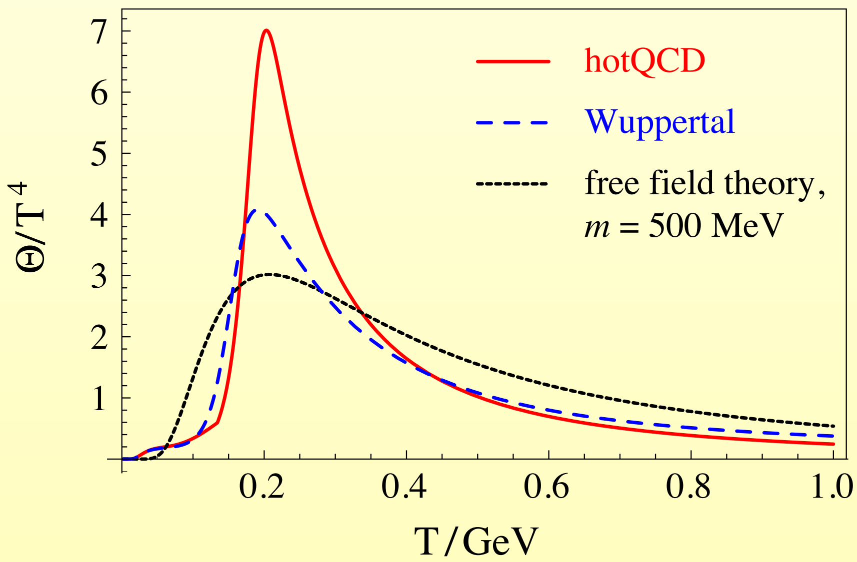
In different phases of SM VE is different

- Main lesson (obvious): VE is different in different phases of the SM

- To truly believe the picture with CC would like to test the effect of VE in phases DIFFERENT from the usual SM phase.

Difficulty of finding effects

Example: QCD PT from lattice



(From Caldwell & Gubser 2013)

Deviation from radiation domination only during short period during PT...

Goal

- Establish experimentally that vacuum energy of microscopic physics is actually what show up in Einstein eq - or there is an adjustment mechanism

What is Vacuum Energy?

- In cosmological context clear - contribution to energy density that DOES NOT dilute with a

$$T_{00} \propto \frac{\lambda^{(0)}_{mat}}{a^3} + \frac{\lambda^{(0)}_{rad}}{a^4} + \Lambda$$

Matter

Radiaton

Vacuum energy

- For a fixed a no way to tell these apart

- Need to follow the expansion of the Universe to be able to separate the three from each other

What is Vacuum Energy?

- Only care about PT's that actually change VEVs of fields

- For example recombinations at z~ 1100 is a PT where $e+p \rightarrow H$, with binding energy 13.6 eV

- Decrease of energy density of matter, but not a change in vacuum energy - this energy density gets diluted with expansion, while ve does not

When is VE important?

- Further complication: neither EW nor QCD PT first order (at least in SM with 125 GeV Higgs) - no gravitational waves produced from bubble collisions...

- NEED: System where vacuum energy $\mathcal{O}(1)$ fraction of total energy

$\downarrow$

Neutron star

Epochs where vacuum energy is comparable to radiation

$\downarrow$

Cosmic phase transitions & effects on primordial gravitational waves

When is VE important?

- Further complication: neither EW nor QCD PT first order (at least in SM with 125 GeV Higgs) - no gravitational waves produced from bubble collisions...

- NEED System where vacuum energy $\mathcal{O}(1)$ fraction of total energy

Neutron star

Focus of this talk

Epochs where vacuum energy is comparable to radiation

Cosmic phase transitions & effects on primordial gravitational waves

Neutron stars for testing vacuum energy

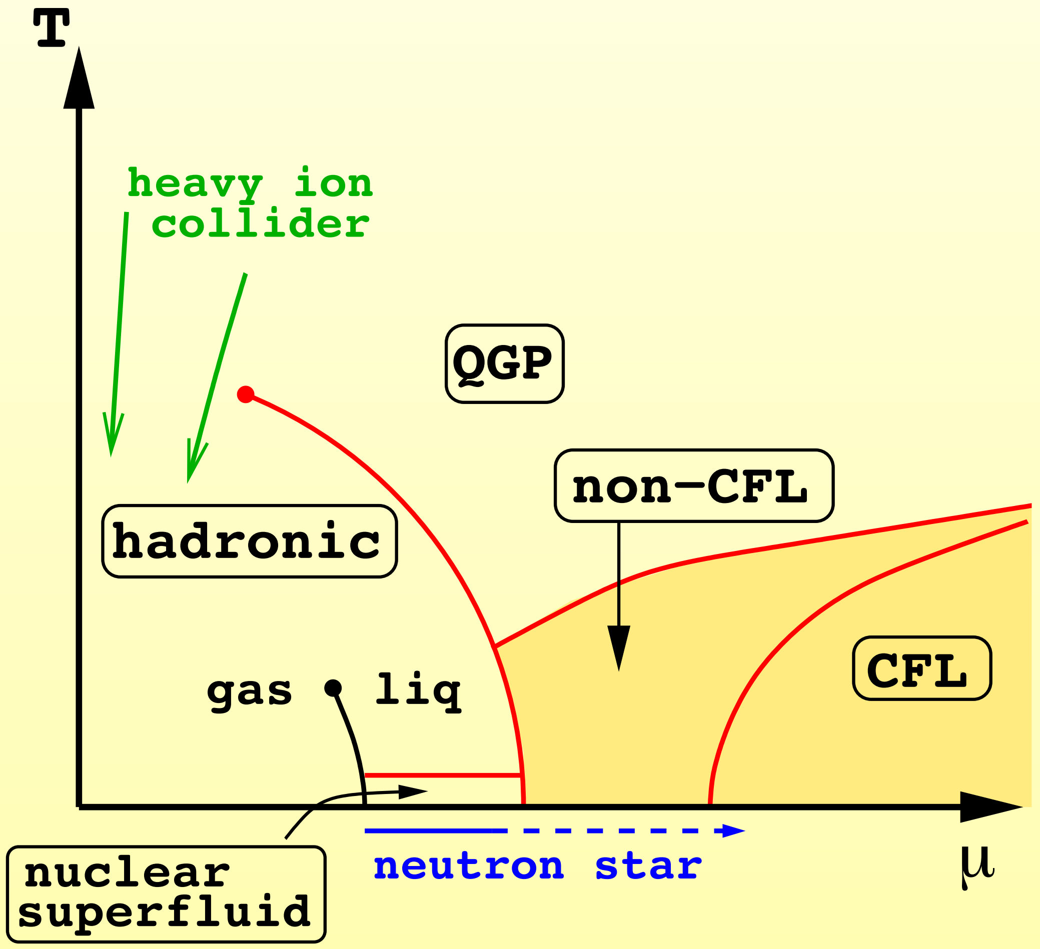
•Need a system which is in different phase of matter

•QCD at large densities probably has those phases: at low T but large chemical potential CFL phase, and non-CFL phase, both with VEVs different from QCD condensates

•Core of neutron star may have this unconventional QCD phase

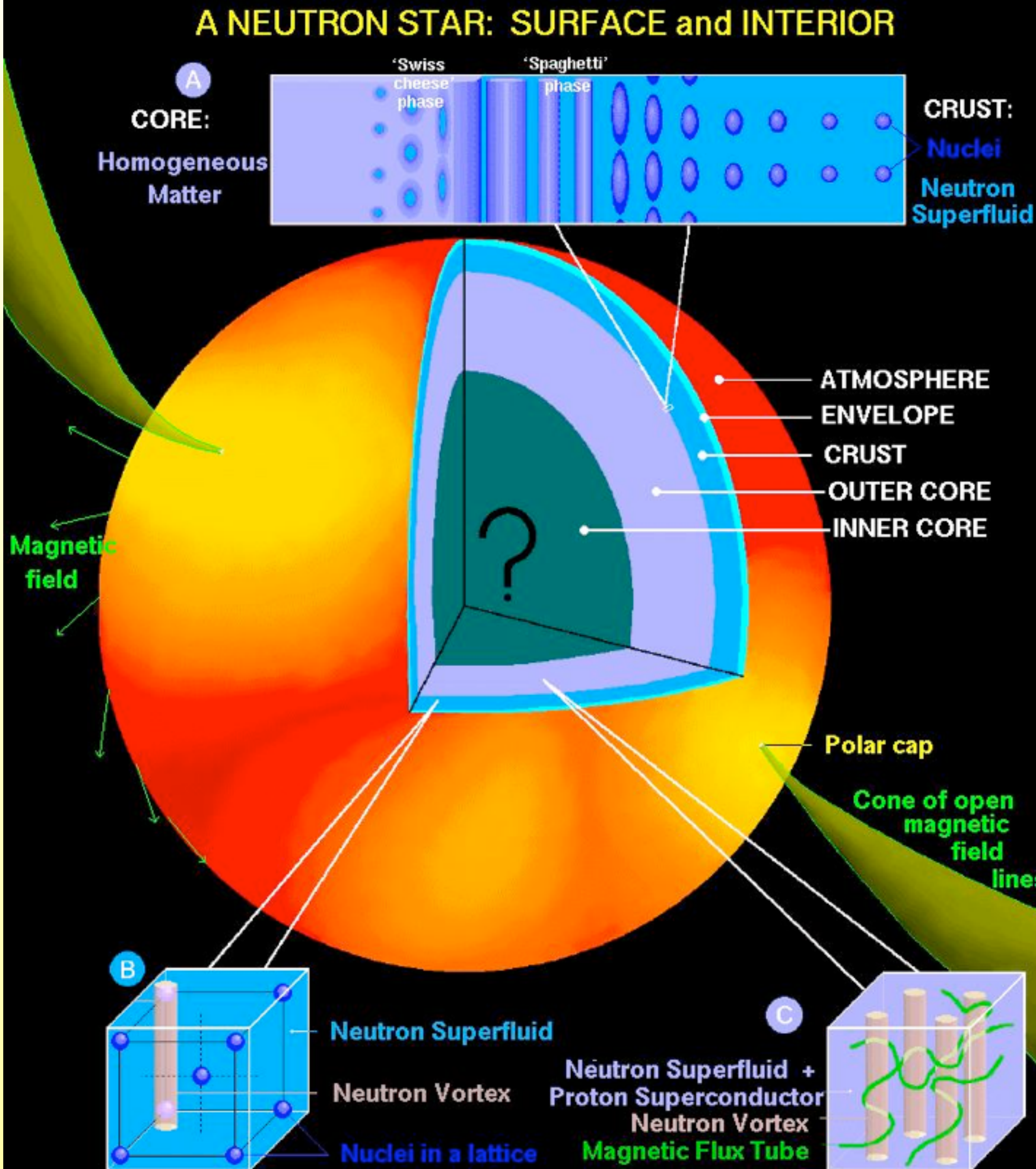
•If adjustment mechanism at play, expect to cancel effect of additional cc in the core. Will modify the structure and M(R) relation of ns's

The phases of QCD



From Alford, Schmitt, Rajagopal, Schaefer 2008

Neutron Stars



From Coleman Miller

Probing VE from NS mergers

(C.C., Eroncel, Hubisz, Rigo, Terning)

- Assume center of NS has new phase of QCD with a VE of order Λ_{QCD}^4

- Solve Einstein equations by patching regions with different equation of state (EoS) together

- Find density profile and M(R) curve and look at effect of VE

- Initially just solved for 2 regions and looked for M(R) with politropic EoS, and phase transition at critical pressure - found there can be sizable effects of VE

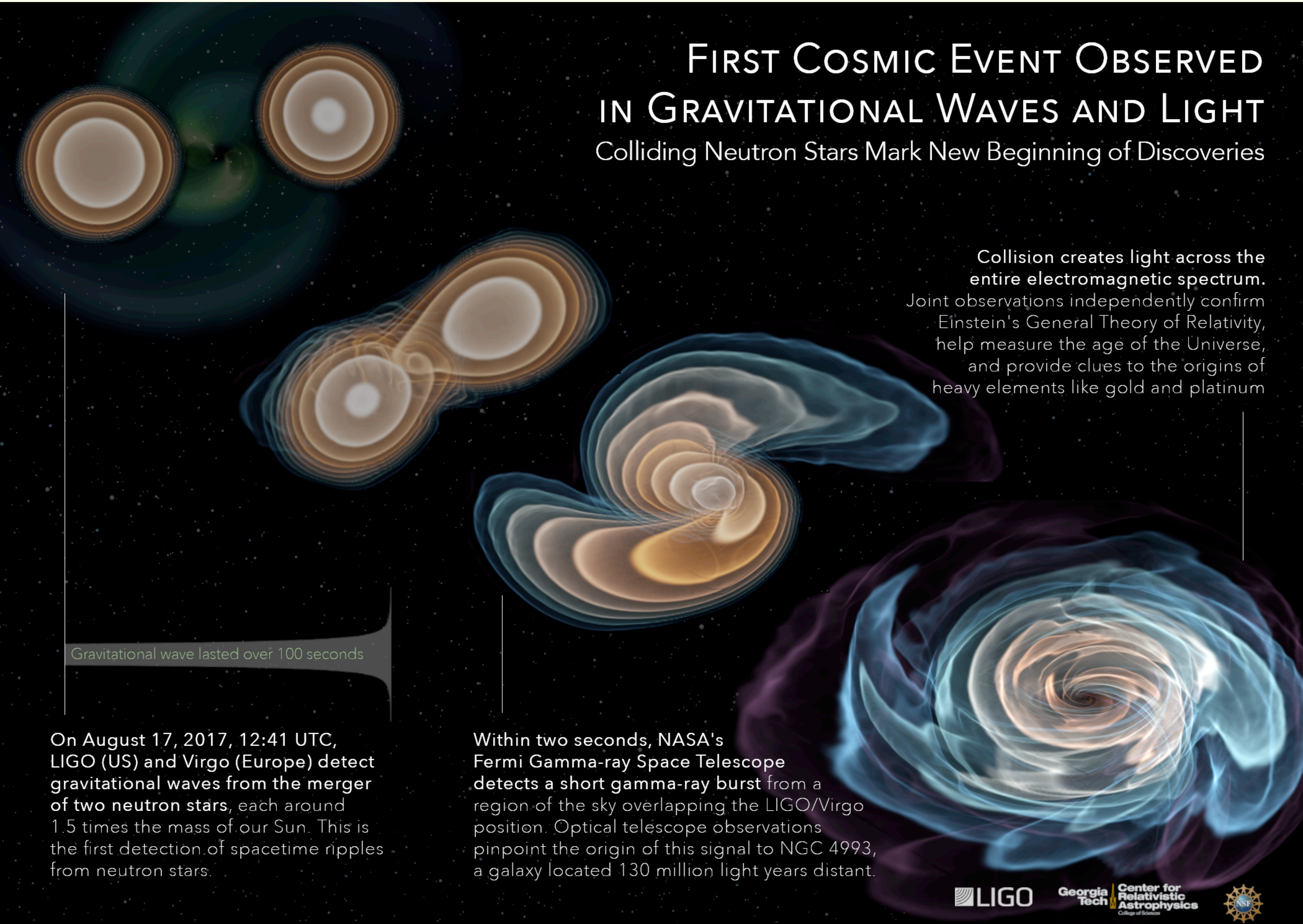
Probing VE from NS mergers

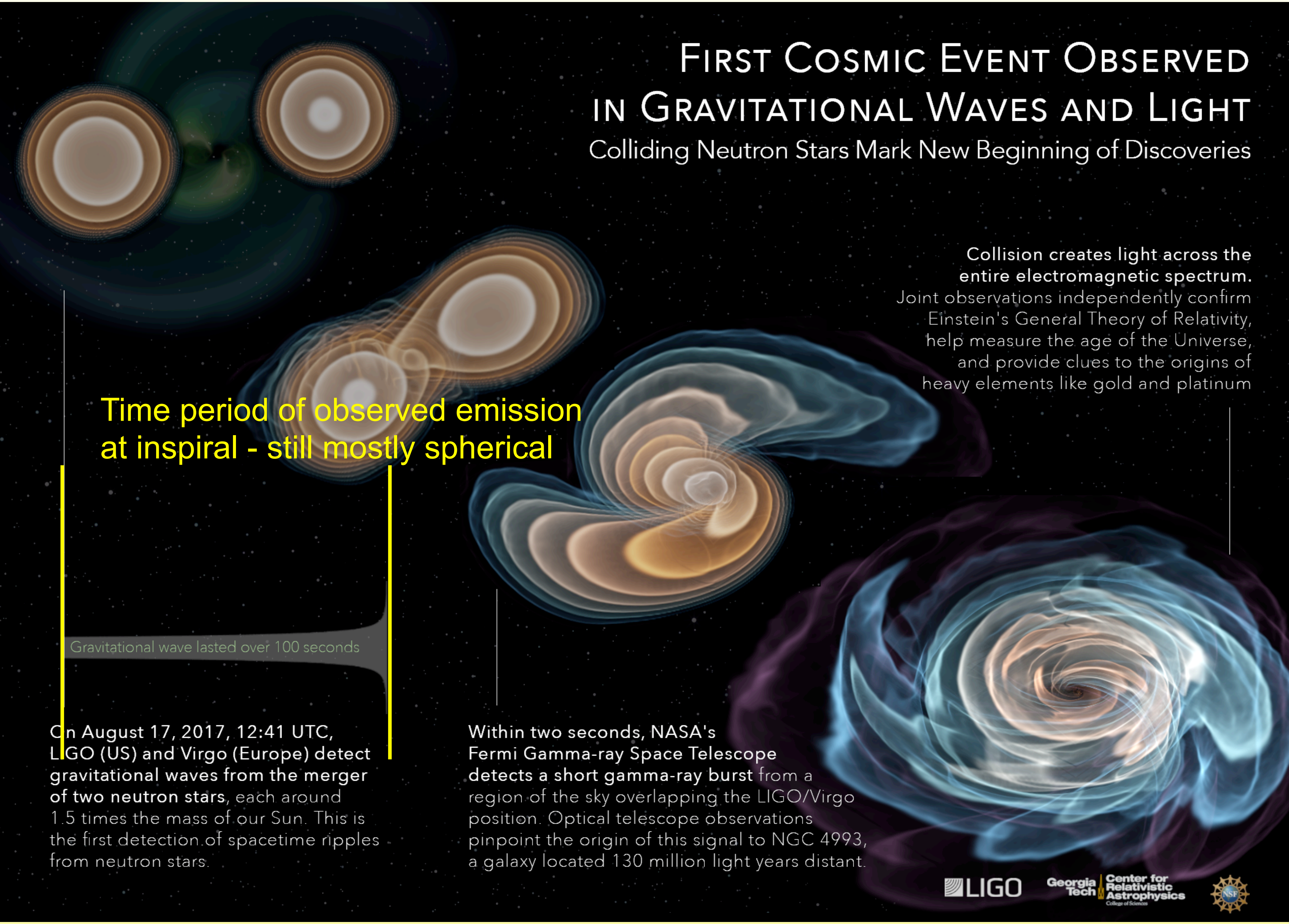
(C.C., Eroncel, Hubisz, Rigo, Terning)

- Rather than just resorting to M(R) curve have more information on internal structure of NS from LIGO/VIRGO gravitational wave measurements for NS merger

- Gravitational wave emitted during inspiral depends on entire $\rho(r)$ profile

- Wave form will contain information on tidal deformability of NS


FIRST COSMIC EVENT OBSERVED
IN GRAVITATIONAL WAVES AND LIGHT
Colliding Neutron Stars Mark New Beginning of Discoveries
Collision creates light across the entire electromagnetic spectrum.
Joint observations independently confirm Einstein's General Theory of Relativity, help measure the age of the Universe, and provide clues to the origins of heavy elements like gold and platinum
Gravitational wave lasted over 100 seconds
On August 17, 2017, 12:41 UTC, LIGO (US) and Virgo (Europe) detect gravitational waves from the merger of two neutron stars, each around 1.5 times the mass of our Sun. This is the first detection of spacetime ripples from neutron stars.
Within two seconds, NASA's Fermi Gamma-ray Space Telescope detects a short gamma-ray burst from a region of the sky overlapping the LIGO/Virgo position. Optical telescope observations pinpoint the origin of this signal to NGC 4993, a galaxy located 130 million light years distant.
LIGO
Georgia Tech
Center for Relativistic Astrophysics
College of Sciences
NSF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Time period of observed emission
at inspiral - still mostly spherical
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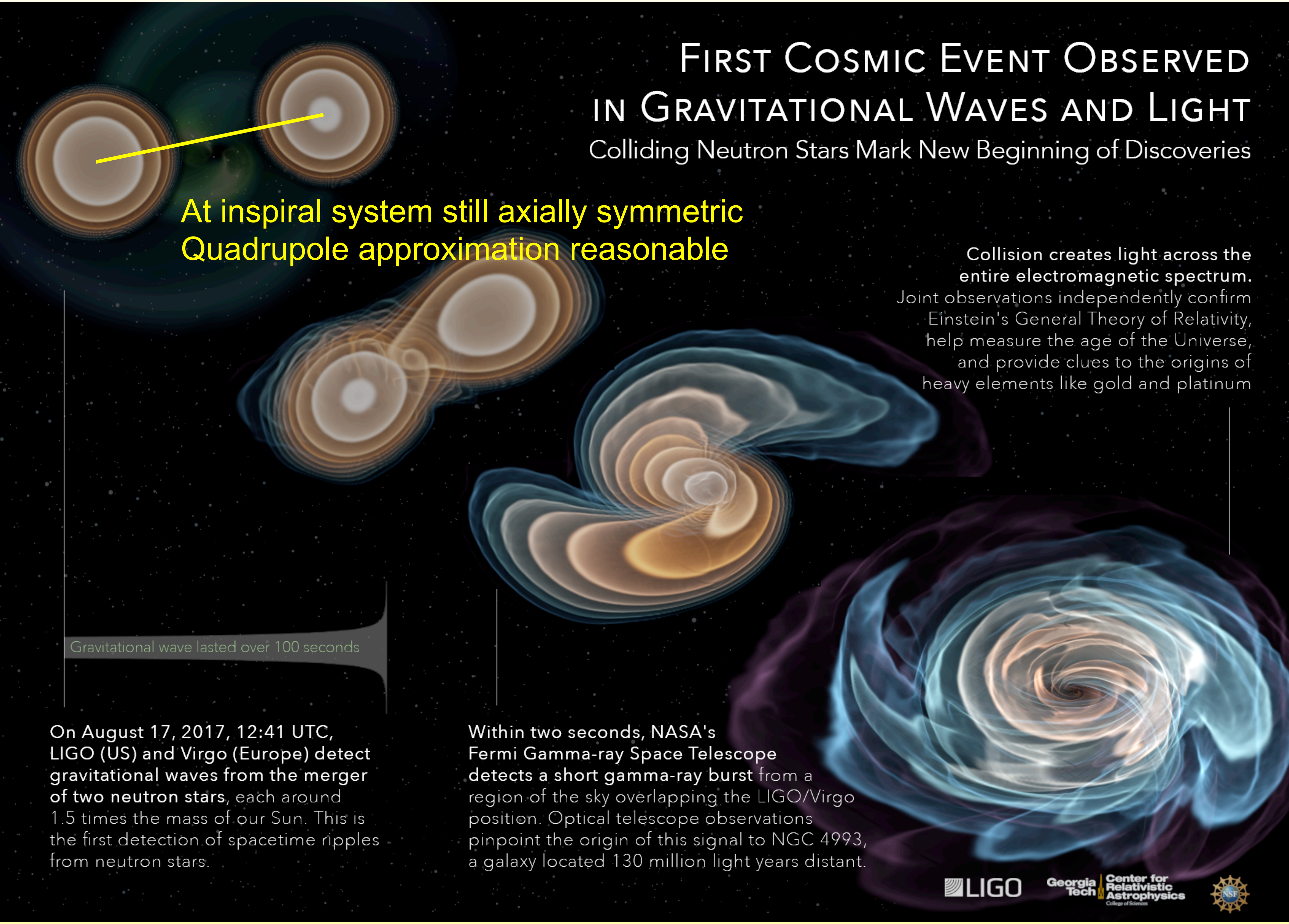
At inspiral system still axially symmetric
Quadrupole approximation reasonable



Modeling QCD matter in the NS interior

(C.C., Eroncel, Hubisz, Rigo, Terning)

- Will try to compare to actual LIGO/VIRGO results - need more precise description of NS

- EoS a complicated function

- Parametrized as piece-wise polytropic

- Usually 7 layers used in state-of-the art NS simulations

- Simply a way of parametrizing a complicated nuclear EoS

Modeling QCD matter in the NS interior

(C.C., Eroncel, Hubisz, Rigo, Terning)

- It is customary to use MASS density as input parameter for EoS $\rho(r) = m\, n(r)$

- Usual parametrization (DIFFERENT model)

$$p = K_i \rho^{\gamma_i}\,, \qquad p_{i-1} \leq p \leq p_i$$

- For the 7 layers i=1,2,…,7, p_i's are the critical pressures where transition to next layer happens

- Need to convert to EoS for energy density ε(r)

$$\epsilon = (1 + a_i)\rho + \frac{K_i}{\gamma_i - 1}\rho^{\gamma_i}$$

Boundary conditions at the layers

- Assume continuity of pressure and mass density for outer 6 layers. Critical pressures p_i and critical mass densities ρ_i

- Continuity of pressure will fix the inner K's:

$$K_i = K_{i-1}\rho_{i-1}^{\gamma_{i-1}-\gamma_i}$$

- Continuity of mass density determines inner a's:

$$a_i = \frac{\epsilon(\rho_{i-1})}{\rho_{i-1}} - 1 - \frac{K_i}{\gamma_i - 1}\rho_{i-1}^{\gamma_i - 1}$$

- For outermost layer ``crust'' $p_0 = 0$ and requiring that $\lim_{\rho\to 0}\frac{\epsilon}{\rho} = 1$ (ordinary matter) fixes $a_1 = 0$

- Input parameters: K_1, γ_i, p_i

Modeling the core and the effect of VE

- We are assuming that core is in a different phase of QCD. Different bound states (if any) but baryon number conserved. Would be best to describe in terms of baryon number density n

$$
\begin{aligned}
p &= \tilde{K}_7 n^{\gamma_7} - \Lambda \, , \\
\epsilon &= (1+\tilde{a}_7) n + \frac{\tilde{K}_7}{\gamma_7 - 1} n^{\gamma_7} + \Lambda
\end{aligned}
$$

- We added effect of VE as an additional piece to pressure/energy as usual

- To more conform to traditional description can introduce fake mass density $\rho = m_n n$ where m_n is neutron mass (real mass density is $\frac{m_b}{m_n}\rho$)

Modeling the core and the effect of VE

- Formally equations will be of the same form

$$
\begin{aligned}
p &= K_7\rho^{\gamma_7} - \Lambda \, , \\
\epsilon &= (1+a_7)\rho + \frac{K_7}{\gamma_7 - 1}\rho^{\gamma_7} + \Lambda \, ,
\end{aligned}
$$

up to corrections from VE

- Pressure must still be continuous (Israel junction)

$$K_7\rho_+^{\gamma_7} - \Lambda = K_6\rho_-^{\gamma_6} = p_6$$

- But now due to presence of Λ mass ρ and energy density ε will have to jump: ρ_+ to ρ_- and ϵ_+ to ϵ_-

Modeling the core and the effect of VE

- Chemical equilibrium implies chemical potential also continuous at the boundary:

$$\frac{\epsilon_+ + p_6}{\rho_+} = \frac{\epsilon_- + p_6}{\rho_-}$$

- Jumps related to each other. One stability condition: $\left(\frac{\partial^2 F}{\partial V^2}\right)_{T,N} > 0$ implies $\left(\frac{\partial p}{\partial n}\right)_{T,N} > 0$

- Jump must be positive: $\rho_+ \geq \rho_-$ and $\epsilon_+ \geq \epsilon_-$

- Will MODEL PT by assuming $\epsilon_+ - \epsilon_- = \alpha|\Lambda|$.

- Input: γ_7, α and Λ and determine K_7 and a_7

Aside: what is VE for NS's

- Equation of state:

$$
\begin{aligned}
p &= K_7 \rho^{\gamma_7} - \Lambda \ , \\
\epsilon &= (1 + a_7)\rho + \frac{K_7}{\gamma_7 - 1}\rho^{\gamma_7} + \Lambda \ ,
\end{aligned}
$$

- Just like for expanding Universe, at fixed **ρ** no way to tell different contributions apart.

- But inside NS the density changes, so will probe entire EOS (like when we follow expansion of Universe)

- Formal definition of VE: the **ρ**→0 limits of the pressure/energy density (even though different phase - really analytic continuation of EOS to **ρ**=0)

Stability conditions

- Need pressure and partial fluid pressure positive:

$$-p_6 < \Lambda$$

- Additional condition: stability of NS: $\dfrac{\partial M}{\partial p_0} > 0$

- At reaching the critical pressure this translates

$$\epsilon_+ - \epsilon_- \geq \frac{1}{2}\epsilon_- + \frac{3}{2}p_6$$

- Interestingly, possible that this condition violated for $p>p_6$ but satisfied again for even larger pressures opening up a disconnected branch (see later)

Neutron star observables

- Again use TOV equations for the model with 7 layers

- Spherically symmetric metric

$$ds^2 = e^{\nu(r)}dt^2 - \left(1 - \frac{2Gm(r)}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

- Einstein equations for spherically symmetric fluid TOV equations

$$\begin{aligned}
m'(r) &= \; 4\pi r^2 \epsilon(r)\,, \\
p'(r) &= \; -\frac{p(r)+\epsilon(r)}{r\left(r-2Gm(r)\right)} G\left[m(r) + 4\pi r^3 p(r)\right]\,, \\
\nu'(r) &= \; -\frac{2p'(r)}{p(r)+\epsilon(r)}\,,
\end{aligned}$$

Tidal deformability and Love number

- The presence of the second neutron star will act as an external perturbation

- Since we are in inspiral phase still relatively far - will use multipole expansion. Due to spherical symmetry of perturbing source no dipole - leading term will be quadrupole

$$\frac{1+g_{tt}}{2} \approx \frac{Gm}{r} + \frac{3GQ_{ij}}{2r^5}x^i x^j - \frac{1}{2}\mathcal{E}_{ij}x^i x^j \ldots$$

- $\mathcal{E}_{ij}$ is the external quadrupole field, and Q_{ij} is the induced term due to the response of the NS.

Tidal deformability and Love number

- For a linear response

$$Q_{ij} = -\lambda\,\mathcal{E}_{ij}$$

- Characteristic dimensionless quantity (R radius):

$$k_2 = \frac{3}{2}\frac{G\lambda}{R^5}$$

- This is l=2 (quadrupole) Love number or tidal deformability. Characterizes the internal structure of the NS beyond just the M(R) relation

- Main additional physical observable

Determining the tidal deformability

- Need to do PT theory around spherically symmetric solution $g_{\alpha\beta} + h_{\alpha\beta}$ with quadrupole deformations

$$h_{\alpha\beta} = \mathrm{diag}\left(e^{\nu(r)} H(r), e^{\mu(r)} H(r), r^2 K(r), r^2 \sin^2\theta K(r)\right) Y_2^0(\theta, \phi)$$

- Two functions H(r) and K(r) related

$$K'(r) \quad = H'(r) + H(r)\nu'(r)$$

- Final equation for H(r)

$$H'' = 2He^{\mu}\left\{-2\pi G\left[5\epsilon + 9p + \frac{d\epsilon}{dp}(\epsilon + p)\right] + \frac{3}{r^2} + 2G^2 e^{\mu}\left(\frac{m(r)}{r^2} + 4\pi r p\right)^2\right\} +$$
$$\frac{2}{r}H'e^{\mu}\left\{-1 + \frac{Gm(r)}{r} + 2\pi G r^2(\epsilon - p)\right\}$$

Determining the tidal deformability

- Solution should have no singularity at r=0

$$H(r) = a\,r^2 + \mathcal{O}(r^4)$$

- Need to numerically solve H(r) given the fixed backgrounds and k_2 will be

$$\begin{aligned}
k_2 =& \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \\
&\times \left\{2C[6-3y+3C(5y-8)] + 4C^3[13-11y+C(3y-2)+2C^2(1+y)]\right. \\
&\left. + 3(1-2C)^2[2-y+2C(y-1)]\log(1-2C)\right\}^{-1},
\end{aligned}$$

- C is compactness $C = \dfrac{GM}{R}$ and $y = \dfrac{RH'(R)}{H(R)}$
- Often different version used: $\bar{\lambda} = \dfrac{2k_2}{3C^5} = \dfrac{\lambda}{G^4 M^5}$

LIGO/VIRGO observables

- Chirp mass $\mathcal{M} = (M_1 M_2)^{3/5}(M_1 + M_2)^{-1/5}$.
- For GW170817 $\mathcal{M} = 1.188 M_\odot$
- Combination of Love numbers:

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1 + 12M_2)M_1^4\bar{\lambda}_1 + (M_2 + 12M_1)M_2^4\bar{\lambda}_2}{(M_1 + M_2)^5}$$

- For GW170817 constraint $\tilde{\Lambda} < 800\ (700)$ (depending on spin of NS's)

The NS models

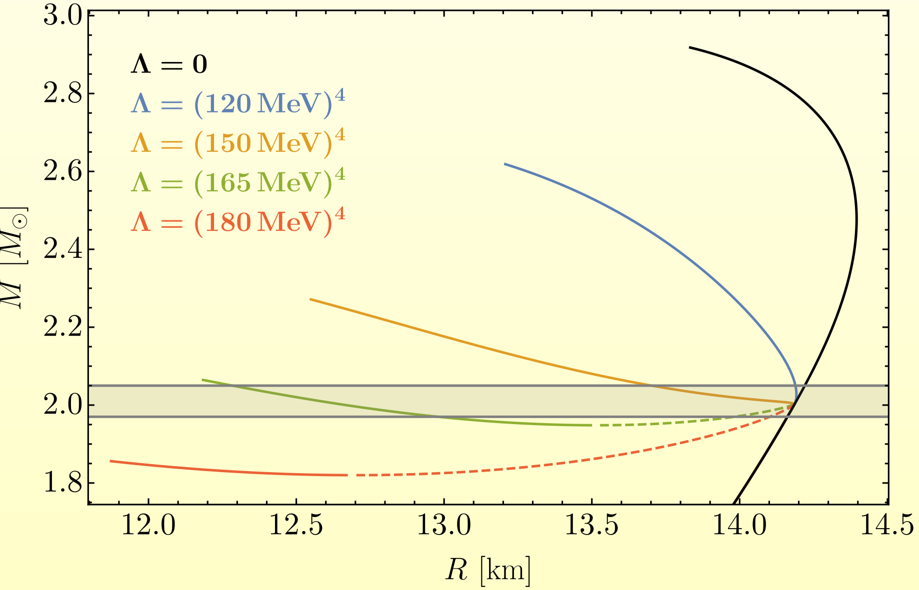
- Use 3 different NS models
- Hebeler et al: most ``aggressive", allows up to $3\ M_\odot$
- AP4
- SLy

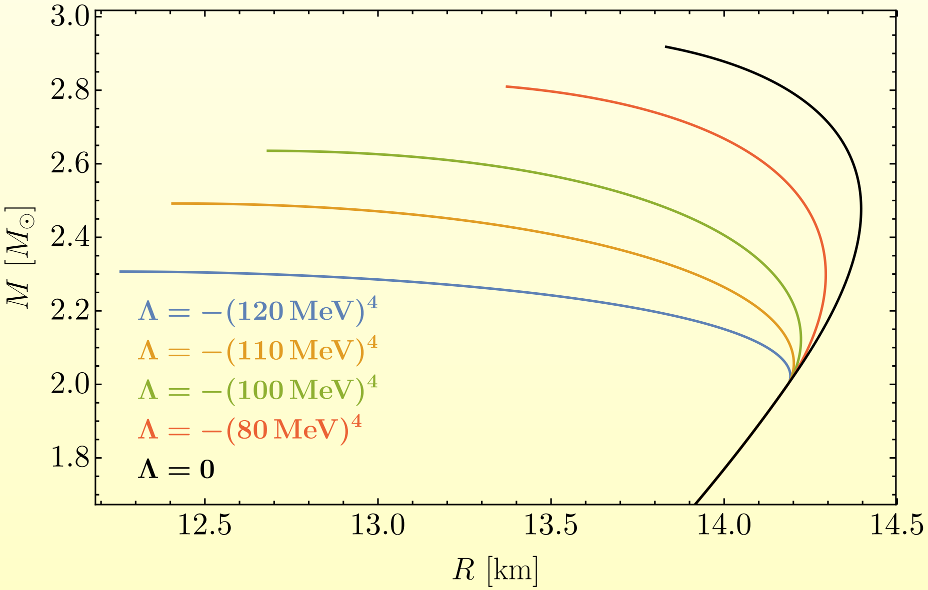
More conservative $\leq 2\,M_\odot$

- Prepared plots for all, here I only show Hebeler
- Results still significant for AP4 and SLy but less dramatic

	SLy	AP4	Hebeler
K_1	9.27637×10^{-6}		See [22]
p_1	$(0.348867)^4$		See [22]
p_2	$(7.78544)^4$		
p_3	$(10.5248)^4$		
p_4	$(40.6446)^4$	$(41.0810)^4$	$(72.2274)^4$
p_5	$(103.804)^4$	$(97.1544)^4$	$(102.430)^4$
p_6	$(176.497)^4$	$(179.161)^4$	$(149.531)^4$
γ_1	1.58425		See [22]
γ_2	1.28733		
γ_3	0.62223		
γ_4	1.35692		
γ_5	3.005	2.830	4.5
γ_6	2.988	3.445	5.5
γ_7	2.851	3.348	3

Results I. Effects of VE on M(R) curves

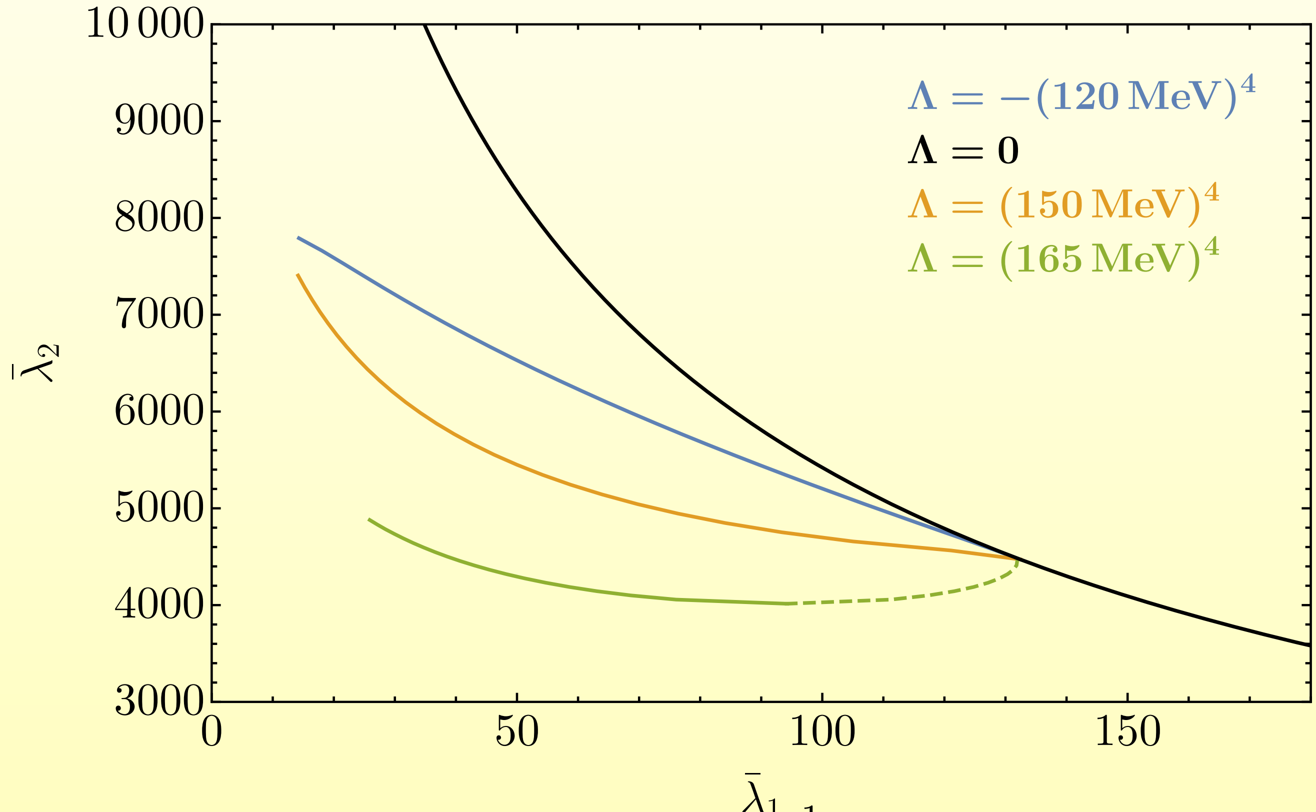




- The currently observed maximal NS mass is around $2M_\odot$

- Picked one set of parameters (Hebeler, Lattimer et al) and kept everything fixed except central pressure. When pressure large enough VE will influence structure

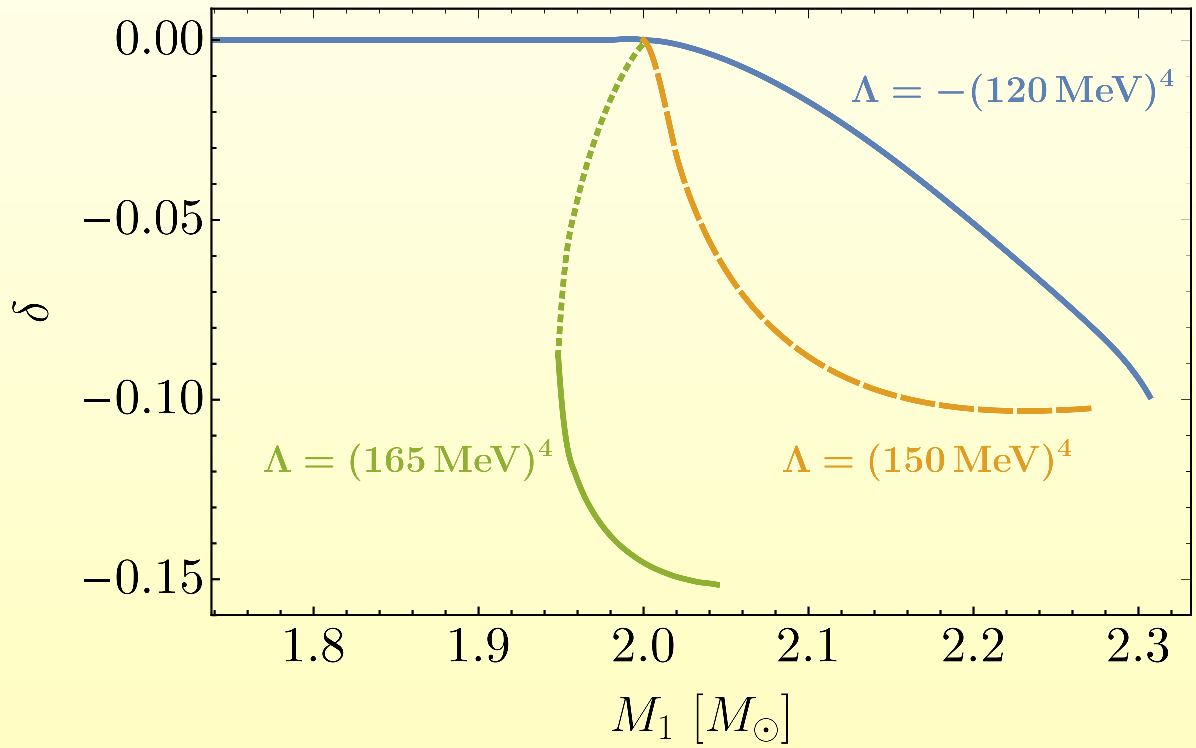
Results II. Effects of VE on tidal deformability



- Chirp mass fixed $\mathcal{M} = 2^{-\frac{1}{5}} 1.4 M_\odot \sim 1.2 M_\odot$

- Picked an aggressive set of parameters (Hebeler, Lattimer et al) that allows large NS masses

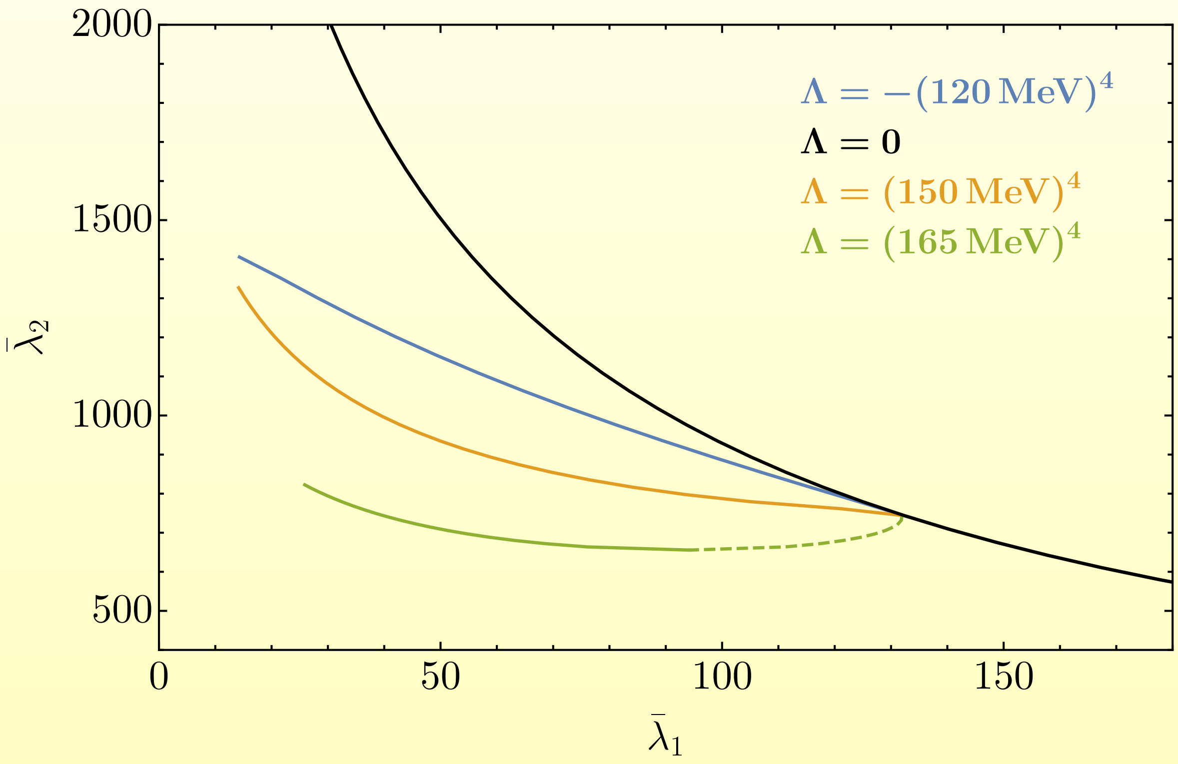
Results II. Effects of VE on tidal deformability



- Chirp mass fixed $\mathcal{M} = 2^{-\frac{1}{5}} 1.4 M_\odot \sim 1.2 M_\odot$

- Percent deviation of deformability of heavier star (lighter star is in normal phase)

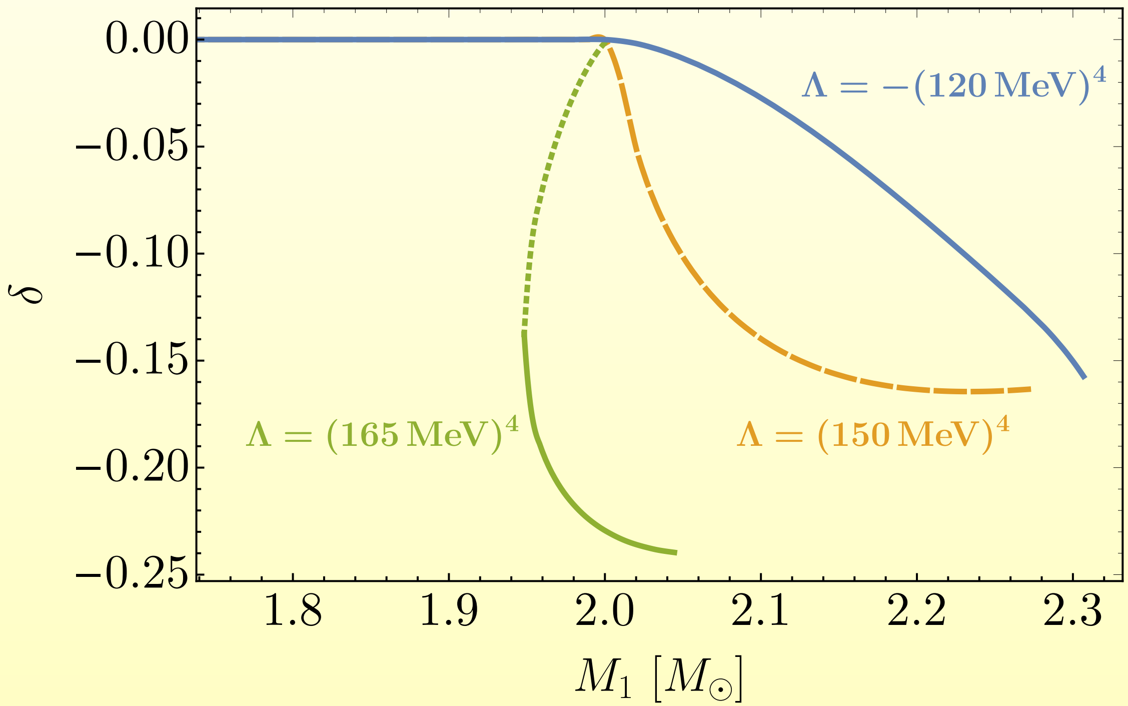
Results II. Effects of VE on tidal deformability



- Chirp mass fixed $\mathcal{M} = 2^{-\frac{1}{5}} 1.7 M_\odot \sim 1.48 M_\odot$

- Picked an aggressive set of parameters (Hebeler, Lattimer et al.) that allows large NS masses

Results II. Effects of VE on tidal deformability



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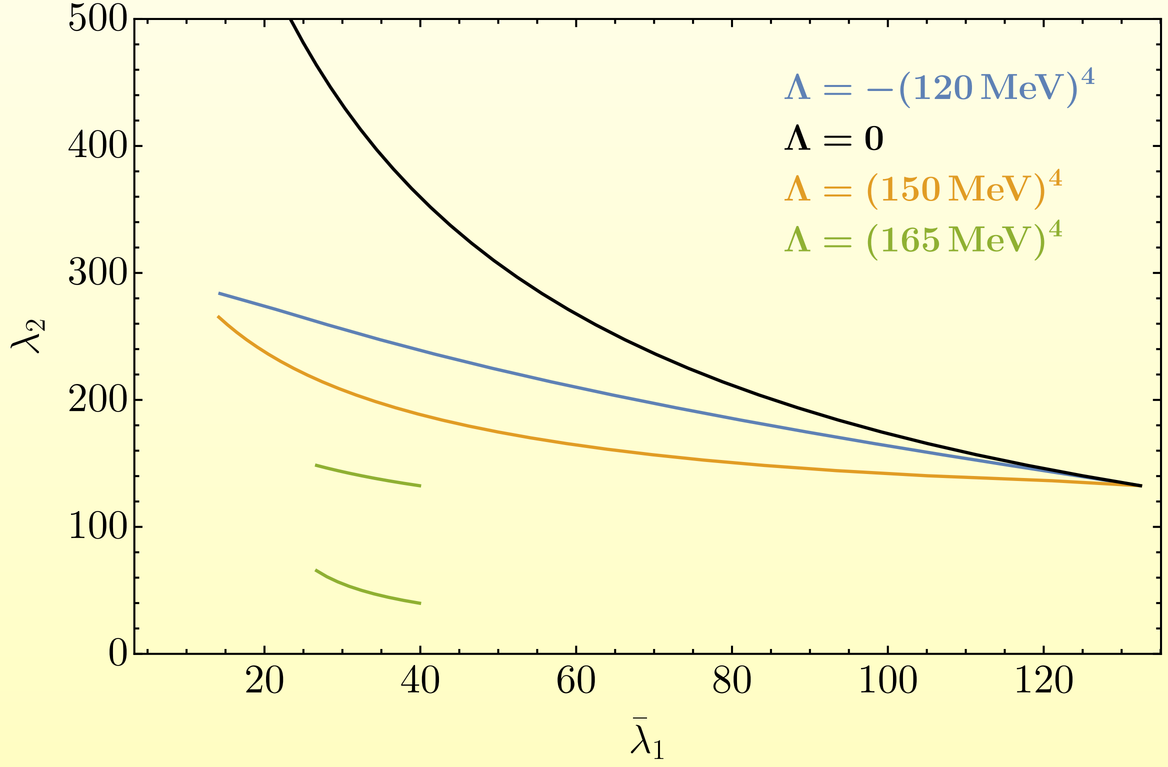
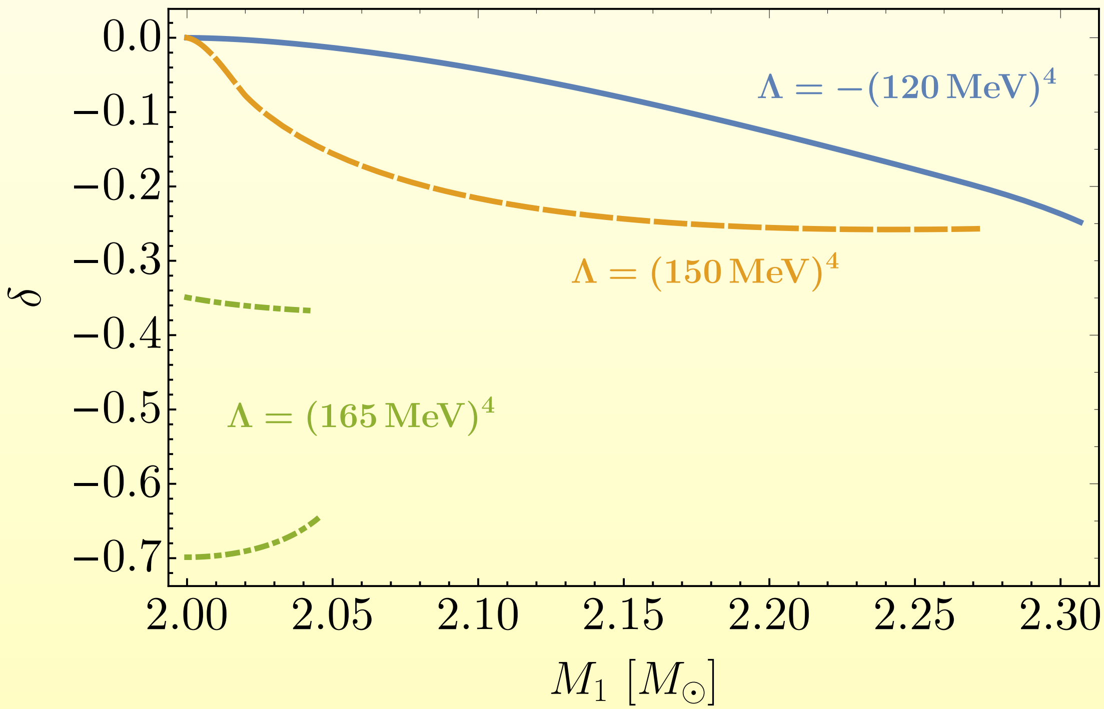
Results II. Effects of VE on tidal deformability



- Chirp mass fixed $\mathcal{M} = 2^{-\frac{1}{5}} 2M_\odot \sim 1.75M_\odot$

- Picked an aggressive set of parameters (Hebeler, Lattimer et al.) that allows large NS masses. Note two branches for large CC

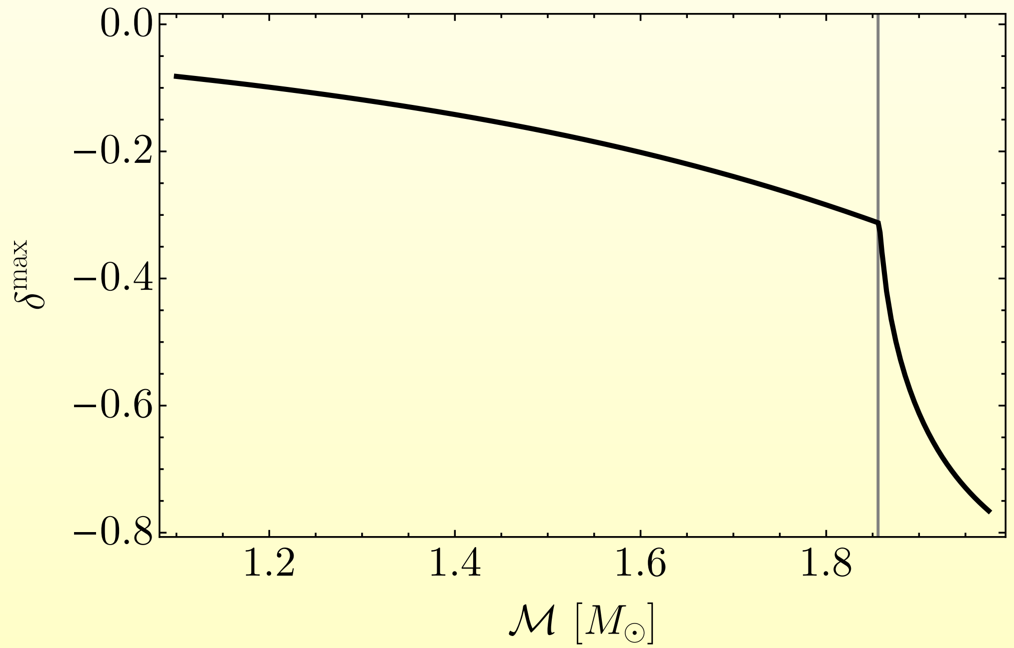
Results II. Effects of VE on tidal deformability



- Chirp mass fixed $\mathcal{M} = 2^{-\frac{1}{5}} 2M_\odot \sim 1.75 M_\odot$

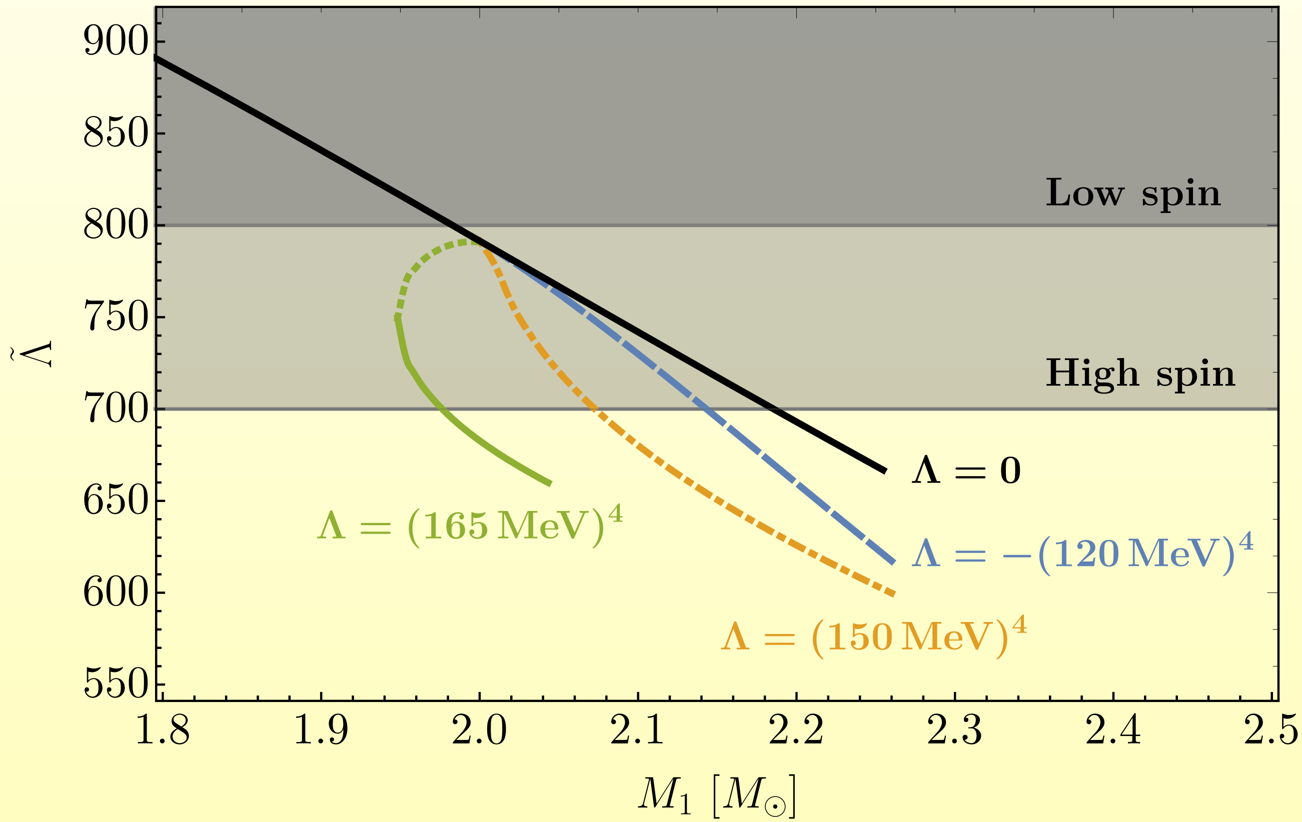
- Percent deviation of deformability of heavier star (lighter star is in normal phase). On second branch of Λ=(165 MeV)^4 both stars in new phase of QCD

Dependence on the chirp mass



- Heavier star's mass fixed $M_1 = 2.27 M_\odot$
- VE fixed Λ=(150 MeV)^4

Effect of Λ on GW170817



- The LIGO bound (for a fixed EoS) on tidal deformability can be evaded by turning on Λ

- Here chirp mass fixed to that of GW170817

Summary

- An important part of our standard picture of cosmology & particle physics: VE should change

- Experimental check important

- Look for systems where vacuum energy is sizable fraction Neutron stars

- Should cause measurable deviation in maximal mass of NS's

- Can affect LIGO observables - tidal deformation

- Other observables?

Backup slides

2. Effect of PT's on Primordial GW's

- Can we possibly say something about the actual vacuum energy of the Universe?

- Need to look for periods around phase transitions

- That is only time when vacuum energy might be sizable

- Especially QCD PT might be interesting

- Case study: look at effect of PT's on primordial gravitational waves, assuming no GW's produced during PT itself

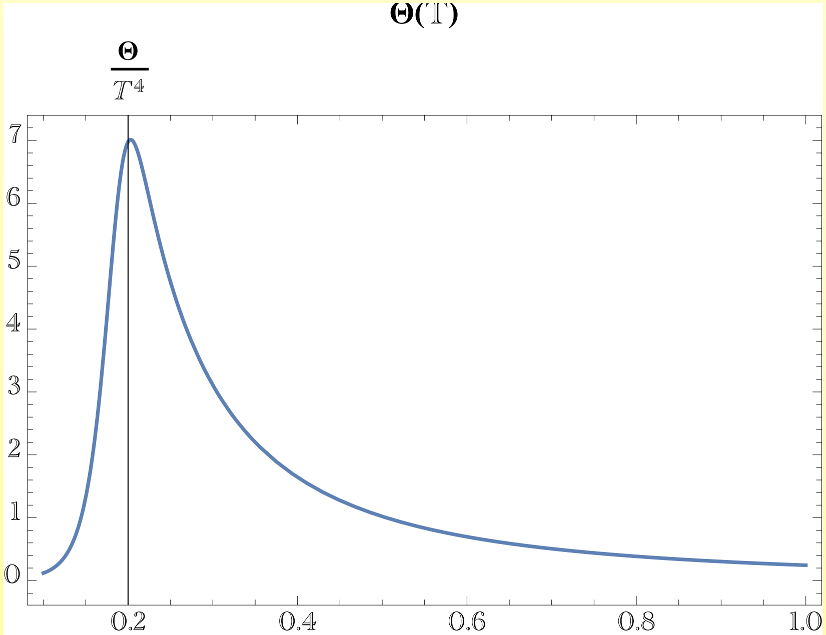
QCD Phase transition

- Numerical evaluation

- Lattice simulations:

$$\Theta = \mathrm{Tr}\, T = T^4 \left(1 - \frac{1}{(1+e^{(T-c_1)/c_2})^2}\right)\left(\frac{d_2}{T^2} + \frac{d_4}{T^4}\right)$$

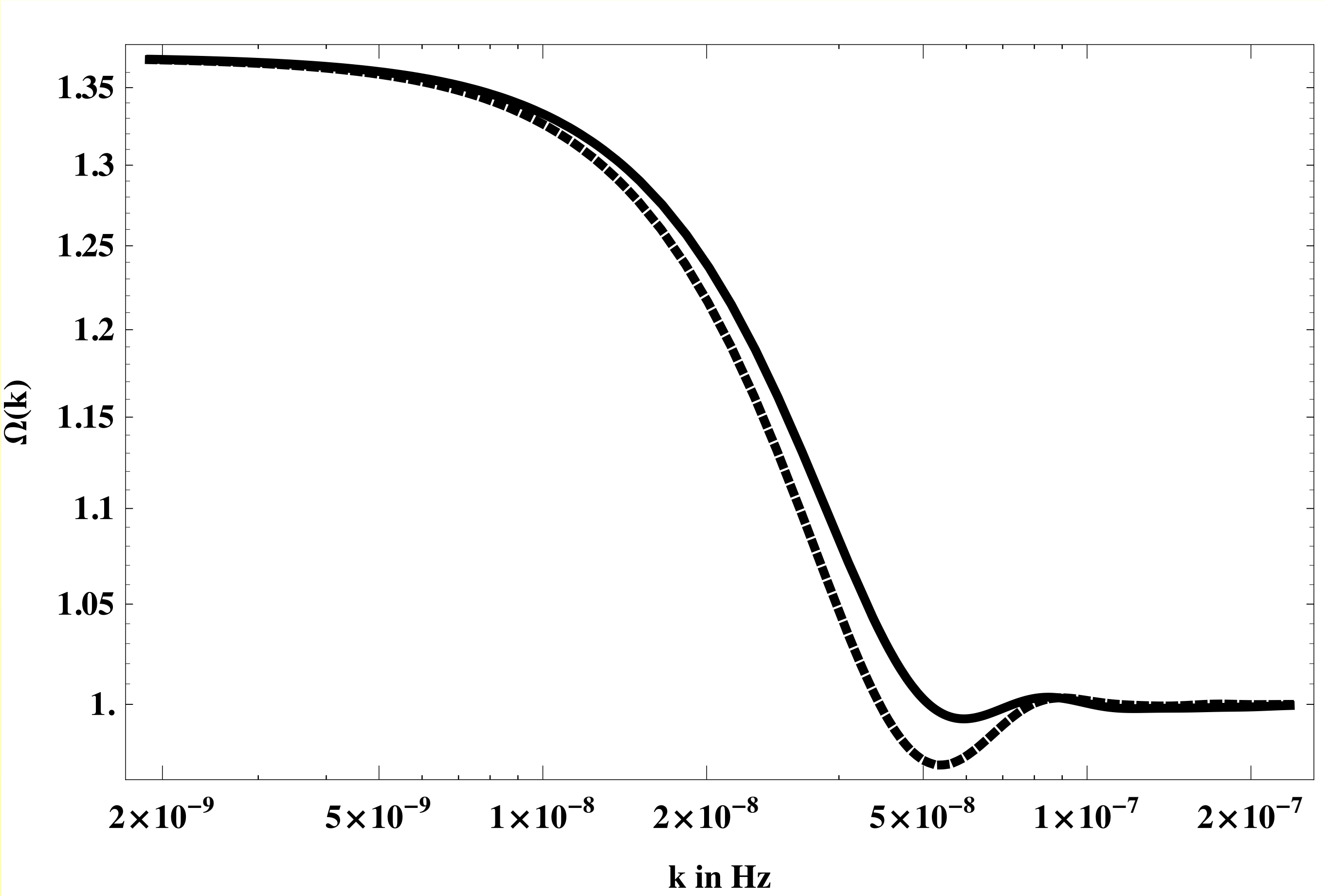
- d_4 is vacuum energy that is changing from $\mathcal{O}(\Lambda^4_{QCD})$ to almost zero



- Valid between 100 MeV and 1 GeV

QCD Phase transition

- Unfortunately here effect very small. A typical result:



- Size of step given by change in DOF 60→20 under QCD, HUGE step

Condition for strong effect of VE

- Simplified discussion: assume VE jumps at some time τ_t, very small before and after transition

- Important quantity: $$\xi = \frac{\rho_\Lambda}{\rho_R} = \frac{\rho_\Lambda}{\bar{\rho}_R} a^4(\tau)$$

- In the presence of VE power spectrum

$$\Omega(k > k_{eq}) \propto a^2(\tau_{hc})k^2 = (1+\xi)\bar{\rho}_R = (1+\xi)g_* T^4 a^4 \propto (1+\xi) g_*^{-\frac{1}{3}}$$

- Since ξ is very small before & after PT, but sizeable during PT - a peak could appear, set by maximal ξ

- Need to compare ξ to size of step

Condition for strong effect of VE

- QCD: $$T_c = 193 \text{ MeV}$$
$$\Lambda = (171 \text{ MeV})^4$$
$$\xi_{QCD} = 0.04$$

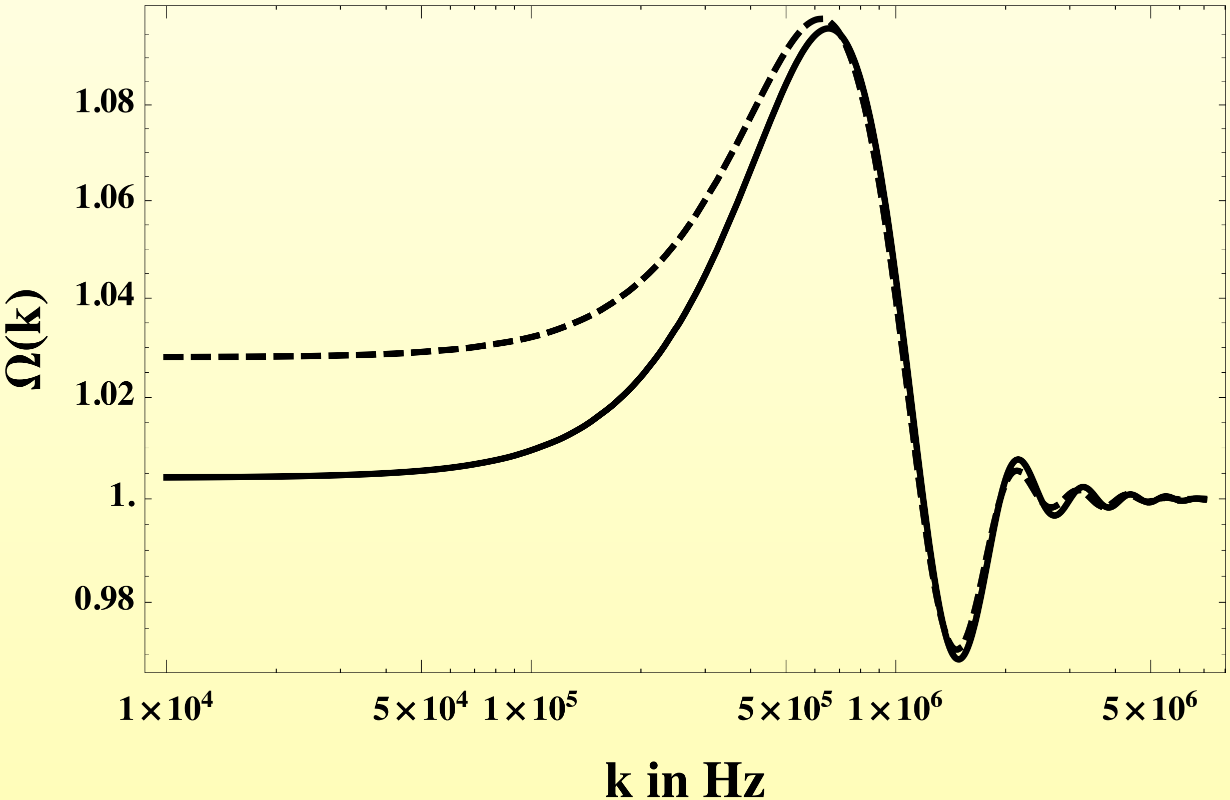
- vs ~ 40 percent step - peak washed out

- EW: $$T_c = 175 \text{ GeV}$$
$$\Lambda = (105 \text{ GeV})^4$$
$$\xi_{EW} = 0.004$$

- vs ~ 7 percent step - again not visible

A peak in the GW spectrum

- An example with peak for hypothetical PT



SU(N)/SU(N-1) $T_c = 10^{11}$ GeV, DOF changes by 10 (dashes) or almost none (solid). Need large quartic coupling on limit of perturbativity!

Effect of adjustment mechanism

- Depends on the time scale for the adjustment

- If very quick - might just set VE to zero always. In this case hard to make any distinction in QCD & EW

- Other possibility: adjustment time scale somewhat larger than that of PT

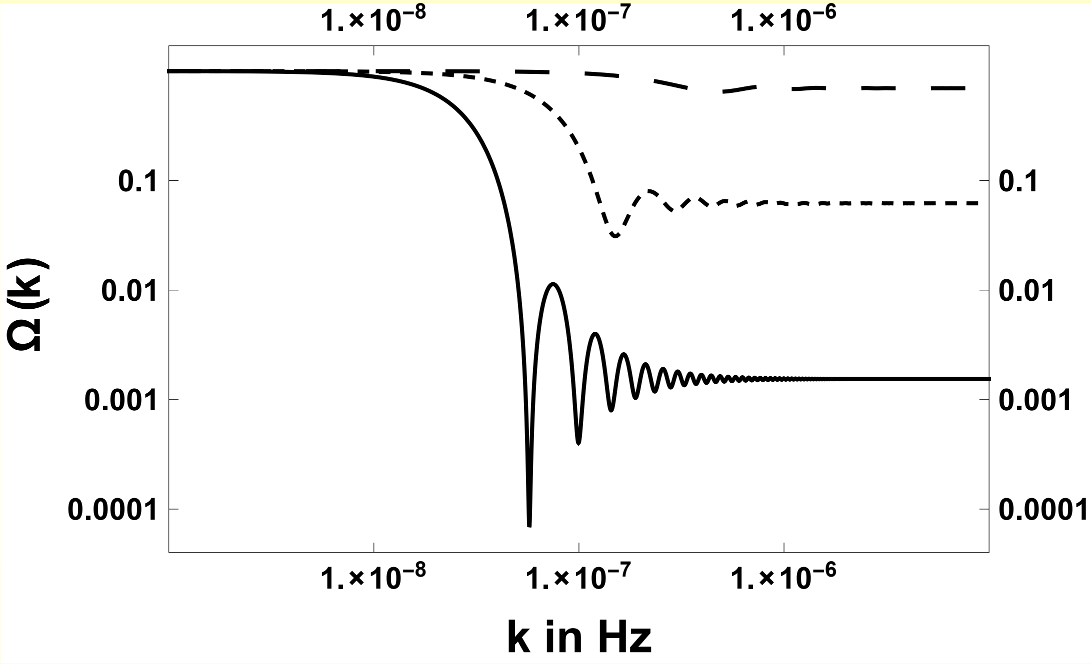
- In this case expect a period where VE dominates after the PT

- Could have a short inflation-like period after PT

Effect of short inflation

- All modes that enter before inflation will be strongly suppressed

- Can get a step much larger than from change of degrees of freedom

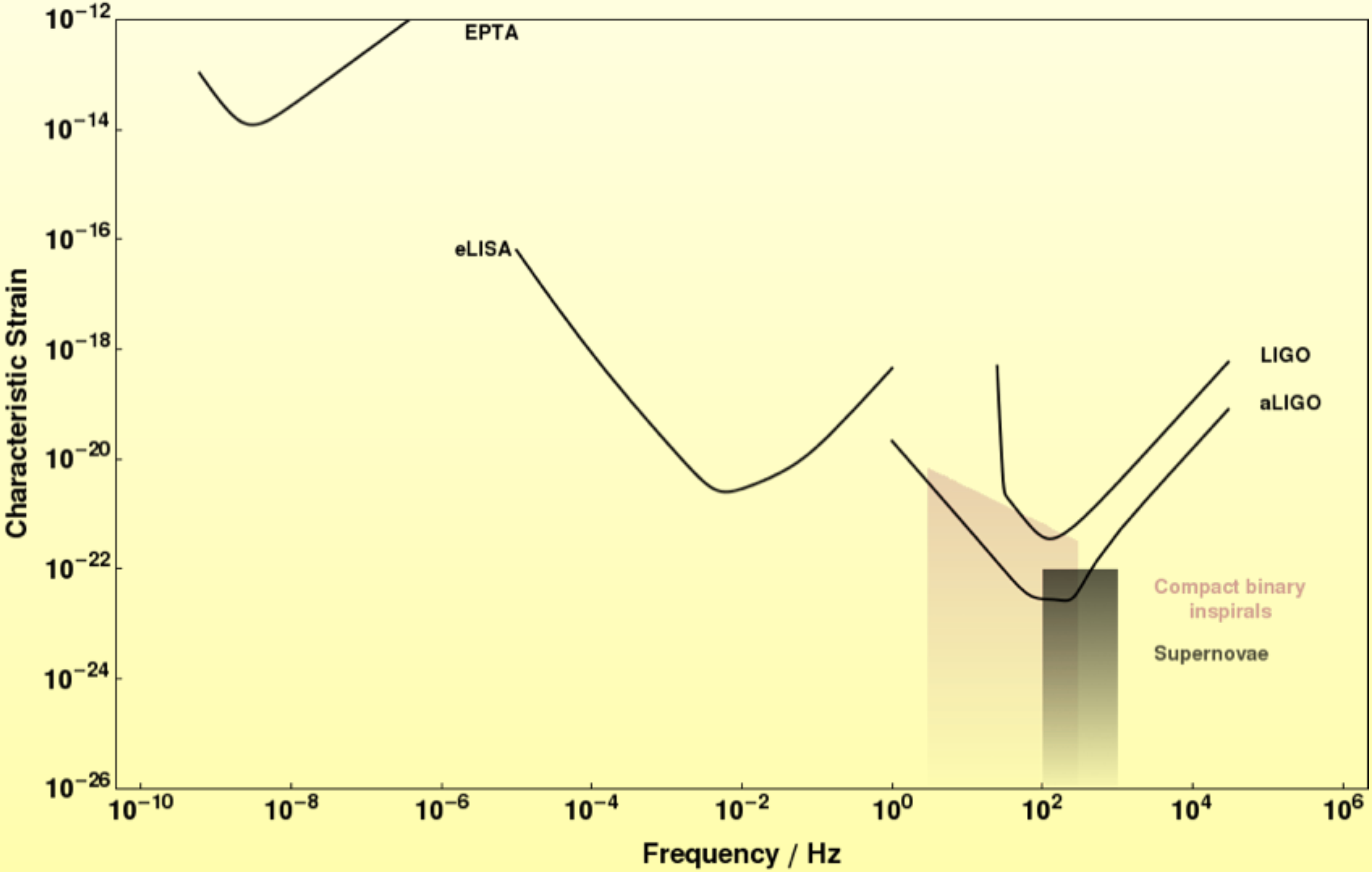


$t_{relax} = t_{QCD}$

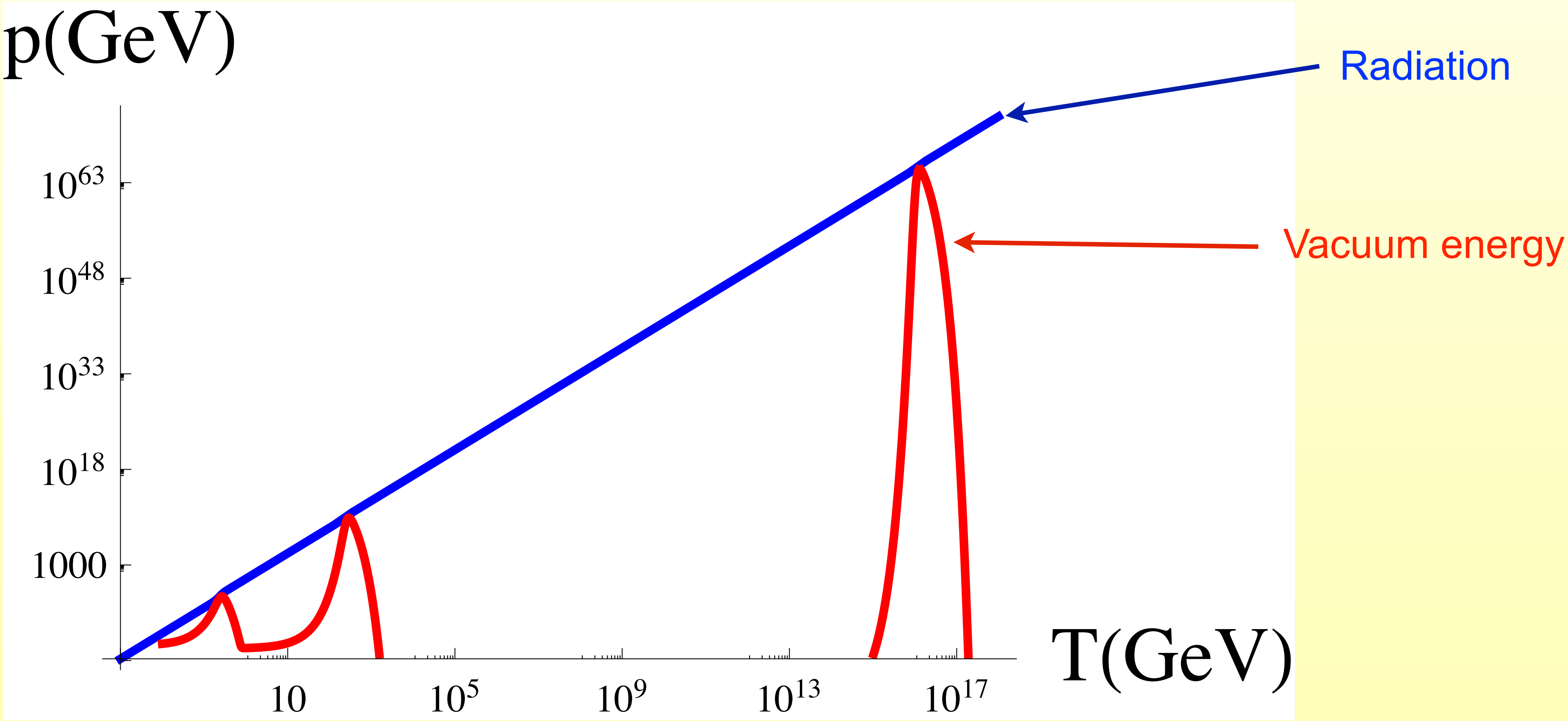
$t_{relax} = 5t_{QCD}$

$t_{relax} = 10t_{QCD}$

Sensitivity of future experiments



Alternative evolution of Λ: with adjustment



Alternative evolution of Λ: with adjustment

- Λ is always small except around PT's
- When PT starts Λ starts growing
- Adjustment mechanism kicks in and drives Λ small again
- Will have its own timescale Δt_{adj}
- Heights will depend on details of adjustment, PT

Steps or adjustment?

- Important goal: to determine experimentally which of these pictures is right one

- If steps: lends more credence to anthropic arguments

- If adjustment need to find mechanism

- Difficulty: Λ always sub-dominant

- Last of these transitions occurred at Λ_{QCD} : Above CMB, BBN, etc. Not much precision results from that period

Propagation of primordial gw's

- Tensor perturbations h_{ij} transverse traceless

$h^i_i = 0, \text{ and } \partial_k h^k_i = 0$

- Perturbation of metric in expanding Universe

$$ds^2 = a(\tau)^2 \left(d\tau^2 - (\delta_{ij} + h_{ij})dx^i dx^j\right)$$

- Usually conformal time τ is used $a(\tau)d\tau = dt$
where expansion equation

$$a' = a\dot{a} = a^2 H \,, \qquad \frac{a''}{a} = a^2 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = \frac{4\pi G}{3} a^2 T^\mu_\mu$$

Propagation of primordial gw's

- Einstein equation: $$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 0$$
- Expand in modes: $$h_{ij} = \sum_{\sigma=+,-} \int \frac{d^3k}{(2\pi)^3} \epsilon_{ij}^{(\sigma)} h_k^{(\sigma)}(\tau) e^{ikx}$$
- Rescaled modes: $$\chi_k \equiv a h_k$$
- Satisfy very simple equation:

$$\chi''_k + (k^2 - \frac{a''}{a})\chi_k = \chi''_k + \left[k^2 - \frac{4\pi G}{3} a^2 T^\mu_\mu\right]\chi_k = 0$$

- Exciting: equation depends on trace of EM tensor!
- Might think (we did for a while) that VE will have big effect before PT - NOT true

Propagation of primordial gw's

$$\chi_k'' + (k^2 - \frac{a''}{a})\chi_k = \chi_k'' + \left[k^2 - \frac{4\pi G}{3} a^2 T^\mu_\mu\right]\chi_k = 0$$

- Interpretation: if $k^2 > \frac{a''}{a}$ just free plane wave for χ
- But actual mode is χ/a getting damped by 1/a
- Interpretation: if $k^2 < \frac{a''}{a}$ then equation $\frac{\chi''}{\chi} = \frac{a''}{a}$

has solution $\chi \propto a$ and actual mode χ/a is frozen

- If mode outside horizon it is frozen. Once it enters horizon it is damped by 1/a

Propagation of primordial gw's

- What sets the horizon?
- Naively: $$\frac{a''}{a} = \frac{4\pi G}{3} a^2 T^\mu_\mu$$
- This horizon is larger than Hubble horizon - suggests can not have any physical effect
- Indeed when entering this ``naive horizon" velocity of solution still very large - will keep expanding until reaches actual Hubble horizon
- Real condition: rate of entering actual horizon

Energy density in GW's

- The physical quantity:

$$\rho_h(\tau) = \frac{1}{16\pi G a^2(\tau)} \int \frac{d^3k}{(2\pi)^3} |h'_{\sigma,k}|^2$$

- The power spectrum:

$$\Delta_h^2 = \frac{4k^3}{2\pi^2}|h_k|^2 \ , \quad |h_k|^2 = |h_{\sigma,k}|^2 \ .$$

- Transfer function $\mathcal{T}$: $\quad h_k(\tau) \equiv h_k^P \mathcal{T}(\tau, k)$

- h_k^P is the primordial amplitude, usually assumed to have constant power

$$(\Delta_h^P)^2 = \frac{4k^3}{2\pi^2}|h_k^P|^2 \simeq \frac{2}{\pi^2}\frac{H_\star^2}{M_P^2}$$

Energy density in GW's

- The energy density can then be written in terms of the transfer function

$$\rho_h(\tau) = \frac{1}{32\pi G a^2(\tau)} \int d\ln k (\Delta_h^P)^2 \mathcal{T}'^2(\tau, k)$$

- The most commonly used quantity: energy density per log scale normalized to critical density

$$\Omega_h(\tau, k) \equiv \frac{\tilde{\rho}_h(\tau, k)}{\rho_c(\tau)} \ , \qquad \tilde{\rho}_h(\tau, k) = \frac{d\rho_h(\tau, k)}{d\ln k}$$

- Most useful expression:

$$\Omega_h(\tau, k) = \frac{(\Delta_h^P)^2}{12} \frac{1}{H^2(\tau)} \frac{1}{a^2(\tau)} \mathcal{T}'^2(\tau, k)$$

Energy density in GW's

- Assuming mode deep inside horizon:

$$\mathcal{T}'^2(\tau, k) \simeq k^2\, \mathcal{T}^2(\tau, k)$$

- Given our previous discussion, after inflation modes start out outside the horizon and are frozen

- Mode enters at $\tau = \tau_{hc}$ after which energy density gets diluted as radiation

$$\mathcal{T}^2(\tau < \tau_{hc}, k) \simeq \frac{a^2(\tau_{hc})}{a^2(\tau)}$$

- Approximate expression:

$$\Omega_h(\tau, k) \simeq \frac{(\Delta_h^P)^2}{12} \frac{k^2}{H^2(\tau)} \frac{a^2(\tau_{hc})}{a^4(\tau)}$$

Modes entering during RD

- This is the most relevant case for studying PT's, both QCD and EW happen in that epoch
- Condition for entering: $(aH)^{-2}(\tau_{hc}) \simeq 1/k^2$
- During RD $H^2 \propto 1/a^4$
- Thus $k^2 a^2(\tau_{hc}) \propto const.$
- Spectrum for modes entering during RD constant!

Effect of Phase transition

- Depart from pure RD during PT

- Traditional description: changing number of rel. degrees of freedom in equilibrium

$$g_{\star,a} \equiv g_\star(\tau > \tau_t) \neq g_\star(\tau < \tau_t) \equiv g_{\star,b}$$

- Assuming PT is second order adiabatic (entropy conserved):

$$S = \frac{\rho + p}{T} a^3 = const.$$

- For radiation

$$\rho + p \propto g_* T^4$$

Effect of Phase transition

- Expansion rate: $a \propto T^{-1} g_*^{-1/3}$
- Hubble: $H^2 \propto \rho \propto \frac{1}{a^4} g_*^{-1/3}$

- Energy density:

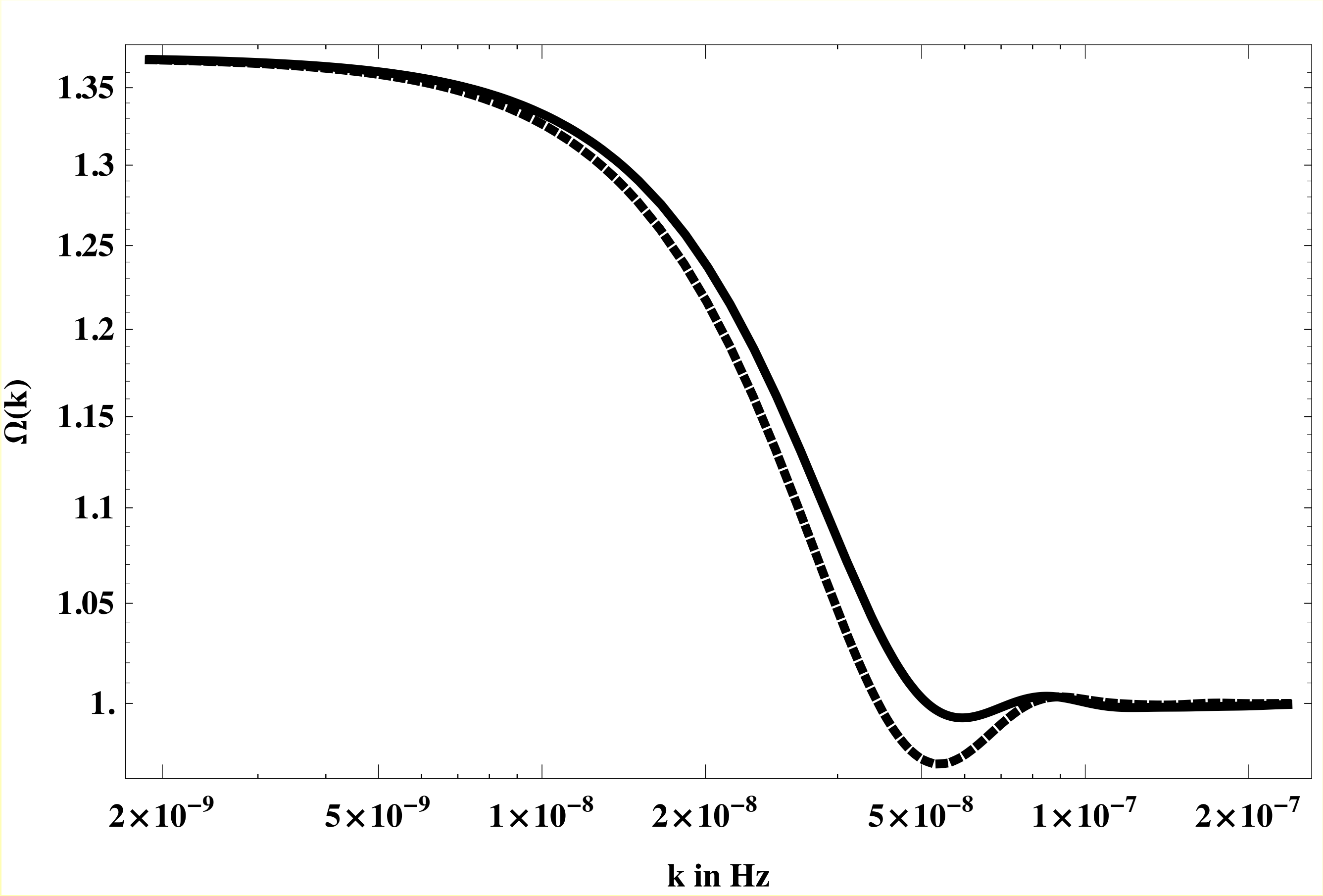
$$\Omega_h \propto k^2 a^2(\tau_{hc}) \propto a^4(\tau_{hc}) H_{hc}^2 \propto g_*^{-1/3}$$

- Depends only on # of DOF's

- Expect to see a step in GW density

QCD Phase transition

- If all VE (ie. change EOS for Θ)



- Almost no difference. Effect of VE (vs. changing DOF's) not measurable in QCD PT

Toy model for neutron stars

- At zero temperature, gravitational pressure balanced by pressure of fluid. Metric:

$$ds^2 = e^{\nu(r)} dt^2 - (1 - 2GM(r)/r)^{-1} dr^2 - r^2 d\Omega^2$$

- Einstein eq's (aka Tolman-Oppenheimer-Volkoff eq):

$$\begin{aligned} M'(r) &= 4\pi r^2 \rho(r) \,, \\ p'(r) &= -\frac{p(r) + \rho(r)}{r^2 (1 - 2GM(r)/r)} \left[GM(r) + 4\pi r^3 p(r) \right] \,, \\ \nu'(r) &= -\frac{2p'(r)}{p(r) + \rho(r)} \,, \end{aligned}$$

Toy model for neutron stars

- Radius determined by position of vanishing pressure p(R)=0

- Assume phase transition happens at p_{crit}

- Two different EOS's

$$
\begin{aligned}
p =& p_{(-)}(\rho)\,, \qquad \rho = \rho_{(-)}\,, \qquad p \geq p_{cr}\,, \qquad r \leq r_{cr} \\
p =& p_{(+)}(\rho)\,, \qquad \rho = \rho_{(+)}\,, \qquad p < p_{cr}\,, \qquad r \geq r_{cr}\,.
\end{aligned}
$$

- Junction condition: $\nu'(r), M(r)$ continuous, thus $p(r)$ also cont.

Toy model for neutron stars

- For inner core use polytropic with cc:

$$p_{(-)}(\rho) = p_f(\rho) - \Lambda = K_- \rho_f^{\gamma_-} - \Lambda$$
$$\rho_{(-)} = \rho_f + \Lambda$$

- For outer core just polytropic

$$p_{(+)}(\rho) = p_f(\rho) = K_+ \rho_f^{\gamma_+}$$
$$\rho_{(+)} = \rho_f \,.$$

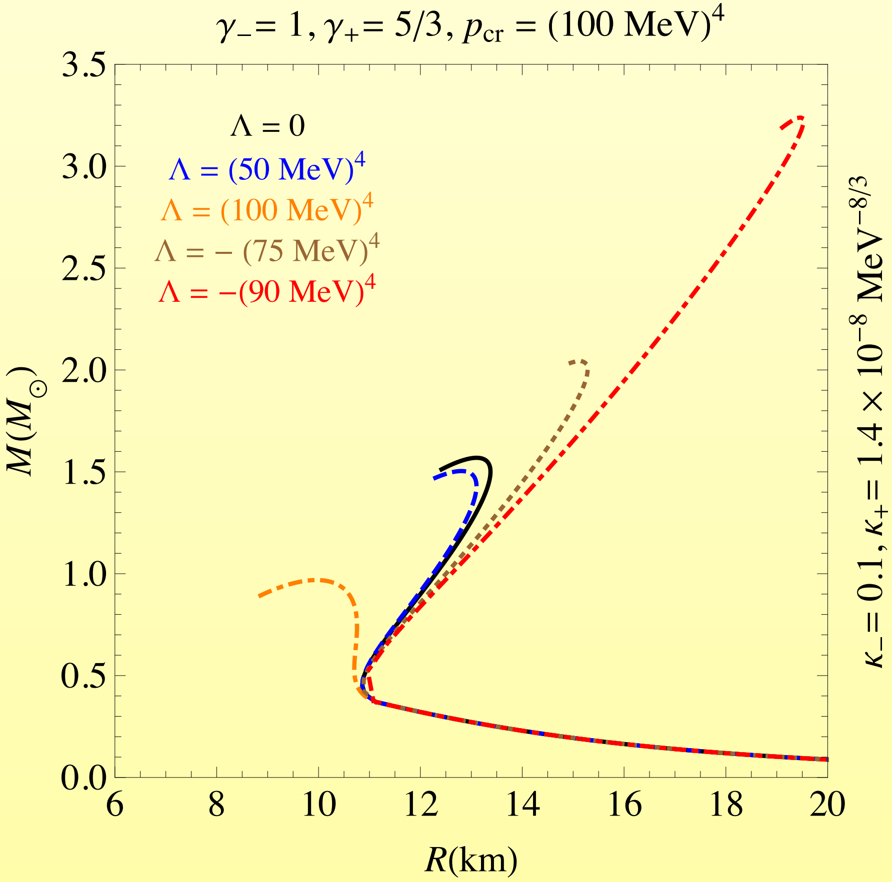
- The value $\gamma_+ = 5/3$ reproduces the small pressure limit of a Fermi fluid

- The cc can not be too large negative: $\Lambda > -p_{cr}$
Otherwise partial pressure of QCD fluid negative

Toy model for neutron stars

- Likely also a thermodynamic upper bound to satisfy $dG = 0$ for Gibbs free energy in equilibrium between phases. Will limit upper value of Λ to few $\cdot 100\ \mathrm{MeV}$

- Checked nicely reproduce the characteristic M(R) curves for neutron stars



Toy model for neutron stars

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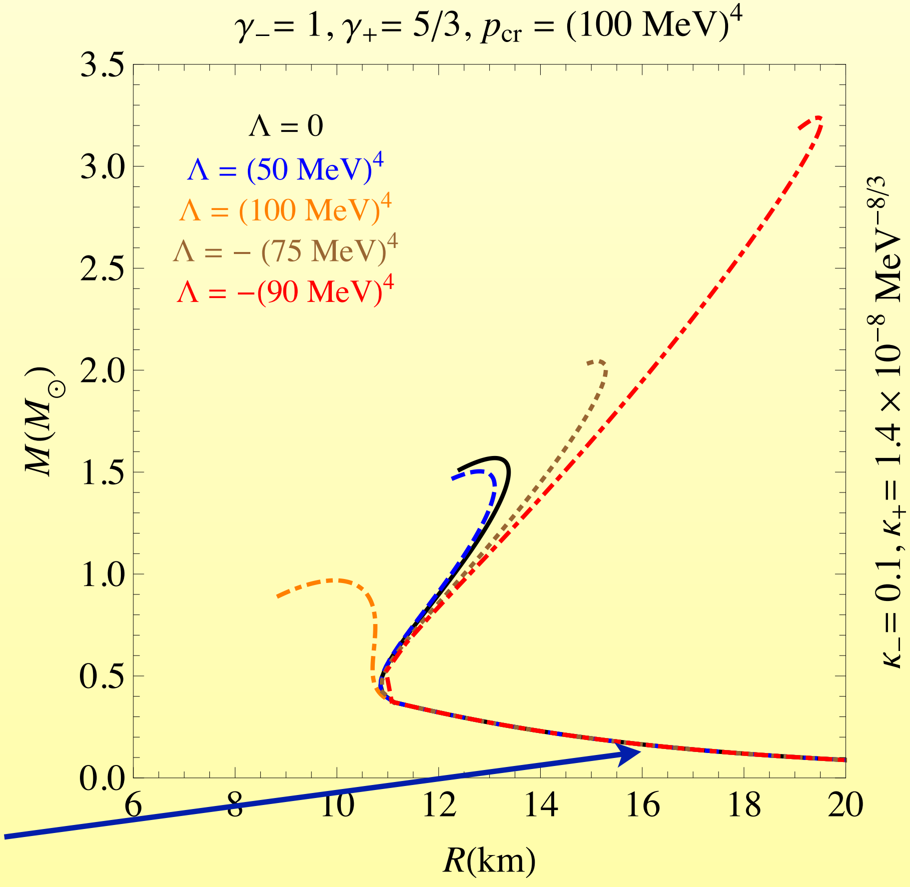
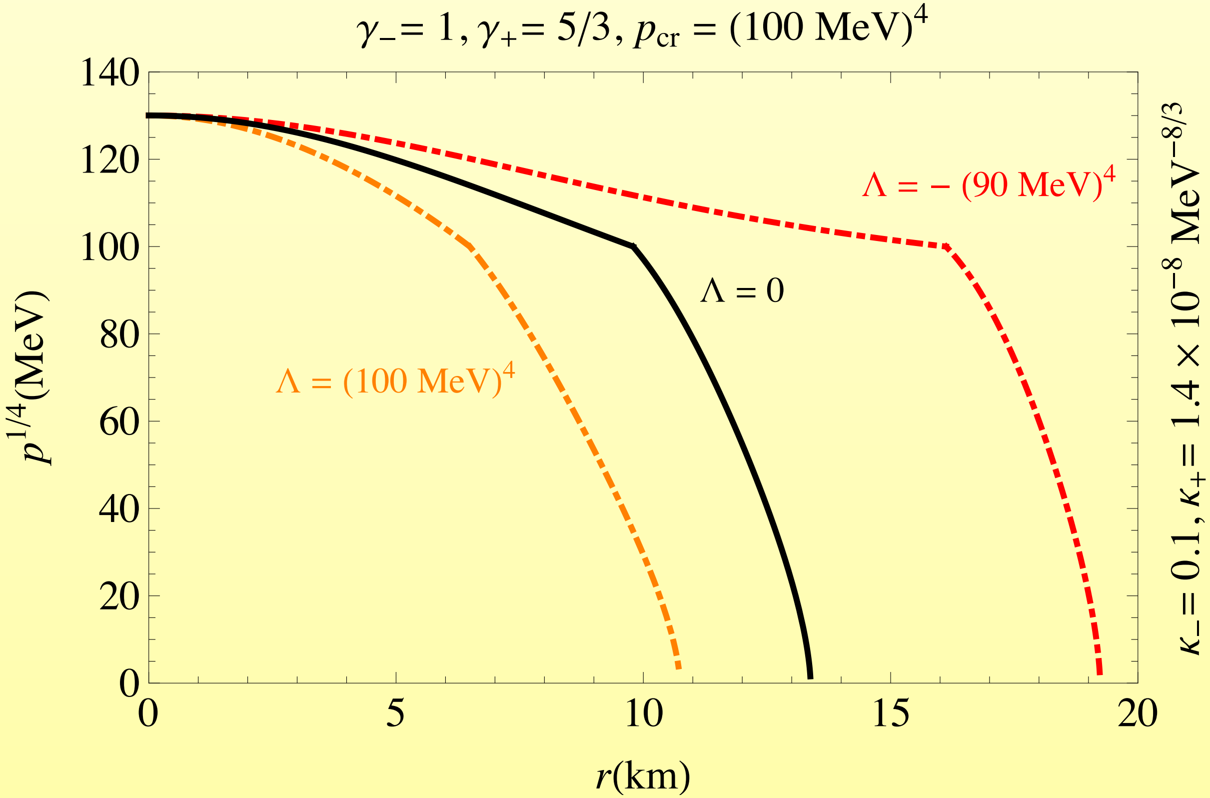


(No inner region here)

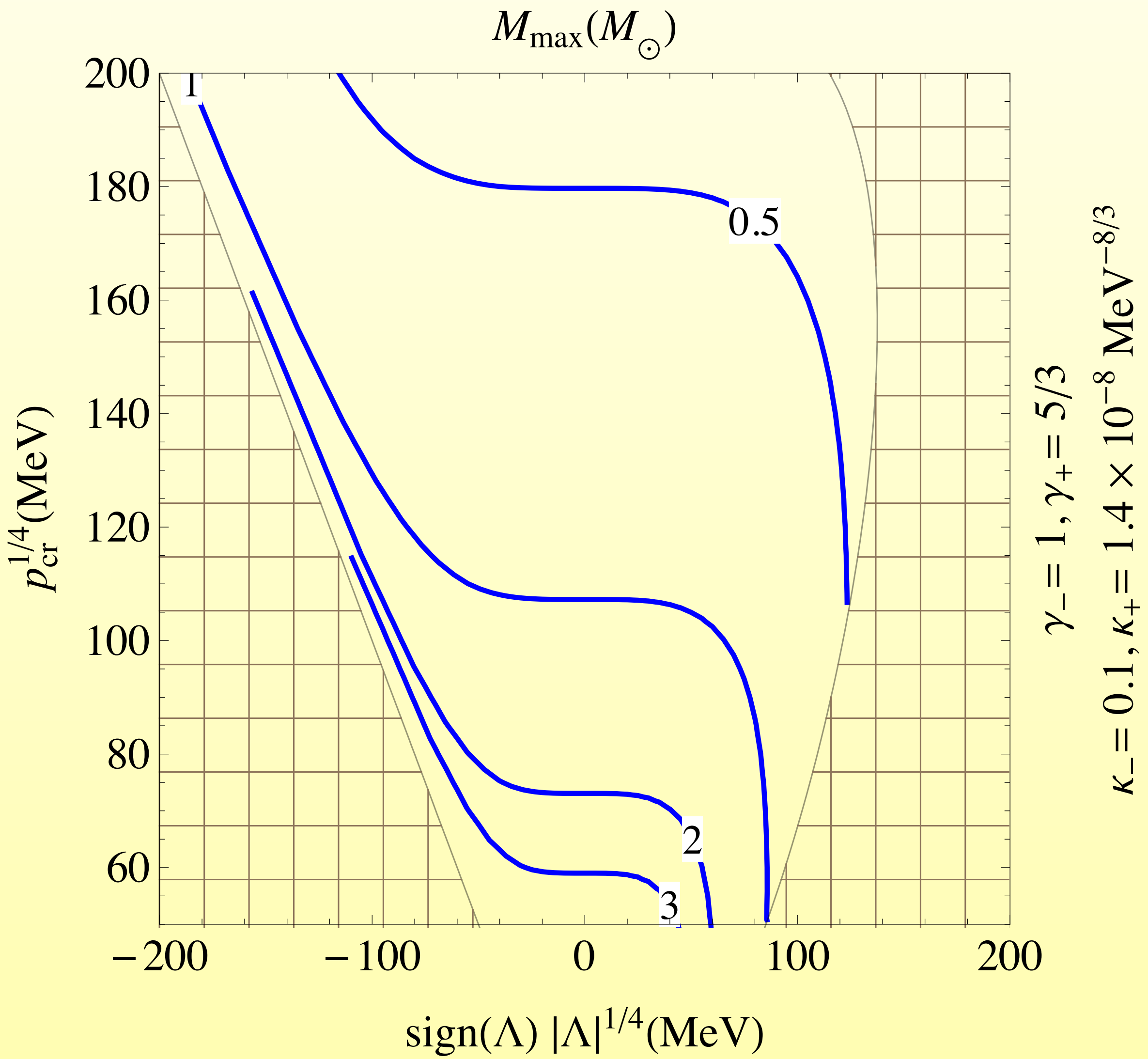
Toy model for neutron stars

- The cross section as function of changing the vacuum energy (while p_{crit} is kept fixed)

- Same central pressure - larger matter pressure - faster evolution - smaller radius

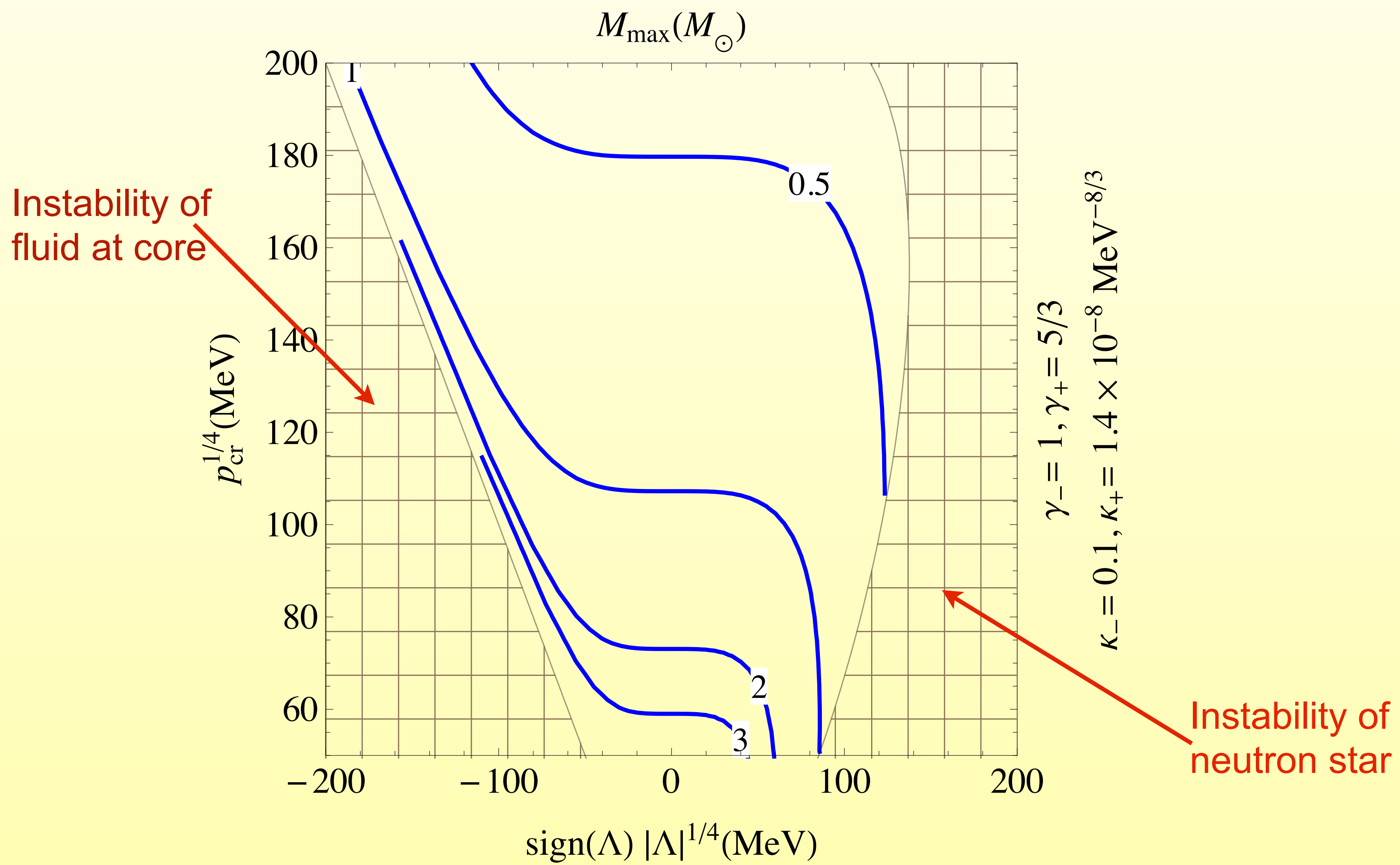


Sensitivities of NS's to vacuum energy



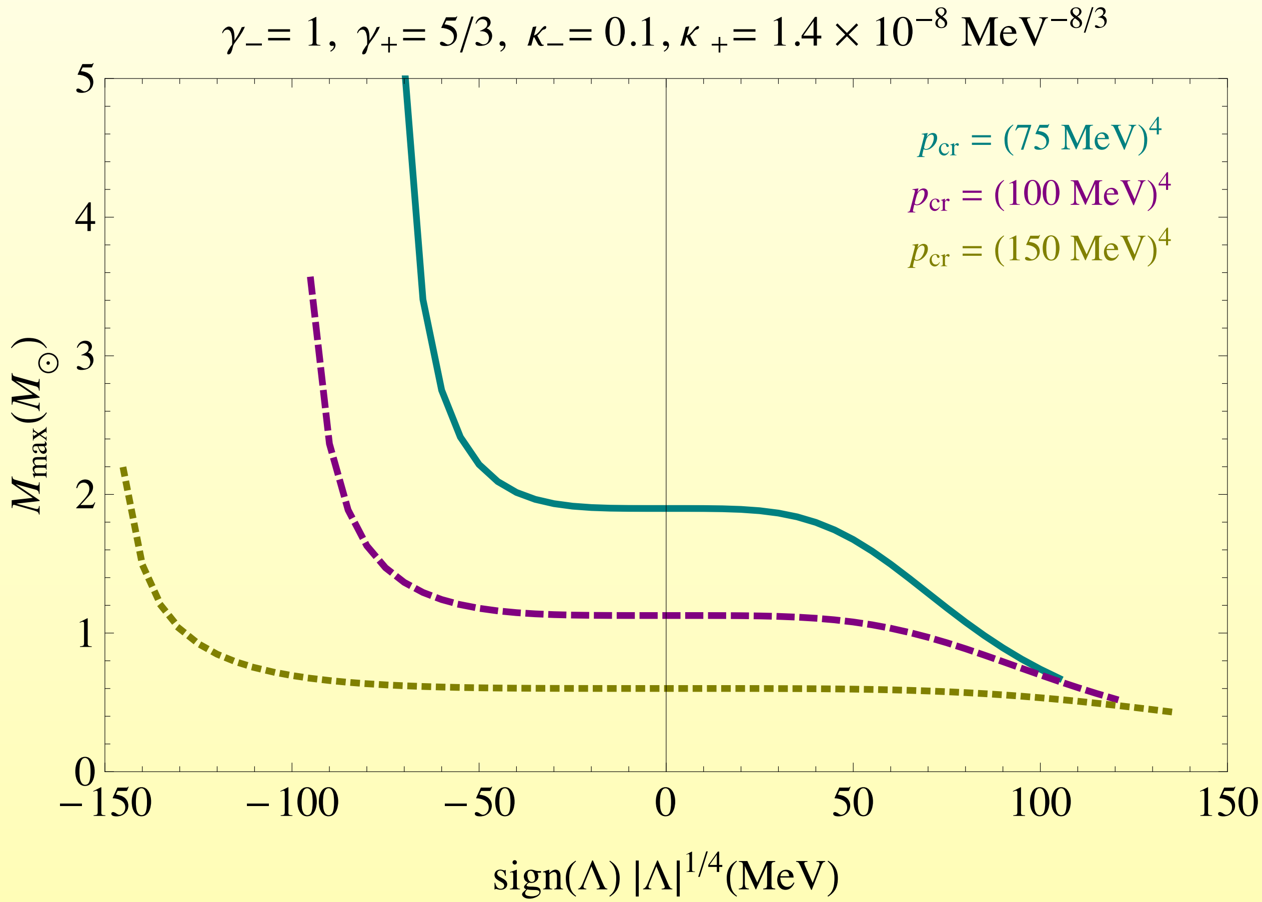
Maximum mass varying Λ and p_{cr}

Sensitivities of NS's to vacuum energy



Maximum mass varying Λ and p_{cr}

Sensitivities of NS's to vacuum energy



Effect on maximal mass by changing Λ for fixed p_{cr}

Sensitivities of NS's to vacuum energy

- Check effect of changing Λ on M(R) curve

- Depending on parameters maximal mass can change significantly

- But depends very strongly on equations of state parameters, critical pressure...

- Status: maximal mass appears to be bigger than $2M_\odot$

- For now radius measurements difficult, only few known from X-ray measurements.