Géodésie chronométrique

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Outline

Some semantics

- Chronometry is the science of the measurement of time
- Chronometric geodesy is sometimes named clock-based geodesy
- Relativistic geodesy is a wider term: it contains all geodesic observables and models (relativistic gravimetry, gradiometry, ...)

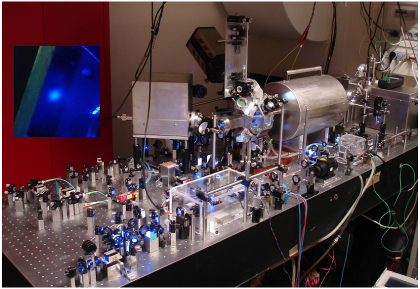


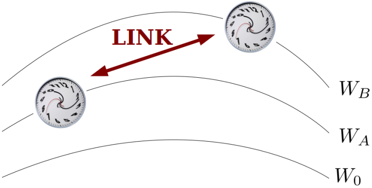
Figure : Strontium clock in SYRTE/Paris Observatory

Basic principle of chronometric geodesy

The flow of time, or the rate of a clock when compared to coordinate time, depends on the velocity of the clock and on the space-time metric (which depends on the mass/energy distribution).

In the weak-field approximation:

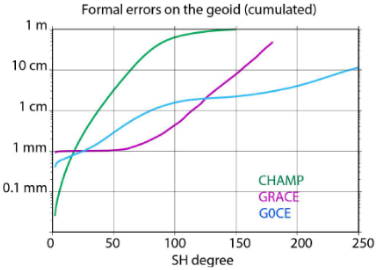
$$\frac{\Delta\tau}{\tau} = \frac{\Delta f}{f} = \frac{U_B - U_A}{c^2} + \frac{v_B^2 - v_A^2}{2c^2} + O(c^{-4})$$
$$= \frac{W_B - W_A}{c^2} + O(c^{-4}) \quad (1)$$

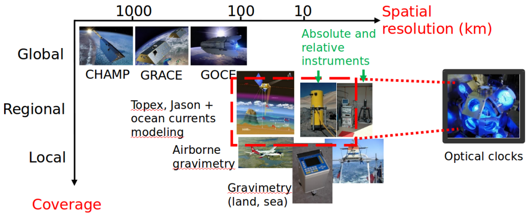


$$1\ \mathrm{cm} \leftrightarrow \frac{\Delta f}{f} \sim 10^{-18} \leftrightarrow \Delta W \sim 0.1\ \mathrm{m^2 s^{-2}}$$

Chronometric observables in geodesy

- Chronometric observables are a completely new type of observable in geodesy: gravity potential differences are directly observed
- Accuracy of optical clocks starts to be competitive with classical methods which have accuracies up to a few centimeters for the static potential at high spatial resolution

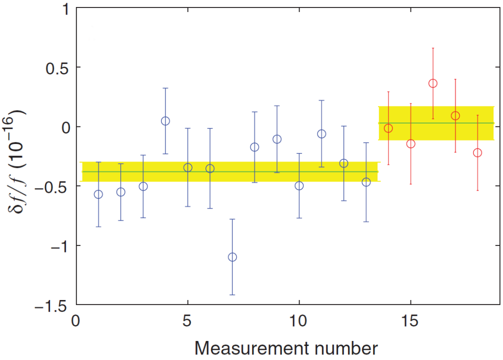




A local comparison

Experimental demonstration of the dependency of clock frequency with local heigth [Chou et al., 2010] with two Al^{+} optical clocks.

Starting at data point 14, one of the clock is elevated by 33 cm. The net relative shift is measured to be $(41 \pm 16) \times 10^{-18}$.



The shape of the Earth

As a proof-of-principle, one can determine (roughly) J_2 with two clocks:

$$\frac{\Delta f}{f} = \frac{W_B - W_A}{c^2} + O(c^{-4}) \ , \ W = U + \frac{v^2}{2}$$

$$U = \frac{GM_E}{r}\left[1 + \frac{J_2 R_E^2}{2r^2}\left(1 - 3\sin(\phi)^2\right)\right]$$

- using INRIM CsF1 vs. SYRTE FO2 comparison we find:

$$J_2 = (1.097 \pm 0.016) \times 10^{-3}$$

- Error of $\sim 1.4\%$ compare to best known value
- However, ground clocks are sensitive to higher order multipoles of the grav. potential

Chronometric levelling

Possibilities for technical realisation of a system for measuring potential differences over intercontinental distances using clock comparisons [Vermeer, 1983]

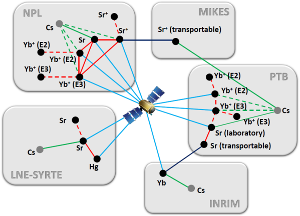
Need accurate clocks

- Hydrogen maser clocks: considered initially, but not accurate
- Cesium clock: accurate by definition but limited to $\sim 1\ m$
- Optical clocks: best knowledge of frequency ratios is needed $\rightarrow$ systematic comparison of optical clocks

EMRP
European Metrology Research Programme
Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

ITOC
International Timescales with Optical Clocks



Main challenge: stable links for frequency comparison

- Satellite (GNSS, TWSTFT): intercontinental but limited to $\sim 10^{-16}$, rather long integration time
- Broadband TWSTFT (ITOC), T2L2 (optical): better stability and faster integration, but still far from what is needed
- ACES Micro-Wave Link (MWL): plan is to achieve 10^{-17} frequency comparisons
- Fibre links: best accuracy ($\sim 10^{-19}$ over thousands of kilometers in just 100 s demonstrated), but limited to continental scales
- Free space coherent optical links through turbulent atmosphere are in their infancy, but show potential for similar performance as fibre links
- Transportable optical clocks are developed (back to the future?)

The chronometric geoid

"The relativistic geoid is the surface where precise clocks run with the same speed and the surface is nearest to mean sea level"
[Bjerhammar, 1985]

- Operational definition based on clock comparisons
- Problem of the realization of the geoid. . .
- . . . solved by the conventions of the IAU in 2000

Isochronometric surfaces

- An isochronometric surface S is a surface where all clocks beat at the same rate:

$$\left.\frac{\mathrm{d}\tau}{\mathrm{d}t}\right|_S = \text{cst}$$

- They are almost equivalent to newtonian equipotential of the gravity field (differences of the order of 2 mm)
- By defining Terrestrial Time (TT) with reference to TCG, the IAU implicitly defined a reference isochronometric surface S_0:

$$\left.\frac{\mathrm{d}\tau}{\mathrm{d}(\mathrm{TCG})}\right|_{S_0} = \text{cst} = 1 - L_G \ , \ L_G = 6.969290134 \times 10^{-10}$$

- The corresponding (newtonian) gravity equipotential is:

$$W_0 \equiv c^2 L_G \simeq 62636856.00 \ \mathrm{m}^2.\mathrm{s}^{-2}$$

- The classical geoid moves away from the reference isochronometric S_0 surface with ~ 2 mm/year speed, i.e. 2×10^{-18} in 10 years

Consortium of the ITOC EMRP project

National Physical Laboratory (NPL, UK)

Cesky Metrologicky Institut (CMI, Czech Republic)

Istituto Nazionale di Ricerca Metrologica (INRIM, Italy)

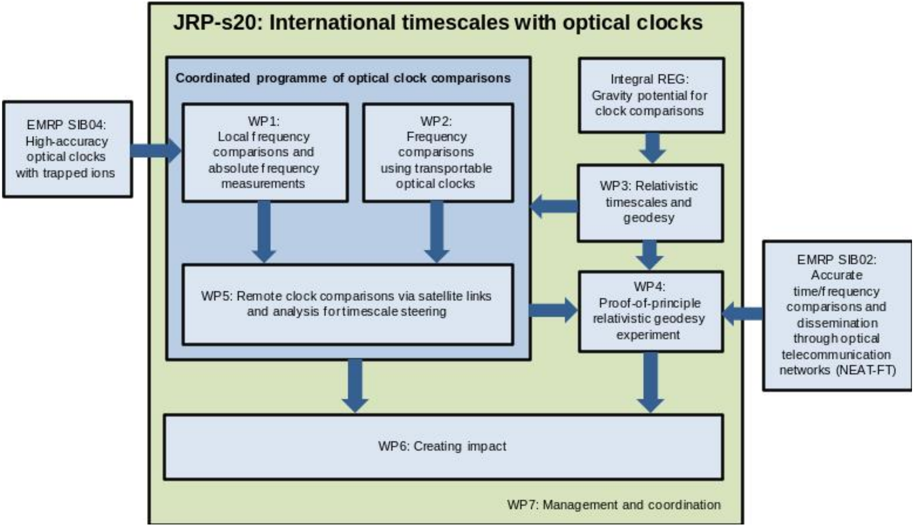
Mittatekniikan Keskus (MIKES, Finland)

Physikalisch-Technische Bundesanstalt (PTB, Germany)

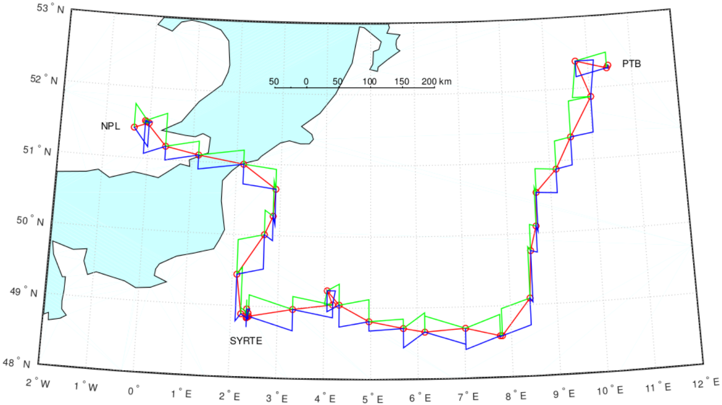
SYRTE – Paris Observatory (France)

IfE – Leibniz Universität Hannover (Germany)

Detailed work plan of the ITOC EMRP project



Relativistic model of the signal propagation in a fibre



fibre link	Length/km	Correction/ps
PTB-SYRTE	1401	3976 ± 27
NPL-SYRTE	813	1214 ± 6

Sagnac delays in the REFIMEVE+ network [Geršl et al., 2015]

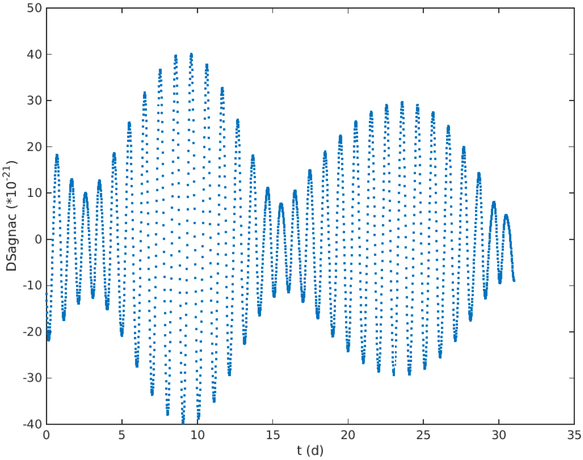
Parameters uncertainty in fibre time transfer

Parameter	Uncertainty
Fibre length (1-way only)	0.2 mm
Refractive index (1-way only)	3×10^{-10}
Fibre endpoints position	200 m
Fibre inner points position	600 m
Fibre velocity in co-rotating frame	9 cm/s
Earth angular velocity	$\sim$ 0.01 % (relative)
Gravitational plus centrifugal potential (1-way only)	$\sim$ 30 % (relative)

Table : Input parameters and their maximal uncertainties sufficient for 1 ps uncertainty in time transfer. The values were obtained for situations where the sensitivity of a correction to a parameter is maximized and they are calculated for 1000 km long fibre [Geršl et al., 2015].

Fibre propagation

The PTB-SYRTE link: Earth rotation signal



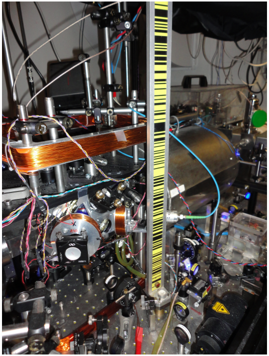
It corresponds to a variation of around 5 fs over 12 hours in the time transfer

Classical levelling of the clocks [Denker, 2013]

- Design of setups to determine the static gravity potential at all clock locations
- Development of a refined European geoid model including new gravity observations around all relevant clock sites (done by IfE/LUH)

SYRTE clocks leveling campaign

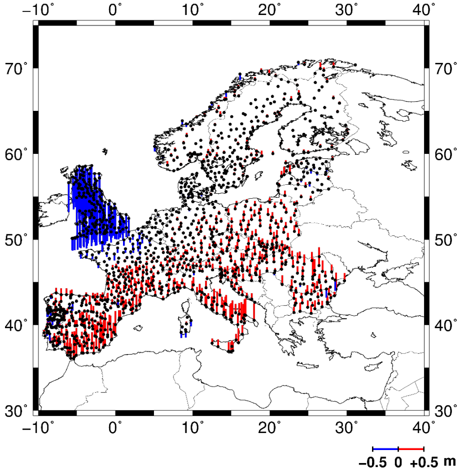
(IGN SGN Travaux Spéciaux)



Differences between GNSS/geoid & geometric levelling approach [Denker, 2015]

$$\Delta C = C^{(\mathrm{GNSS/geoid})} - C^{(\mathrm{lev})}$$

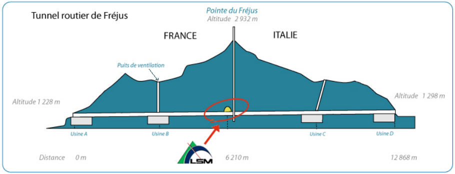
NMI	ΔC (10^{-2} m^2.s^{-2})
INRIM	2.3
LSM	-8.3
NPL	-15.3
OBSPM	-11.3
PTB	-2.3

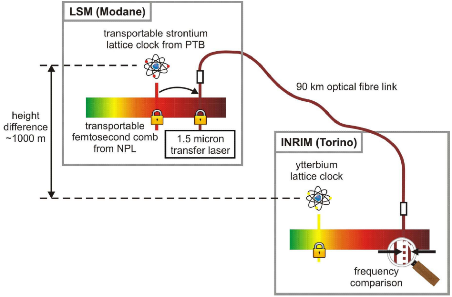


Large-scale demonstration of chronometric geodesy

Demonstrate that optical clocks can be used to measure gravity potential differences over medium-long baselines with high temporal resolution

- Height difference ~ 1 km $\Rightarrow$ Gravitational redshift $\sim 10^{-13}$
- Target $\rightarrow$ resolution of tens of cm in a few hours

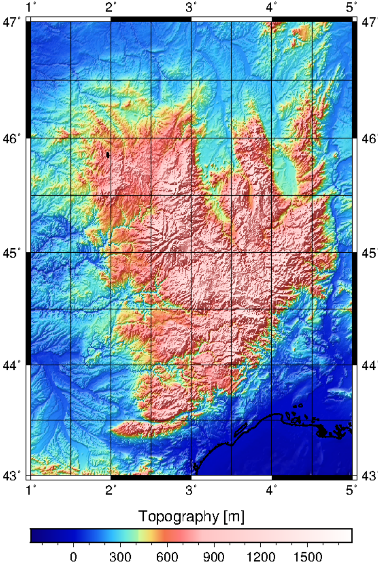


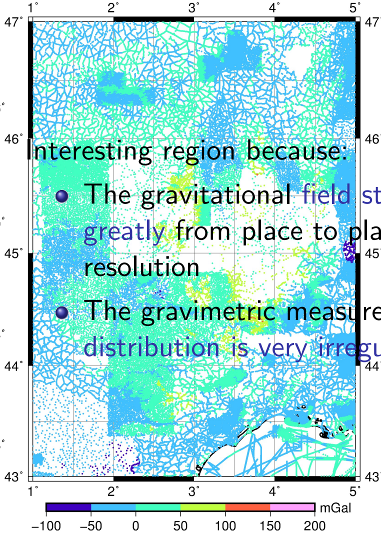


Chronometric geodesy for high resolution geopotential

- Collaboration between SYRTE/Obs.Paris, LAREG/IGN and LKB, with the support of GRAM, First-TF and ERC grants
- Goals
  - evaluating the contribution of optical clocks for the determination of the geopotential at high spatial resolution
  - Find the best locations to put optical clocks to improve the determination of the geopotential
- Results obtained by Guillaume Lion (post-doc)

The Auvergne region in France





Interesting region because:

- The gravitational field strength varies greatly from place to place at high resolution
- The gravimetric measurements distribution is very irregular

Global methodology

STEP 1: build synthetic field model

STEP 2: add simulated noise and choose distribution

STEP 3: Estimation of reference model from simulated measurements

Reference model
gravity anomaly $\delta\bar{g}$
potential anomaly $\delta\bar{T}$

Simulated measurements
δg (gravimeters)
and δT (clocks)

Estimated model
$\delta\tilde{g}$ and $\delta\tilde{T}$

residuals

STEP 1: build synthetic field model

1. Global gravity model at 10 km resolution (EIGEN-6C4, Förste et al. 2014)
2. Removal of low frequencies (covered by satellites)
3. Correction from topography contribution (dV_ELL_RET2012, Claessens & Hirt 2013)

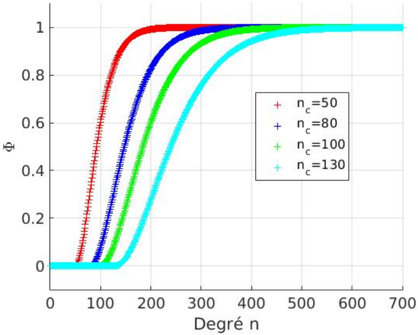


Figure : Filtre based on Poisson wavelets at order 3 (Holschneider et al. 2003)

STEP 1: build synthetic field model

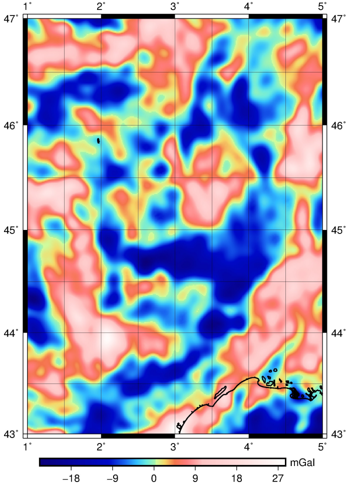


Figure : Reference gravity anomaly

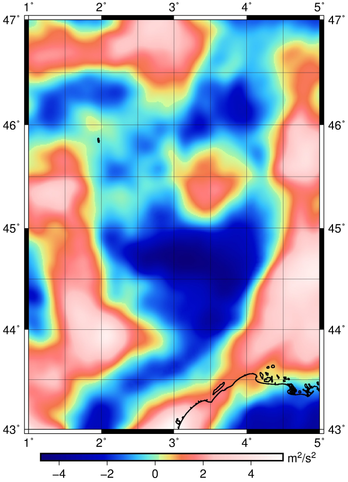
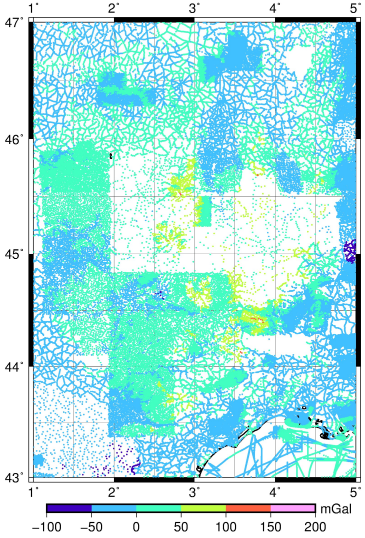
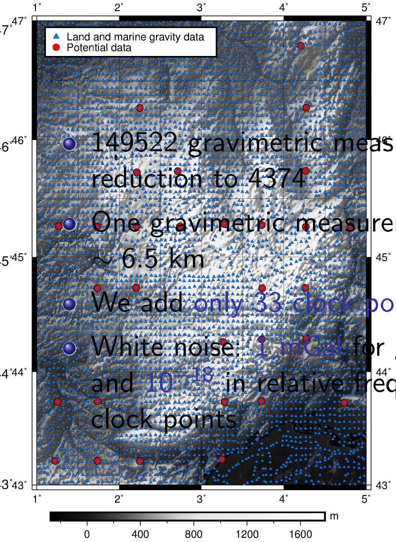


Figure : Reference potential anomaly

STEP 2: add noise and choose observables distribution





- 149522 gravimetric measurements → reduction to 4374
- One gravimetric measurement every ∼ 6.5 km
- We add only 33 clock points
- White noise: 1 mGal for gravimetry and 10^{-18} in relative frequency for clock points

STEP 3: estimation of reference model

1. Prior on field regularity: estimation of a 3D covariance function from the simulated gravimetric measurements
2. Estimation of the potential on a 10x10 km grid with least-squares collocation (Moritz, 1980):
    - from gravimetric data only
    - from gravimetric and clock data

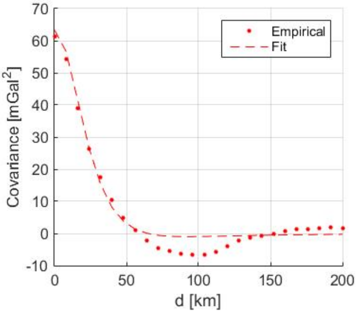
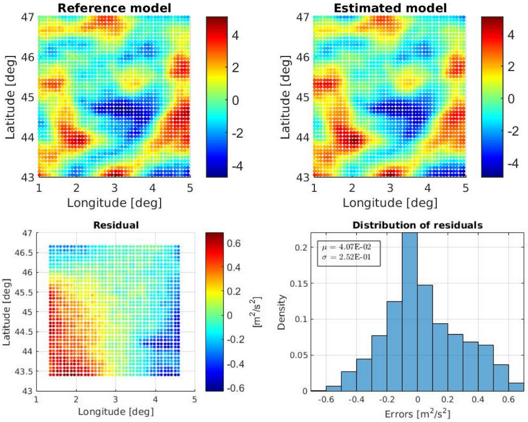


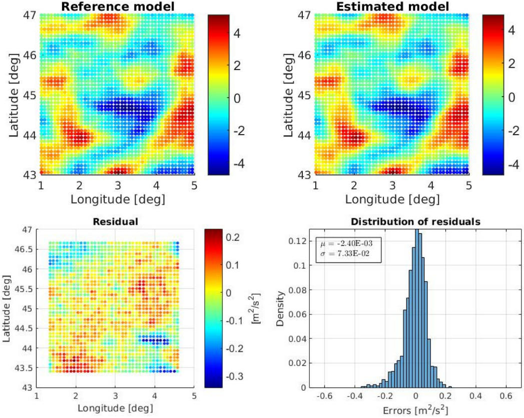
Figure : Fit: logarithmic covariance model by Forsberg (1987); Empirical: empirical covariance. Correlation length is $\sim$ 20 km

Estimation of potential from gravimetric data



- 4374 simulated gravimetric measurements
- Potential anomaly residuals:
  - Standard deviation $\sigma = 0.25\ \mathrm{m}^2.\mathrm{s}^{-2}$ ($\sim$ 2.5 cm on geoid heights)
  - Mean $\mu = -0.04\ \mathrm{m}^2.\mathrm{s}^{-2}$ ($\sim$ 4 mm on geoid heights)
- Trend from West to East of the residuals

Estimation of potential from gravimetric and clock data



- 4374 simulated gravimetric measurements + 32 clock comparisons
- Potential anomaly residuals:
  - Standard deviation $\sigma = 0.07$ m^2.s^{-2} (~ 0.7 cm on geoid heights)
  - Mean $\mu = -0.002$ m^2.s^{-2} (~ 0.2 mm on geoid heights)
- The residual trend disappeared

Conclusion

- Atomic clocks are rapidly improving in accuracy and stability
- Chronometric Geodesy: directly measure gravity potential differences with clock comparisons (accuracy few cm); and variations of gravity potential differences (stability $\sim$1 cm @ 7h)
- The ITOC EMRP project:
  - New model for propagation of signal in an optical fibre [Geršl et al., 2015]
  - New geoid of reference for atomic clocks [Denker, 2015]
  - On-going large scale demonstration of chronometric geodesy
- High resolution potential determination:
  - Only a few clock comparisons can significantly improve the determination of the geopotential at high resolution
  - Improvement from $\sim$ 2.5 cm standard deviation on the geoid heigths to $\sim$ 0.7 cm with only 32 clock comparisons
- Other projects linked to chronometric geodesy: ACES, applications to geophysics

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