Anisotropic diffusion in the RAMSES code

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Outline

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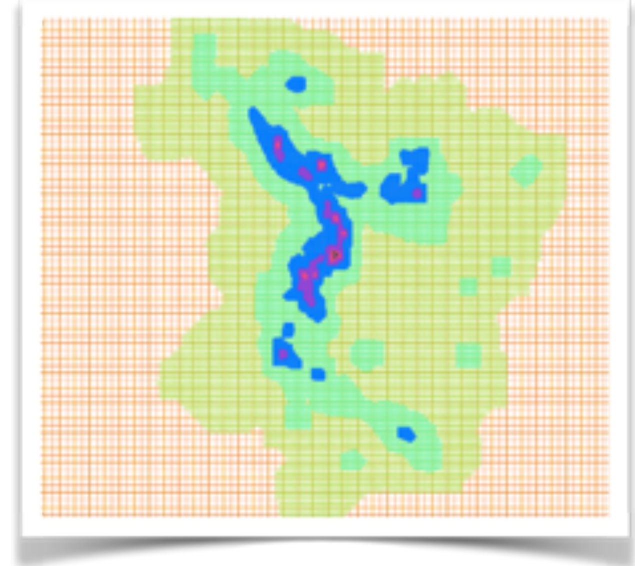
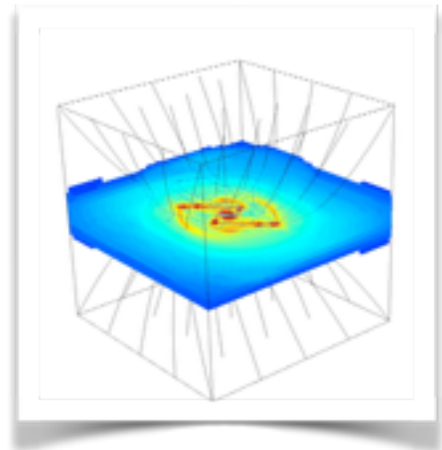
2. **Anisotropic diffusion**
   - Implicit scheme
   - Adaptive time steps
   - Problems
3. **Tests**
   - Shock tube
   - Step function diffusion
   - Diffusion along circular B field

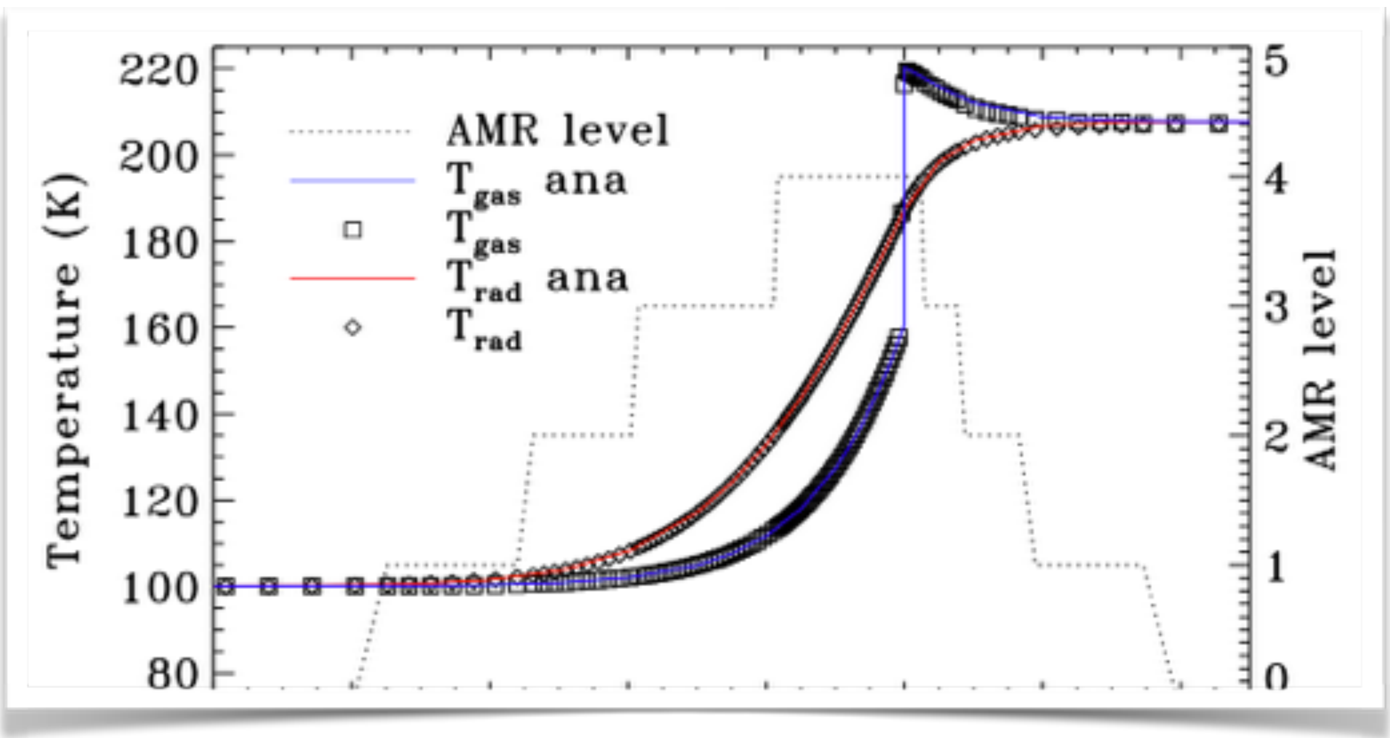
RAMSES AMR code

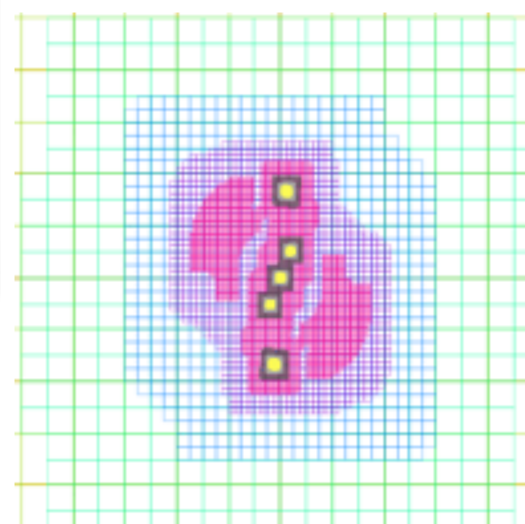
✓ **RAMSES code** *(Teyssier 2002)*

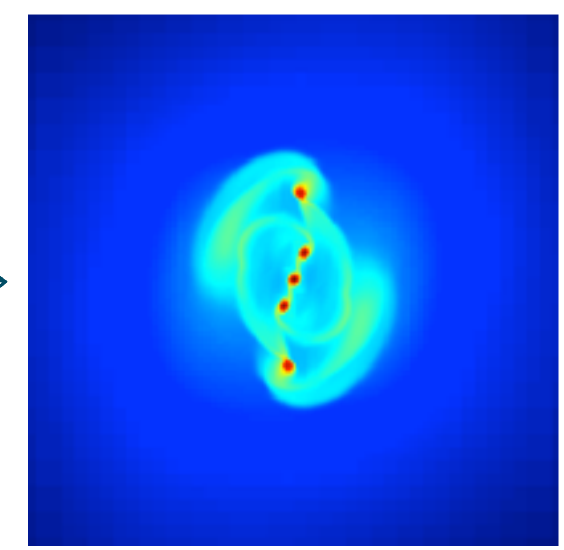
- AMR code, 2nd order Godunov scheme
- non-ideal MHD solver (*Fromang et al. 2006, Masson et al. 2012*)
- Radiation Hydrodynamics solver (*Commerçon et al. 2011, 2014*)
- Refinement criterion based on user defined criteria .e.g. hydro variable gradient, Jeans length, mass, etc…











Anisotropic diffusion in RAMSES

➡ Fluid of gas + cosmic ray with total energy $e = e_{\rm int} + \rho u^2/2 + B^2/2 + e_{\rm cr}$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{u}) = 0\,,$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla.\left(\rho \boldsymbol{u}\boldsymbol{u} + p_{\rm tot} - \frac{\boldsymbol{B}\boldsymbol{B}}{4\pi}\right) = 0\,,$$

$$\frac{\partial e}{\partial t} + \nabla.\left((e + p_{\rm tot})\boldsymbol{u} - \frac{\boldsymbol{B}(\boldsymbol{B}.\boldsymbol{u})}{4\pi}\right) = -\nabla.(-\boldsymbol{\kappa}.\nabla T) - \nabla.(-\boldsymbol{D}_{\bf cr}.\nabla e_{\rm cr})\,,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) = 0\,,$$

$$\frac{\partial e_{\rm cr}}{\partial t} + \nabla.(e_{\rm cr}\boldsymbol{u}) = -p_{\rm cr}\nabla.\boldsymbol{u} - \nabla.(-\boldsymbol{D}_{\bf cr}.\nabla e_{\rm cr})\,,$$

Conservative terms (blue circles)

Source terms (red circles)

➡ modified sound speed $\tilde{c}_{\rm s} = \sqrt{c_{\rm s} + \gamma_{\rm cr}(\gamma_{\rm cr} - 1)e_{\rm cr}}$

➡ anisotropic diffusion tensor…

Implicit integration of CR diffusion equation

$$\frac{\partial e_{\rm cr}}{\partial t} = \nabla \cdot (D_\| \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \cdot \nabla e_{\rm cr}) + \nabla \cdot [D_\perp (\boldsymbol{I} - \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}) \nabla e_{\rm cr}]$$

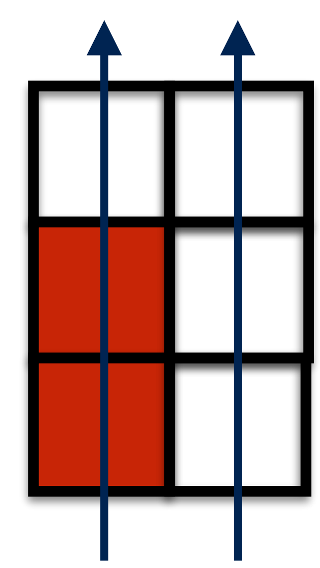
- CFL condition is $\Delta t_{\rm exp} < \frac{\Delta x^2}{2D_\|}$ $\qquad D_\perp = 0.01 D_\|$

✓ *But* implicit scheme is unconditionally stable

➡ **Implicit discretization using a centred symmetric scheme:**

$$e_{i,j}^{n+1} = f(e_{i,j}^{n}, e_{i-1,j}^{n+1}, e_{i+1,j}^{n+1}, e_{i,j-1}^{n+1}, e_{i,j+1}^{n+1})$$

+ easy to solve using conjugate gradient
- does not preserve monotonicity (negative values)



➡ **implicit adaptive time-step** *(Commerçon et al. 2014)*

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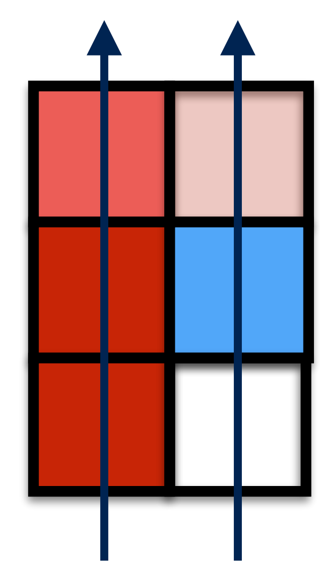

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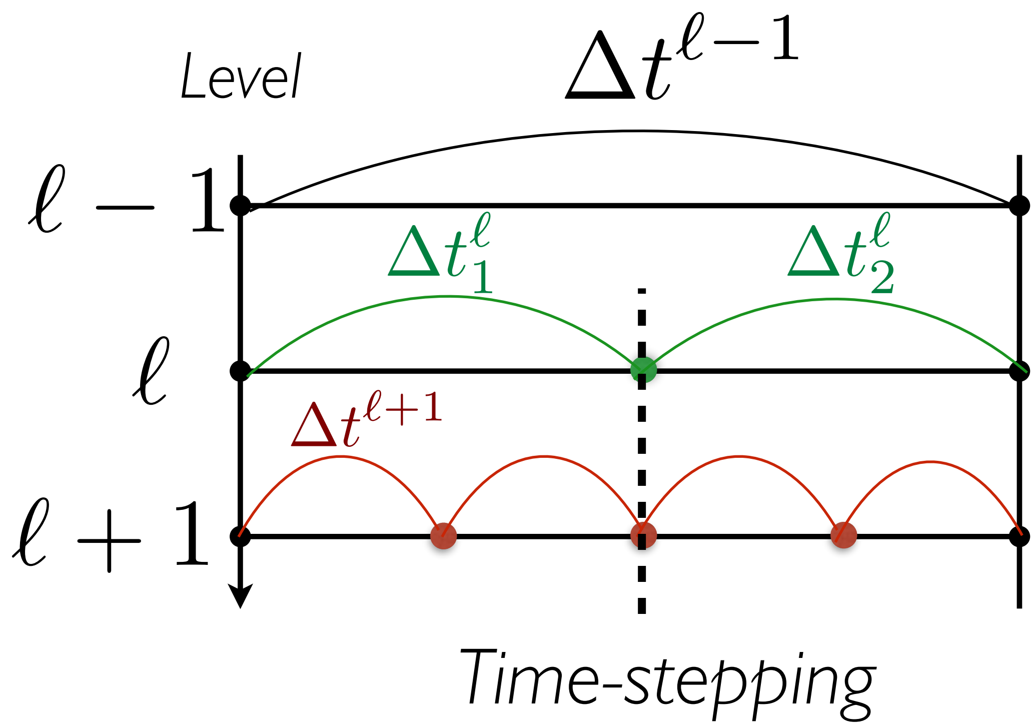
Adaptive time-stepping on AMR grid



Straightforward for explicit scheme at coarse-to-fine interface:

$$F_{i+1/2}^{n+\Delta t^{\ell-1}} = \frac{1}{\Delta t_1^{\ell} + \Delta t_2^{\ell}} \left(\Delta t_1^{\ell} F_{i+1/2}^{n+\Delta t_1^{\ell}} + \Delta t_2^{\ell} F_{i+1/2}^{n+\Delta t_1^{\ell}+\Delta t_2^{\ell}} \right)$$

+ energy is conserved
+ highly efficient for hydrodynamics

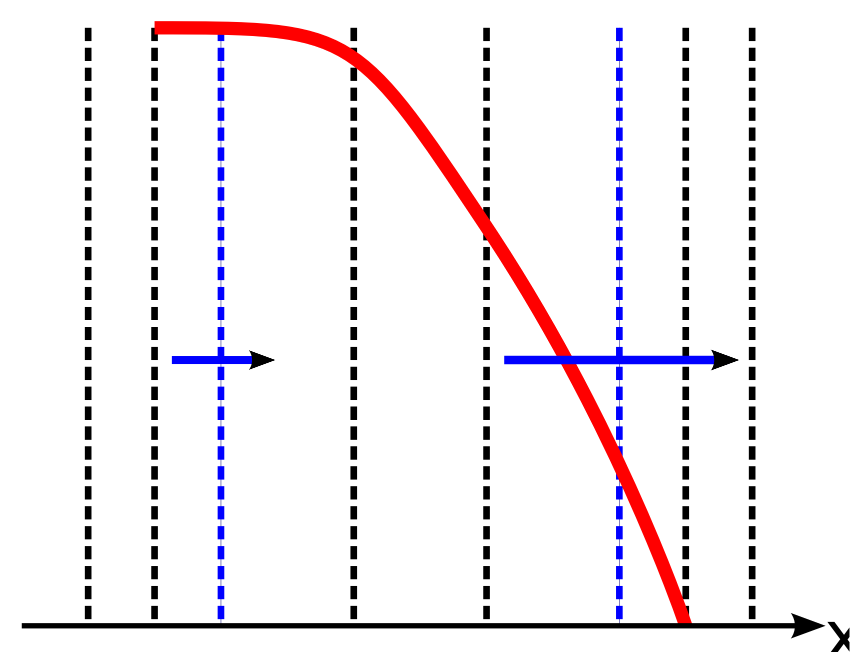
but….

energy does not propagate more than one cell (CFL condition)

=> What happens for implicit schemes when flux are stored?

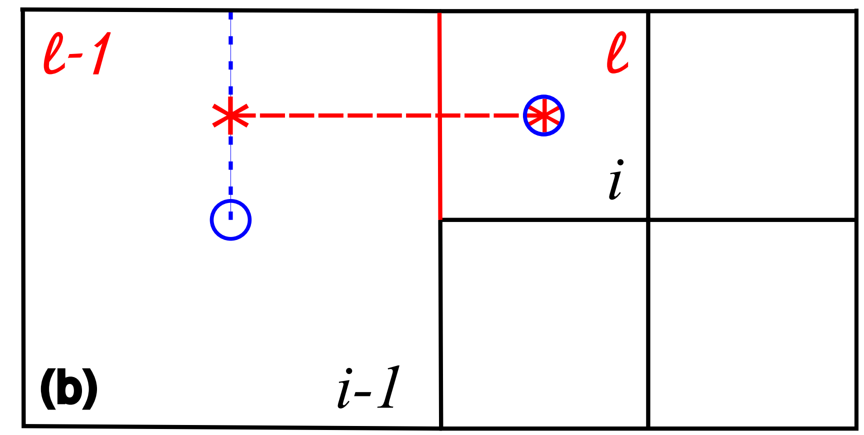
NEGATIVE ENERGY!!!!

again :-(



Grid configuration and boundary conditions

- **Dirichlet:** imposed boundary value ($e_{cr}=e_{cr,b}$)
  - ➡ robust , but energy is not conserved
- **Neumann:** imposed flux condition ($\mathbf{F}_{cr}=\mathbf{F}_{cr,b}$)
  - ➡ energy is conserved (e.g. *Howell & Greenhough 2003*)
- **Robin:** mix between Dirichlet and Neumann, weighted by a parameter α

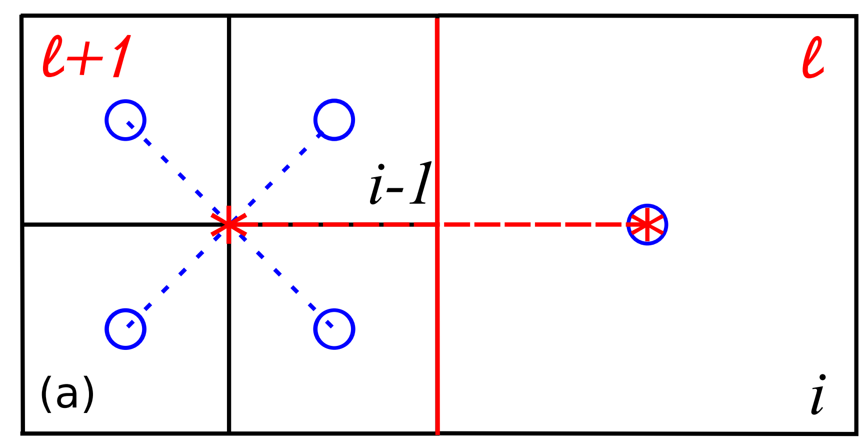


Neighbor is less refined

- **Fine-to-coarse interface:** Dirichlet BC

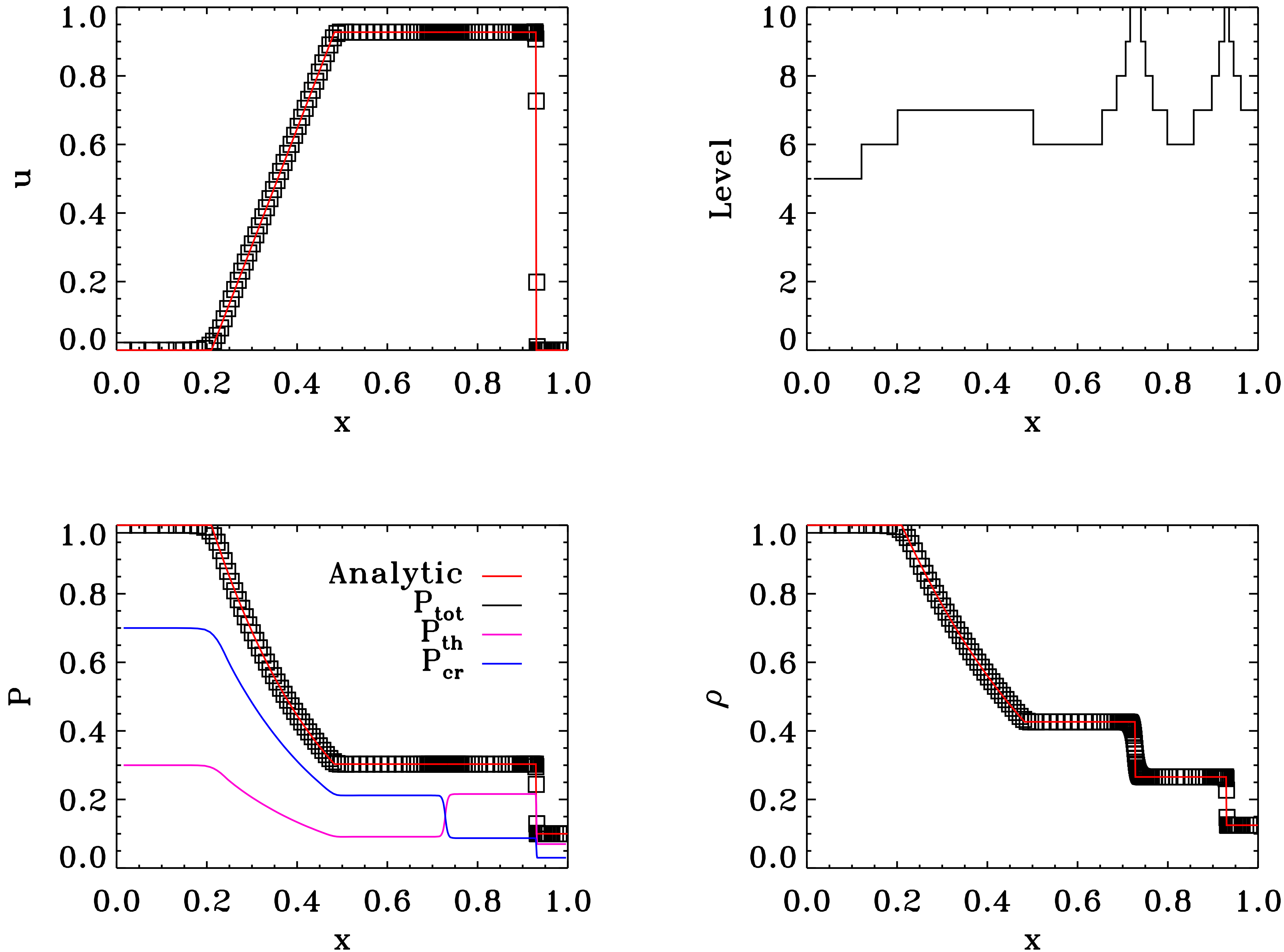
$$\tilde{F}_{i-1/2} = -K_{i-1/2}\frac{E^{n+1}_{\mathrm{r},i} - E^{n}_{\mathrm{r},i-1}}{\frac{3}{2}\Delta x}$$

- **Coarse-to-fine:** 3 possibilities
  Dirichlet BC

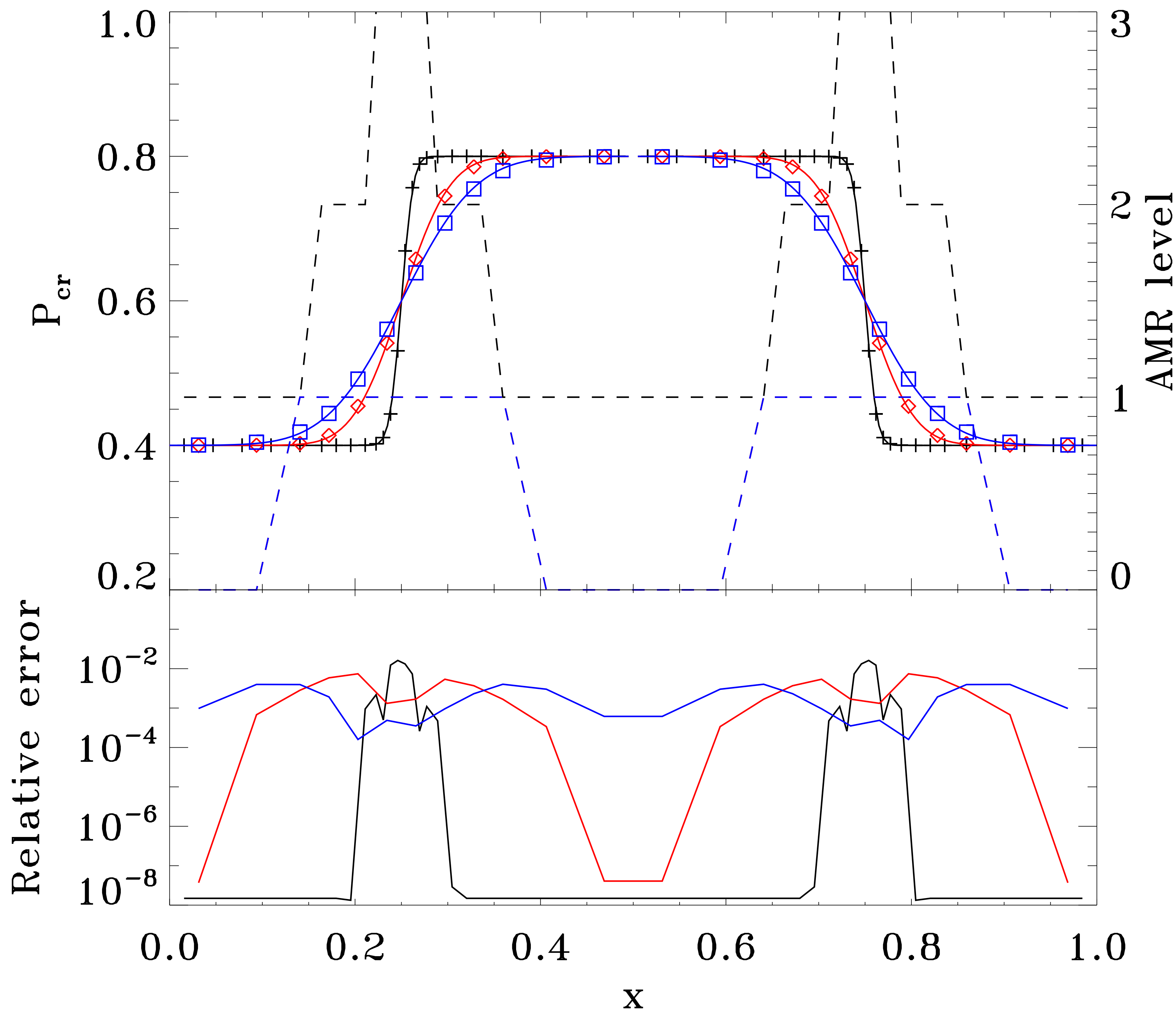


Neighbor is more refined

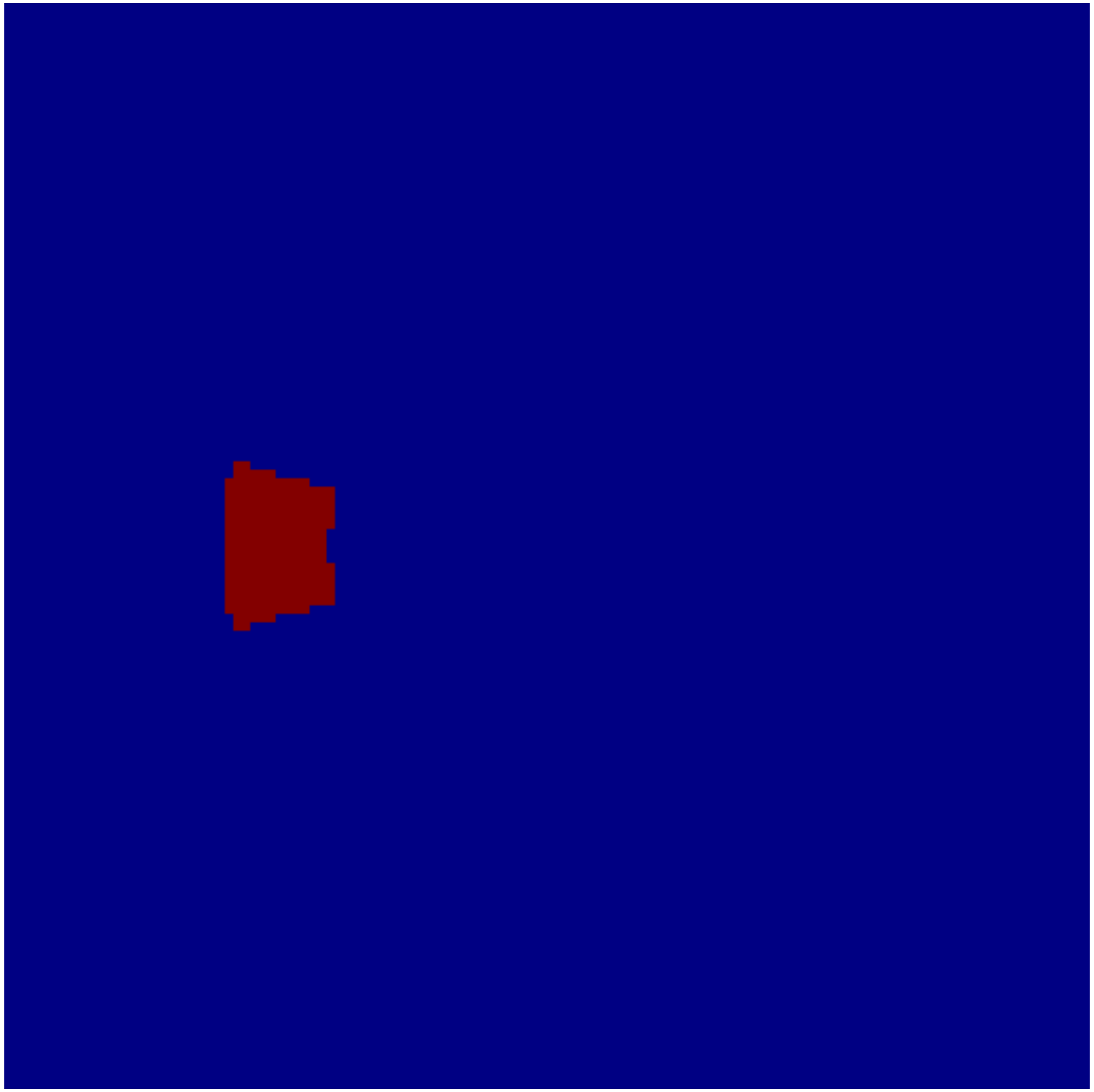
Test 1d: shock tube



Test 1d: diffusion of a step function



Test 2d: diffusion along circular B field



Energy diffusion along circular B field

Conclusion

- ☑ **Implicit Adaptive Time steps** using Dirichlet BC at level interface is simple, fast, and robust
- ☑ **CR diffusion and anisotropic conduction**
- ☑ New solver combining conservative part (classical Godunov explicit scheme) and diffusion using implicit scheme

➡ **more tests?**

➡ **applications!**

➡ **negative energy problem? slope limiters (BiCG)?**

➡ **extension to multiple CR energy bins, mirroring, heating**