Propagation of cosmic rays in the vicinity of their acceleration sites

Nava, Gabici, Marcowith, Morlino, Ptuskin, in preparation

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The context

To study the diffusion of a population of CRs after their escape from the acceleration site

We consider a situation where:

- **the transport of CRs is regulated by the scattering off Alfven waves**
- **a CR of energy *E* resonates with waves of wave number $k = 1/r_L(E)$**
- **quasi linear theory $\delta B/B_0 << 1$**

ambient magnetic field

Energy density *W(k)* of Alfven waves

$$\frac{\delta B^2}{8\pi} = \frac{B_0^2}{8\pi} \int W(k) \mathrm{d} \ln k$$

- perpendicular diffusion coefficient is suppressed
  the problem is one dimensional

Bohm diffusion coefficient

Parallel diffusion coefficient:

$$D = \frac{4\, c\, r_L(E)}{3\pi\, W(k_r)} = \frac{D_B(E)}{W(k_r)}$$

The context

$$W(k, z, t)$$

$$P_{CR}(p, z, t)$$

- the main source of Alfvenic turbulence is the streaming of CRs

Growth of turbulence by CR:
(resonant streaming instability)

$$\Gamma_{growth} = -V_A \frac{\partial P_{CR}}{\partial z} \frac{1}{W}$$

- turbulence damping mechanisms Γ_{damp}

Coupled equations to be solved

$$\frac{\partial P_{CR}}{\partial t} + V_A \frac{\partial P_{CR}}{\partial z} = \frac{\partial}{\partial z}\left(\frac{D_B}{W}\frac{\partial P_{CR}}{\partial z}\right)$$

$$\frac{\partial W}{\partial t} + V_A \frac{\partial W}{\partial z} = (\Gamma_{growth} - \Gamma_{damp})W + Q$$

Skilling 1970

$$\frac{\partial P_{CR}}{\partial t} + V_A \frac{\partial P_{CR}}{\partial z} = \frac{\partial}{\partial z}\cancel{\left(\frac{D_B}{W}\frac{\partial P_{CR}}{\partial z}\right)}$$

$$\cancel{\frac{\partial W}{\partial t}} + \cancel{V_A \frac{\partial W}{\partial z}} = (\Gamma_{growth} - \boxed{\Gamma_{damp}})W + Q$$

waves reach an equilibrium over a time scale much shorter than the CR transport time

slow crossing time: advection term can be neglected

constant in space and time

$$D = \frac{D_B \Gamma_{damp}}{\cancel{Q} - \cancel{V_A \partial P_{CR}/\partial z}}$$

$\boxed{V_A \partial P_{CR}/\partial z \ll Q}$

Level of turbulence determined by equilibrium between external injection and damping:

$W_0 = D_B/D = Q/\Gamma_{damp}$

$P_{CR} \propto t^{-1/2} \exp(-z^2/D_{ISM}\, t)$

Test-particle (TP) case

$\boxed{V_A \partial P_{CR}/\partial z \gg Q}$

Waves grow very quickly: large level of turbulence. CRs are locked to waves and only an unimportant amount of diffusion occurs.

CR-locked case

Ptuskin et al. 2008

$$\frac{\partial P_{CR}}{\partial t} + \cancel{V_A \frac{\partial P_{CR}}{\partial z}} = \frac{\partial}{\partial z}\left(\frac{D_B}{W}\frac{\partial P_{CR}}{\partial z}\right)$$

stationarity

$$\cancel{\frac{\partial W}{\partial t}} + \cancel{V_A \frac{\partial W}{\partial z}} = (\Gamma_{growth} - \boxed{\Gamma_{damp}})W + \cancel{Q}$$

no source of turbulence

slow crossing time

Kolmogorov damping

$$\boxed{\Gamma_{growth} = \Gamma_{damp} = \Gamma_{Kol}}$$

self-similar solution (with a variable $x=z/t^{3/2}$) which describes the non-stationary evolution of the cloud of relativistic particles confined in the magnetic field flux tube

Compared to the ordinary diffusion with constant *D*, the considered non-linear transport is characterized by a relatively slow expansion of the particle distribution around the source

Malkov et al., 2013

$$\boxed{\begin{aligned}\frac{\partial P_{CR}}{\partial t} + \cancel{V_A \frac{\partial P_{CR}}{\partial z}} &= \frac{\partial}{\partial z}\left(\frac{D_B}{W}\frac{\partial P_{CR}}{\partial z}\right)\\ \frac{\partial W}{\partial t} + \cancel{V_A \frac{\partial W}{\partial z}} &= (\Gamma_{growth} - \cancel{\Gamma_{damp}})W + \cancel{Q}\end{aligned}}$$

no source of turbulence

slow crossing time

no wave damping

Method: they solve the two coupled equations and derive an analytic approximated solution

Conclusions: solution depends on two main parameters, W_0 and Π.

Π: field-line-integrated CR pressure

$$\boxed{\Pi = \frac{V_A}{D_B}\int_0^\infty P_{CR}\,dz}$$

- The case $\Pi < 1$ is equivalent to the TP case.
- The case $\Pi > 1$ growth of waves is important

The meaning of Π

$$\Pi = \frac{V_A}{D_B} \Phi_{CR}$$

$$\Phi_{CR} = \int_0^\infty \mathrm{d}z \, P_{CR}$$

Consider the initial setup of the problem: CRs are localized in a small region of size Δz. If the CR pressure within Δz is $P_{CR,0}$ then

$$\Phi_{CR} = P_{CR,0} \, \Delta z$$

growth time: $(V_A / W_0 \, \partial P_{CR} / \partial z)^{-1} \approx W_0 \Delta z / V_A P_{CR,0}$

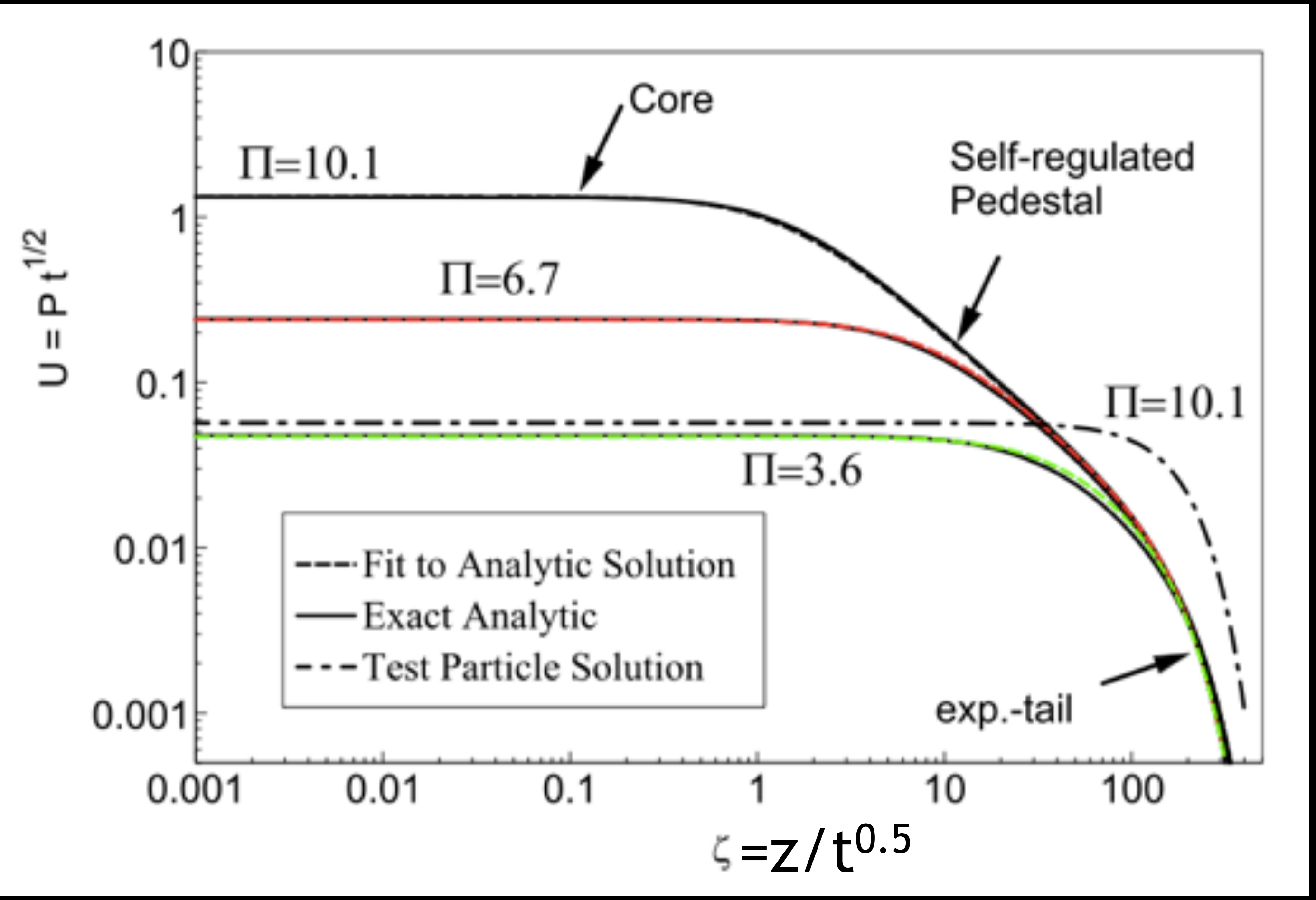
To have a significant growth of waves due to CR streaming, the growth time must be CR shorter than the time it takes the CR cloud to spread due to diffusion

$$\Delta z^2 / D \approx \Delta z^2 W_0 / D_B$$

The initial diffusion coefficient is equal to D_B / W_0.

Such condition can be rewritten as $\Pi > 1$

Malkov et al., 2013



Self-similar solution for the CR pressure for different values of the Π parameter.

Zone 1: Core	*Zone 2: intermediate*	*Zone 3: exponential cutoff*
z < z D_{NL}	z_1	z>z
P_{CR} >>	P	P_{CR}

One major caveat with Malkov's approach:
Π can be in some cases too large and limit the applicability

ISM phases

We consider 2 different ideal phases:

Warm neutral and *Warm ionized* medium

[Jean+09]

phase properties	Warm neutral WNM	Warm ionized WIM
Hydrogen density	0.2-0.5	0.2-0.5
temperature (K)	6000-10000	8000
ionization fraction	0.007-0.05	0.6-0.9
magnetic field (μG)	5	5

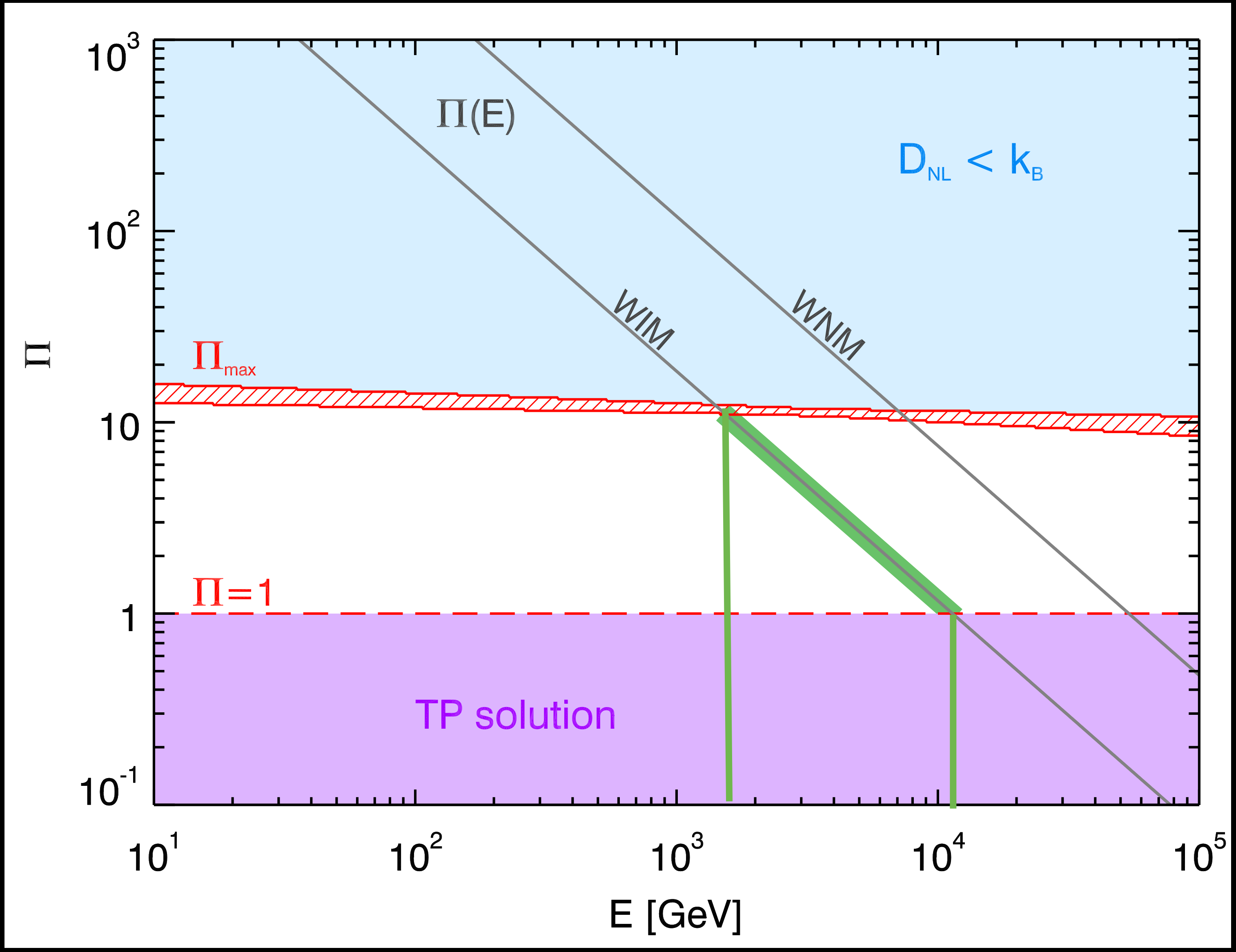
OUR WORK

To quantify the range of applicability of Malkov+13 we explicitly estimate Π

$$\Pi(E; n, R_{esc}, \alpha) \propto E^{1-\alpha} n^{-1/2} R_{esc}^{-2} \xrightarrow{\text{assumptions}} R_{esc} = 20\,\mathrm{pc},\ \alpha = 2.2$$

There is an upper limit to Π: $D_{NL}<D_B$

$$\Pi_{max} = \Pi_{max}(E; D_{ISM}, B) \xrightarrow{\text{assumptions}} D_{ISM} = 10^{28} \left(\frac{E}{10\,GeV}\right)^{0.5} cm^2 s^{-1}$$



Shaded blue:
Not allowed region:
$\Pi > \Pi_{max}$

Hatched red: Maximum Π ($D_{NL}<D_B$) limit of the quasi-linear calculations of Malkov+13.

Shaded purple:
Test Particle solution, no need to apply Malkov+13

$$\frac{\partial P_{CR}}{\partial t} + V_A \frac{\partial P_{CR}}{\partial z} = \frac{\partial}{\partial z}\left(\frac{D_B}{W}\frac{\partial P_{CR}}{\partial z}\right)$$

$$\frac{\partial W}{\partial t} + V_A \frac{\partial W}{\partial z} = (\Gamma_{growth} - \Gamma_{damp})W + Q$$

— Numerical procedure —

Initial conditions:

- $P_{CR}(t=0, z>R_{esc}) = P_{CR,back}$ and $P_{CR}(t=0, z<R_{esc})$ prescribed imposing that 10% of SN energy into CRs
- $D(t=0,z)=D_{ISM}=10^{28}\,[E/10GeV]^{0.5}\ cm^2/s$

Boundary conditions:

- CR and wave fluxes vanish at $z=0$

Solving scheme:

- Explicit finite differences (conditions for accuracy and stability required)

Computing performances:

- Computation time on a standard workstation few minutes/hours depending on the particle energy and spatial resolution

OUR WORK

Turbulence damping processes considered

Non-linear Landau damping (Γ_{NLL}): occurs due to the energy exchange between waves and particles. High-frequency waves are damped by the presence of low-frequency waves and the presence of thermal particles.

$$\Gamma_{NLL} = -\frac{1}{2}\sqrt{\frac{\pi}{2}\frac{k_{boltz}T}{m_p}}\frac{W}{r_L}$$

[Kulsrud 1978; Volk & Cesarsky 1982; Felice & Kulsrud 2001]

Farmer & Goldreich (Γ_{FG}): wave damping by background MHD turbulence.

MHD turbulence act as a damping mechanism for CR-generated waves

$$\Gamma_{FG} = \frac{V_A}{\sqrt{L_{MHD} r_L}}$$

[Yan & Lazarian 2002; Farmer & Goldreich 2004]

Kolmogorov (Γ_{Kol}): Non-linear Kolmogorov-type wave interaction. Energy cascade of Alfvenic waves to large wave numbers is anisotropic: the main part of energy density in this turbulence is concentrated perpendicular to the local B.

$$\Gamma_{Kol} = 0.05\frac{V_A}{r_L}\sqrt{W}$$

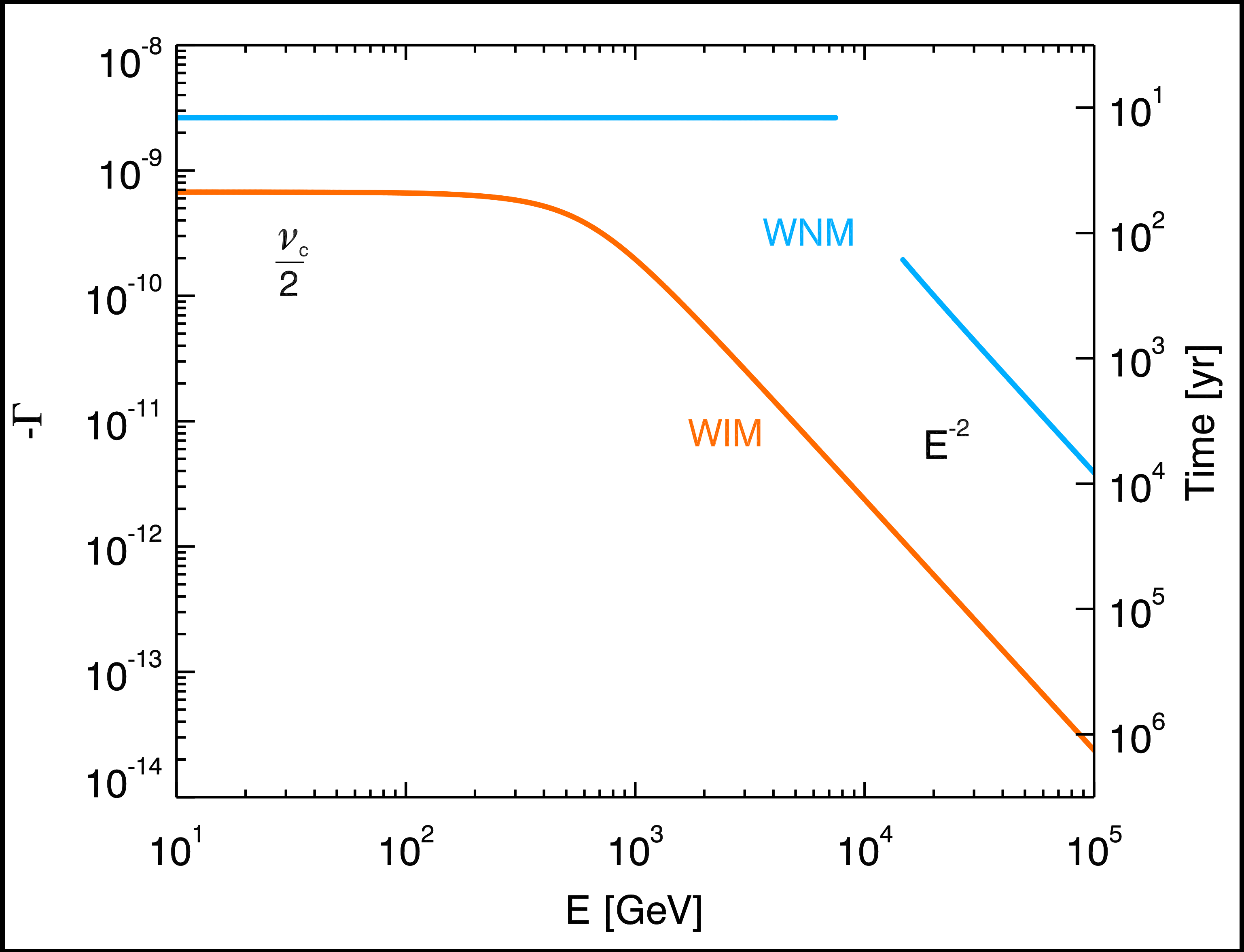
[Ptuskin & Zirakashvili 2003, 2005]

Ion-neutral collisions (Γ_{IN}): momentum-exchanging collisions between ions and neutral particles

[Kulsrud & Pierce'69; Zweibel & Shull'82]

Ion-neutral damping

OUR WORK



collision frequency

$$\nu_c = 1.68 \times 10^{-8} \left(\frac{T}{10^4\,K} \right)^{0.4} n_n\, s^{-1}$$

wave frequency

$$\omega \propto 1/r_L \propto E_{CR}^{-1}$$

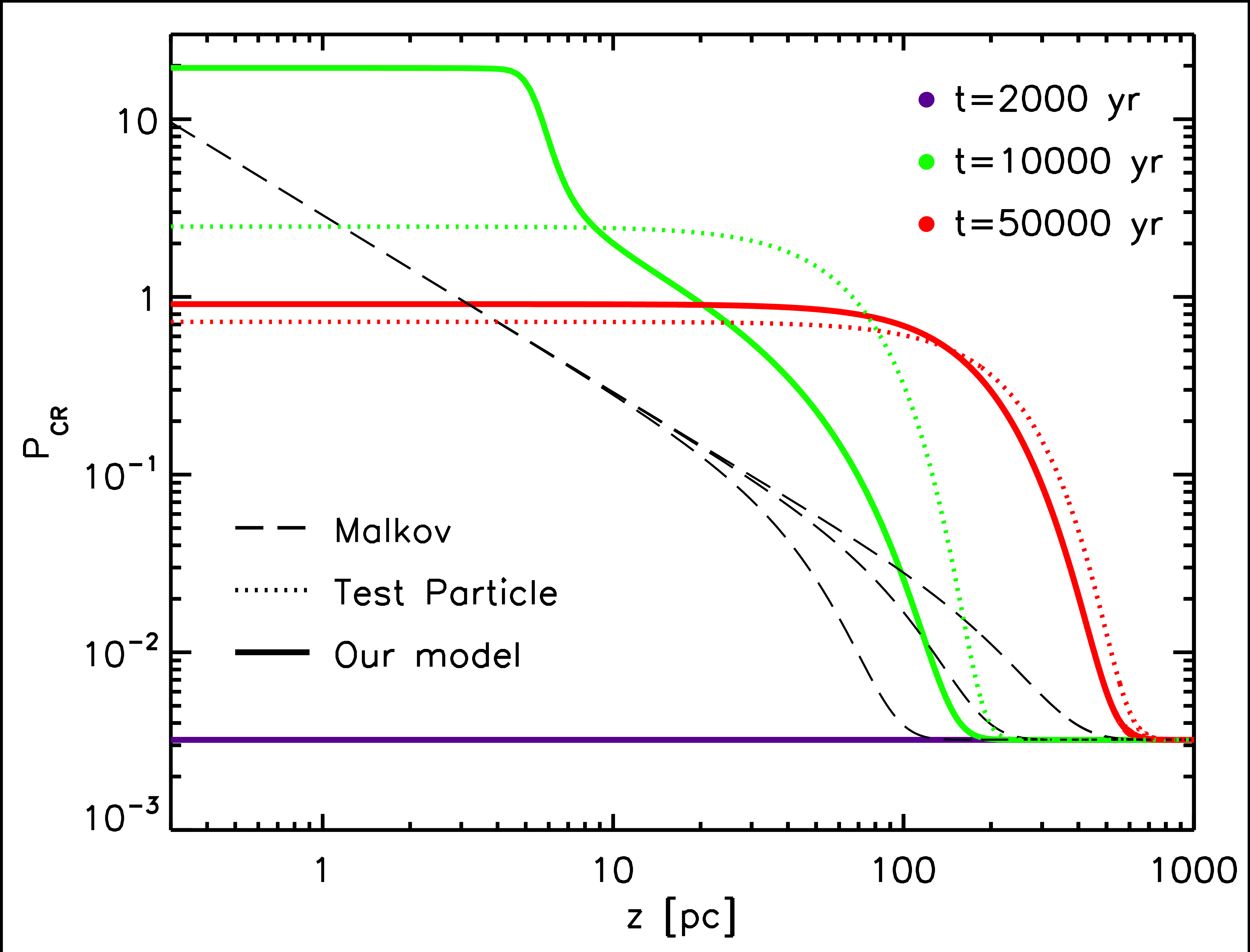
$$\Gamma_{IN} = -\frac{\omega^2}{2\nu_c} \quad \text{for} \quad \omega \ll \nu_c$$

$$\Gamma_{IN} = -\frac{\nu_c}{2} \quad \text{for} \quad \omega \gg \nu_c$$

Frequent collisions reduce the Alfven speed to a value determined by the total mass density instead of the ionized mass density

$$V_A = \frac{B}{\sqrt{4\pi m_p n_i}} \longrightarrow V_A = \frac{B}{\sqrt{4\pi m_p n}}$$

1 TeV — WNM



t=2000 yr
t=10000 yr
t=50000 yr
Malkov
Test Particle
Our model
P_{CR}
z [pc]
10
1
10^{-1}
10^{-2}
10^{-3}
1
10
100
1000

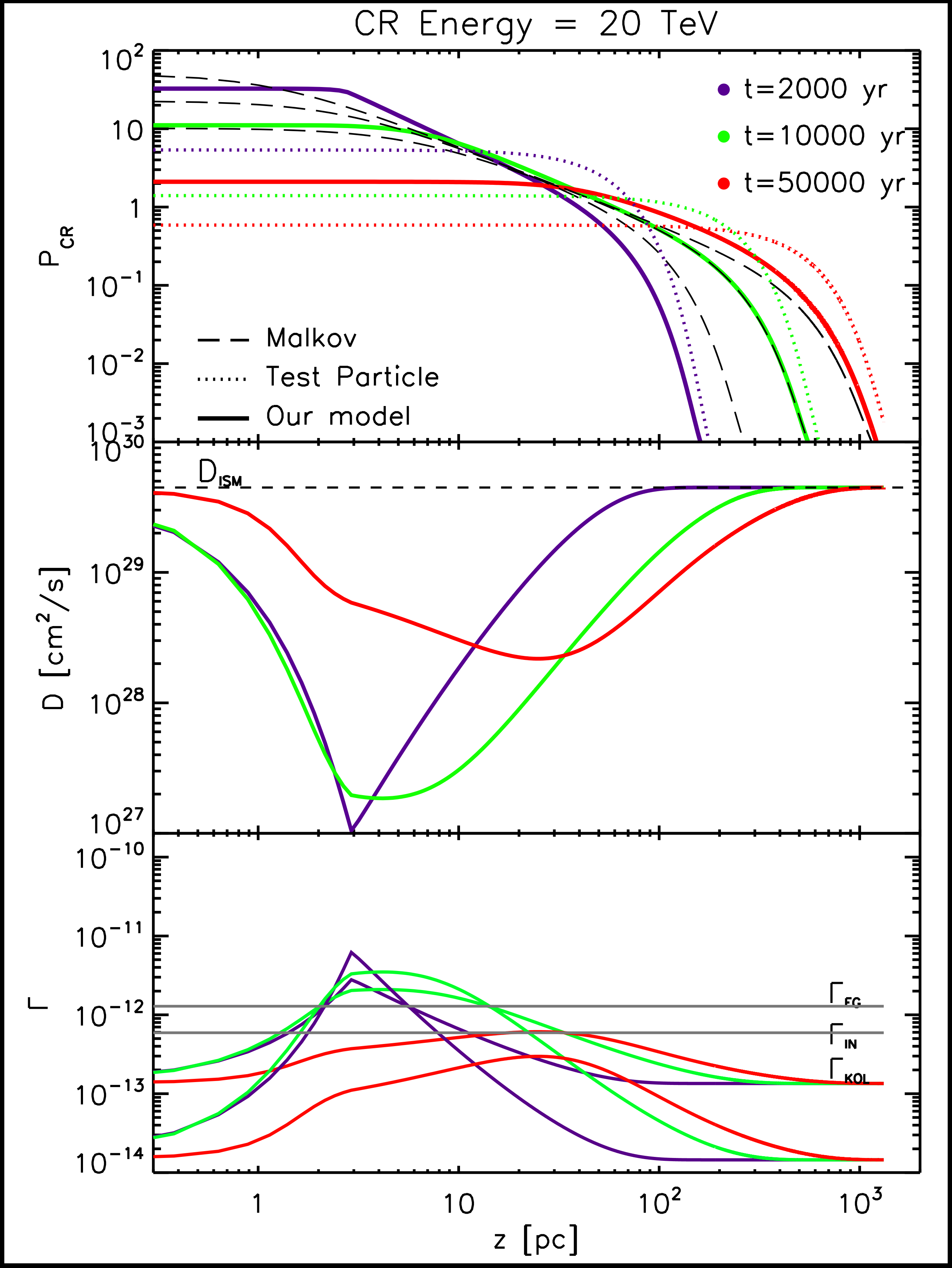
Some preliminary results

Nava, Gabici, Marcowith, Morlino & Ptuskin 14, in preparation

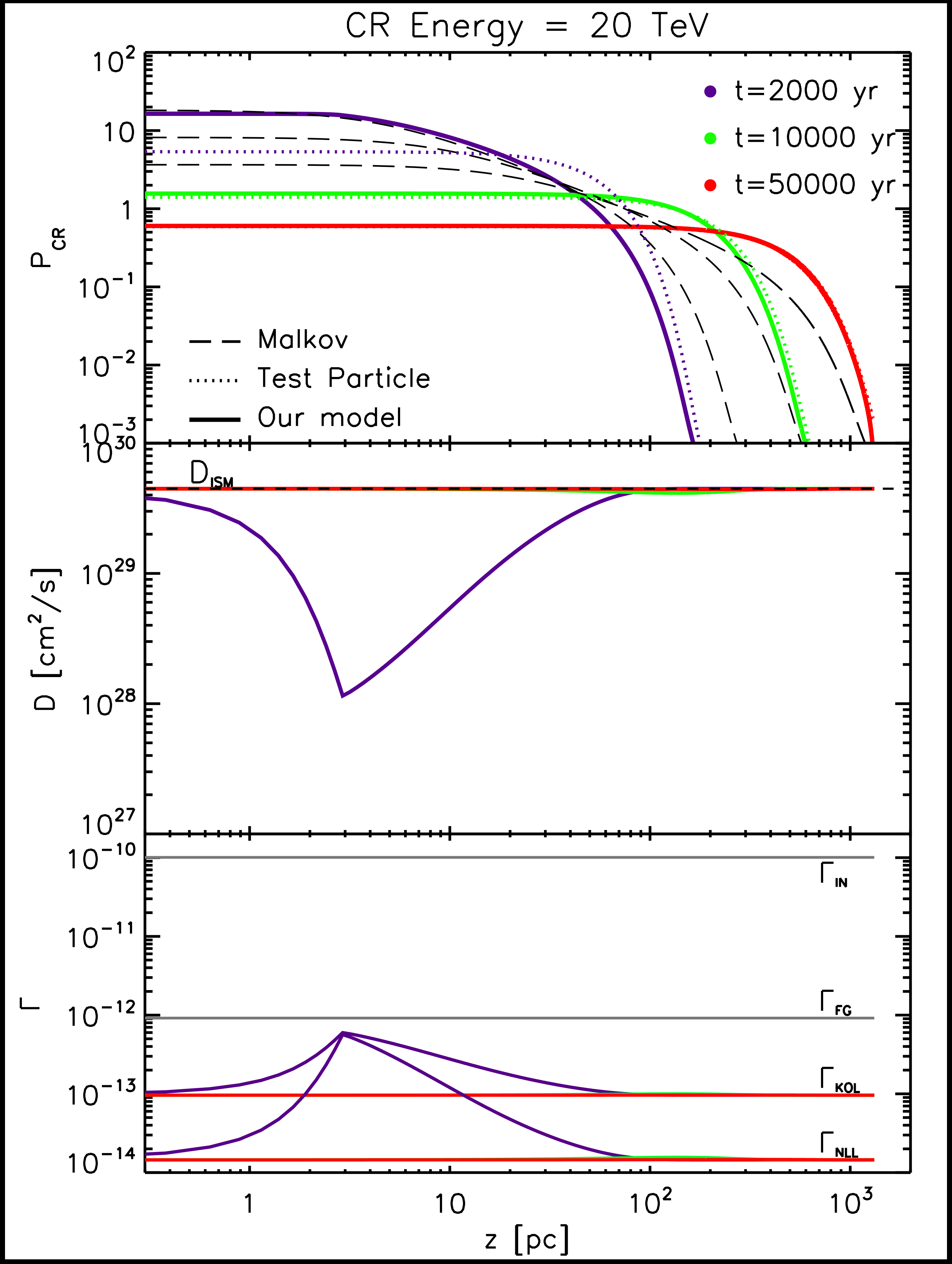
- 2 ISM phases: *WIM* and *WNM*
- 3 Energies: *20 GeV*, *1 TeV*, *20 TeV*
- 3 times: *2 kyr*, *10 kyr*, *50 kyr*

20 TeV

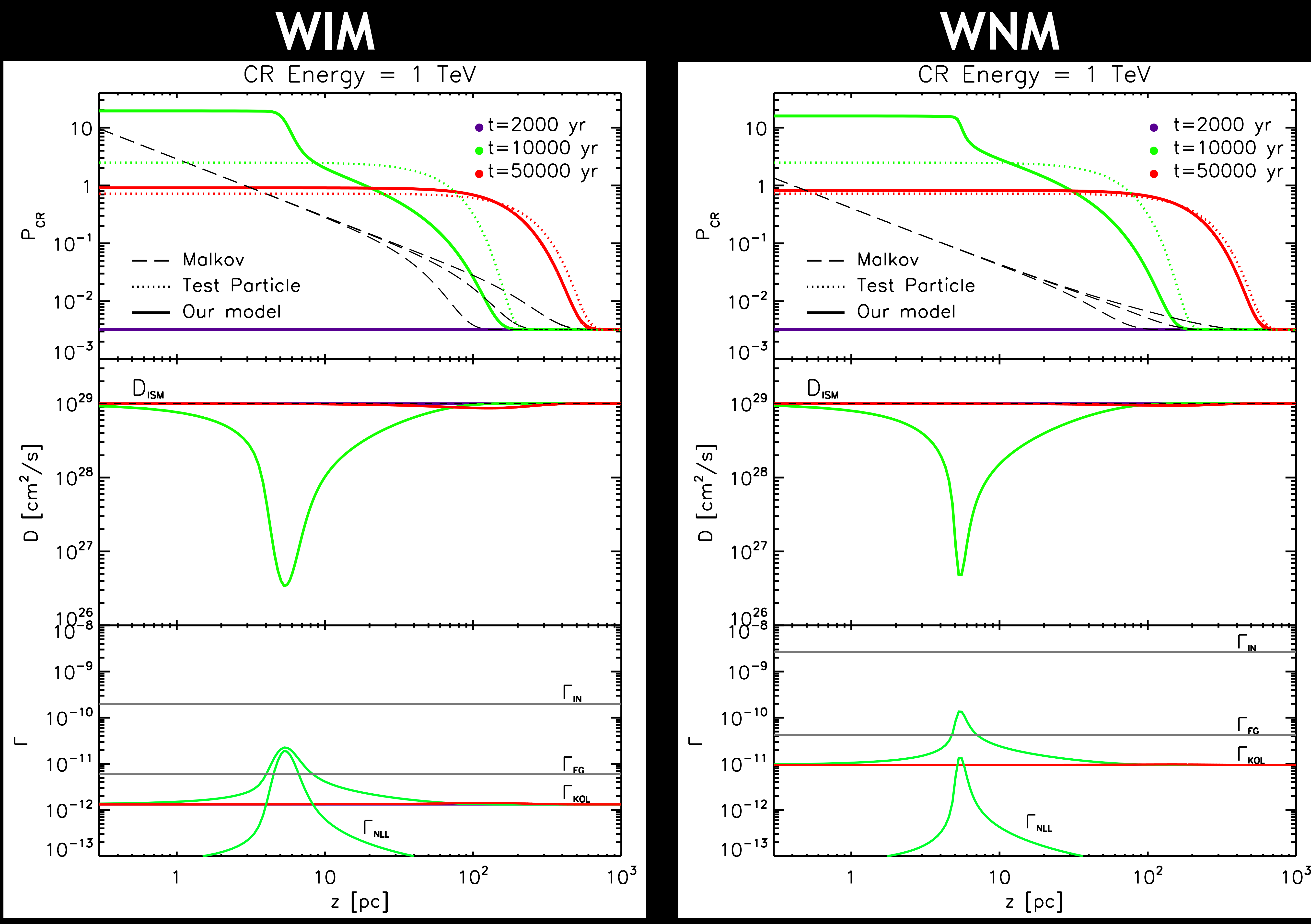
WIM



WNM



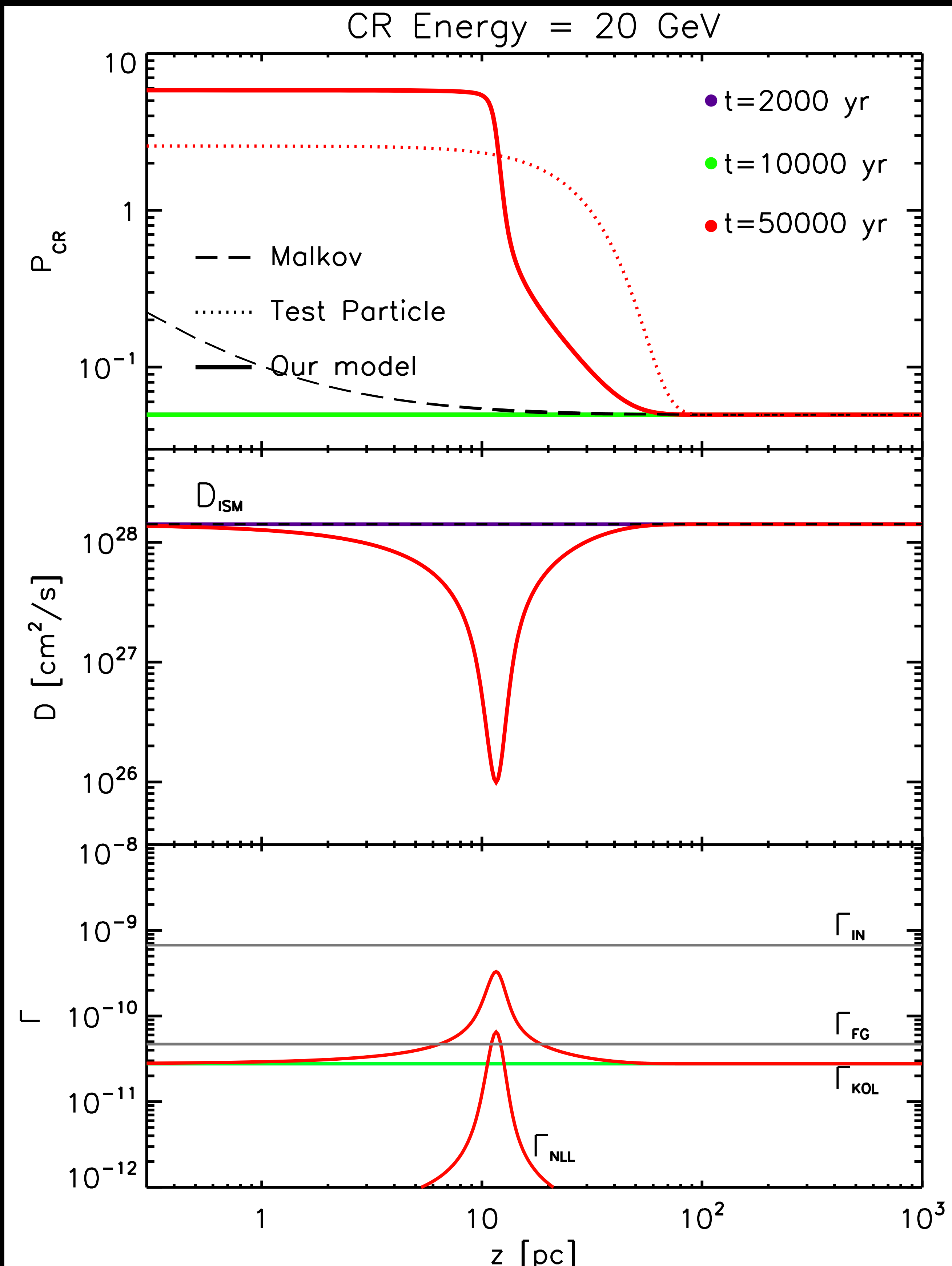
1 TeV



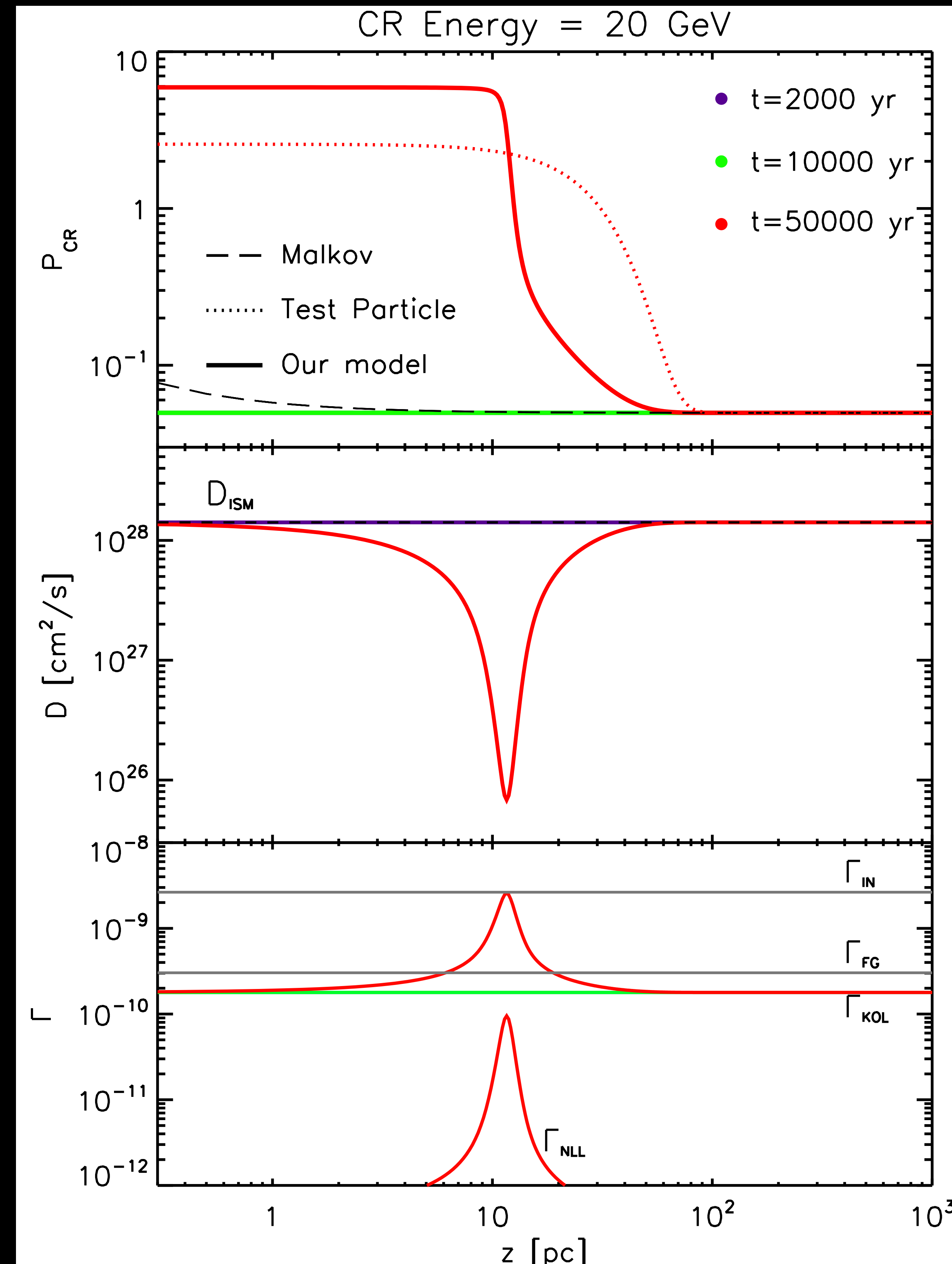
WIM
WNM
CR Energy = 1 TeV
t=2000 yr
t=10000 yr
t=50000 yr
Malkov
Test Particle
Our model
P_{CR}
D_{ISM}
D [cm^2/s]
Γ
Γ_{IN}
Γ_{FG}
Γ_{KOL}
Γ_{NLL}
z [pc]

20 GeV

WIM



WNM



Summary

- Self-consistent solutions of D and P_{CR} in the quasi-linear limit
- Streaming instability as source of turbulence
- Different collisional and collisionless damping
- Two ISM phases: WNM & WIM
- Deviation from the test particle solution at E_{CR}<1 TeV
- Strong self-confinement of CRs of GeV CRs, even at late times

Further developments:

- CR spectra and gamma-ray spectra: constraints from gamma-ray observations, gamma-ray production from clouds