Component separation forecasts

- Tucci et al., MNRAS (2005)
  - Assume templates of synchrotron and dust at 60 and 140 GHz, CMB measured at 100 GHz
  - For a full sky coverage and a degree resolution, we obtain a limit of **$r \approx 10^{-4}$**. This value can be improved by high resolution experiments and, in principle, no clear fundamental limit on the detectability of gravitational wave polarisation is found.

- Amblard, Cooray & Kaplinghat, Phys. Rev. D (2007)
  - The optimal configuration requires at least 5 channels between 30 GHz and 500 GHz with substantial coverage around 150 GHz. With a low-resolution space experiment using 1000 detectors to reach a noise level of about 1000 nK^2 on roughly 66% of the sky the minimum detectable level is **$r \approx 0.002$** at the 99% CL for $\tau \approx 0.1$.

- Betoule et al., A&A (2009)
  - With a future dedicated space experiment, such as EPIC, we can measure $r = 0.001$ at ~6σ for the most ambitious mission designs (i.e. EPIC-CS, 8 channels, sensitivity of order 3 μK.arcmin for cosmological channels).

Component separation forecasts

- Dunkley et al., AIPC (2009)
  - The prospects for detecting an r = 0. 01 signal including degree-scale measurements appear promising, with **$\sigma_r \approx$ 0. 003** forecast from multiple methods.

Method	Average dust pol fraction (%)	Description	$\ell < 15$	$\ell < 150$
Fisher	0	No foregrounds	0.0015	0.00046
Fisher	5	10% residual	0.014	0.00052
Parametric	1	Fixed spectral indices	0.0015	0.0005
Parametric	1	Power-law indices	0.0025	—
Parametric	5	Power-law indices	0.003	—
Blind	4	SMICA	—	0.00055

Table 4: The forecast 1σ uncertainties on the tensor-to-scalar ratio for fiducial model $r = 0.01$, assuming other cosmological parameters are known, for a set of foreground assumptions. The 'Fisher' and 'Blind' tests include a residual noise signal from lensing. The 'Parametric' tests ignore lensing as a contaminant, a poor approximation at $\ell > 15$.

Component separation forecasts

- Katayama & Komatsu, ApJ (2011)
  - Even for the simplest three-frequency configuration with 60, 100, and 240 GHz, the residual bias in r is as small as **$\Delta r \approx 0.002$**. This bias is dominated by the residual synchrotron emission due to spatial variations of the synchrotron spectral index. With an extended mask with $f_{sky} = 0.5$, the bias is reduced further down to **< 0.001**.

- Bonaldi & Ricciardi, MNRAS (2011)
  - For 'conservative' errors in the spectral indices, the best results are obtained with the restricted set of channels. **B-mode polarization can be detected down to r = 0.01** even with the 'conservative' errors in the spectral indices. The minimum value of r that can be reached depends on our ability to estimate the mixing matrix.

- Errard & Stompor, (2012)
  - We have found that though the **foreground residuals** are likely to be a major driver in defining the sensitivity requirements for such experiments, they **do not** on their own **lead to any fundamental lower limits on detectable r** , at least as long as sufficiently precise frequency scaling models are available.

How much can we trust component separation forecasts?

Jacques Delabrouille

Laboratoire APC, Paris

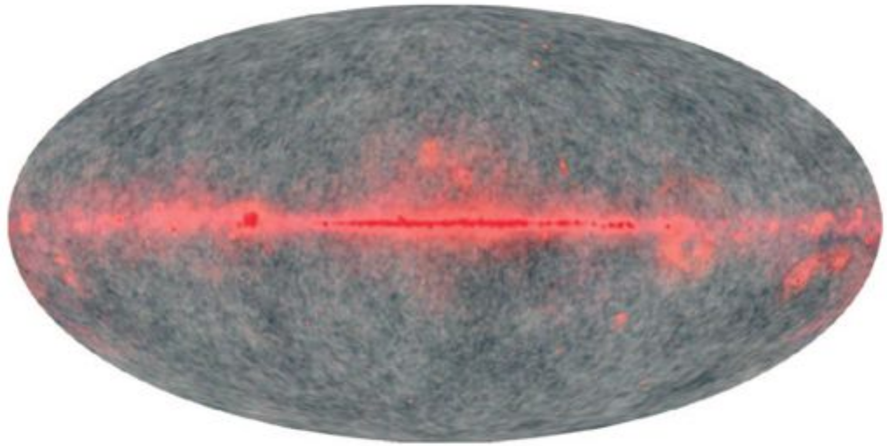
Outline

- The component separation problem
- Key ideas
- Lessons learnt from temperature
- WMAP polarisation
- Conclusion

CMB contamination by foregrounds

- The sky emission, at a given frequency, is a superposition of emissions from different sources
  - Different emission processes (thermal, synchrotron, Bremsstrahlung, …)
  - Different media/objects (Milky way ISM, CMB, clusters of galaxies)
- Has always been an issue, since early measurements of CMB anisotropies!

Multi frequency observations allow us to check that observed anisotropies have the correct emission law



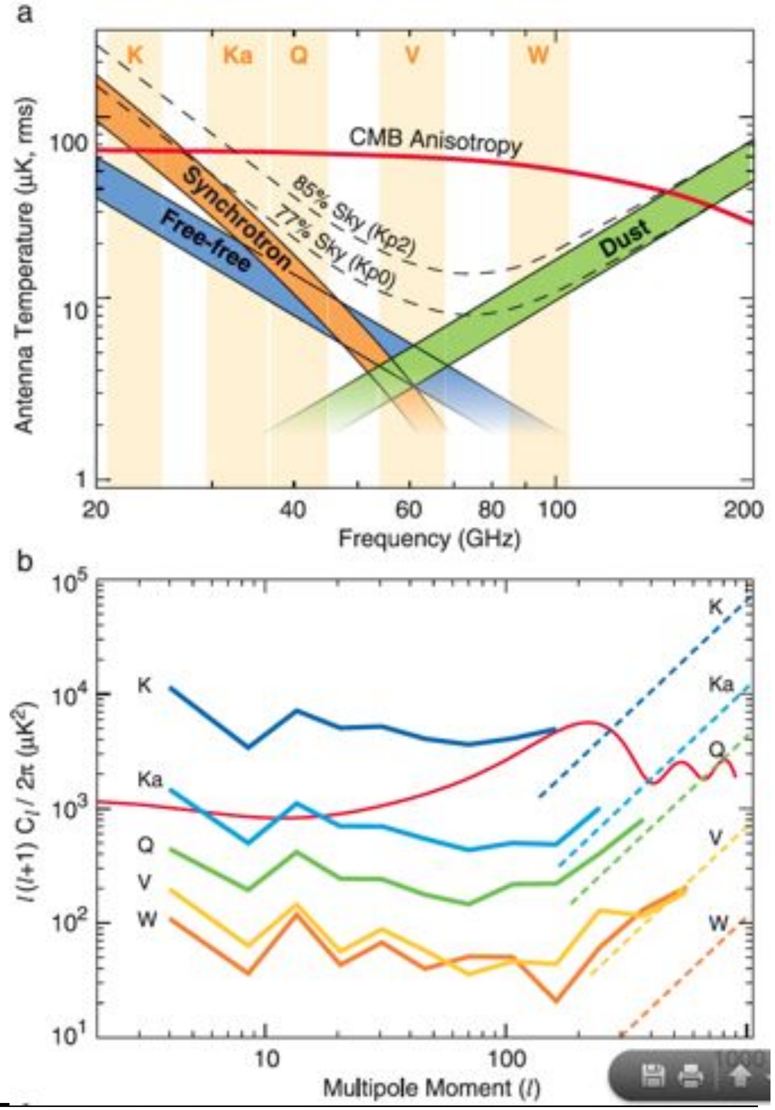
Bennett et al., ApJSS Volume 148, Issue 1, pp.97-117 (2003)

The ultimate frontier

- Astrophysical confusion is the ultimate frontier when instrumental noise becomes vanishingly small…

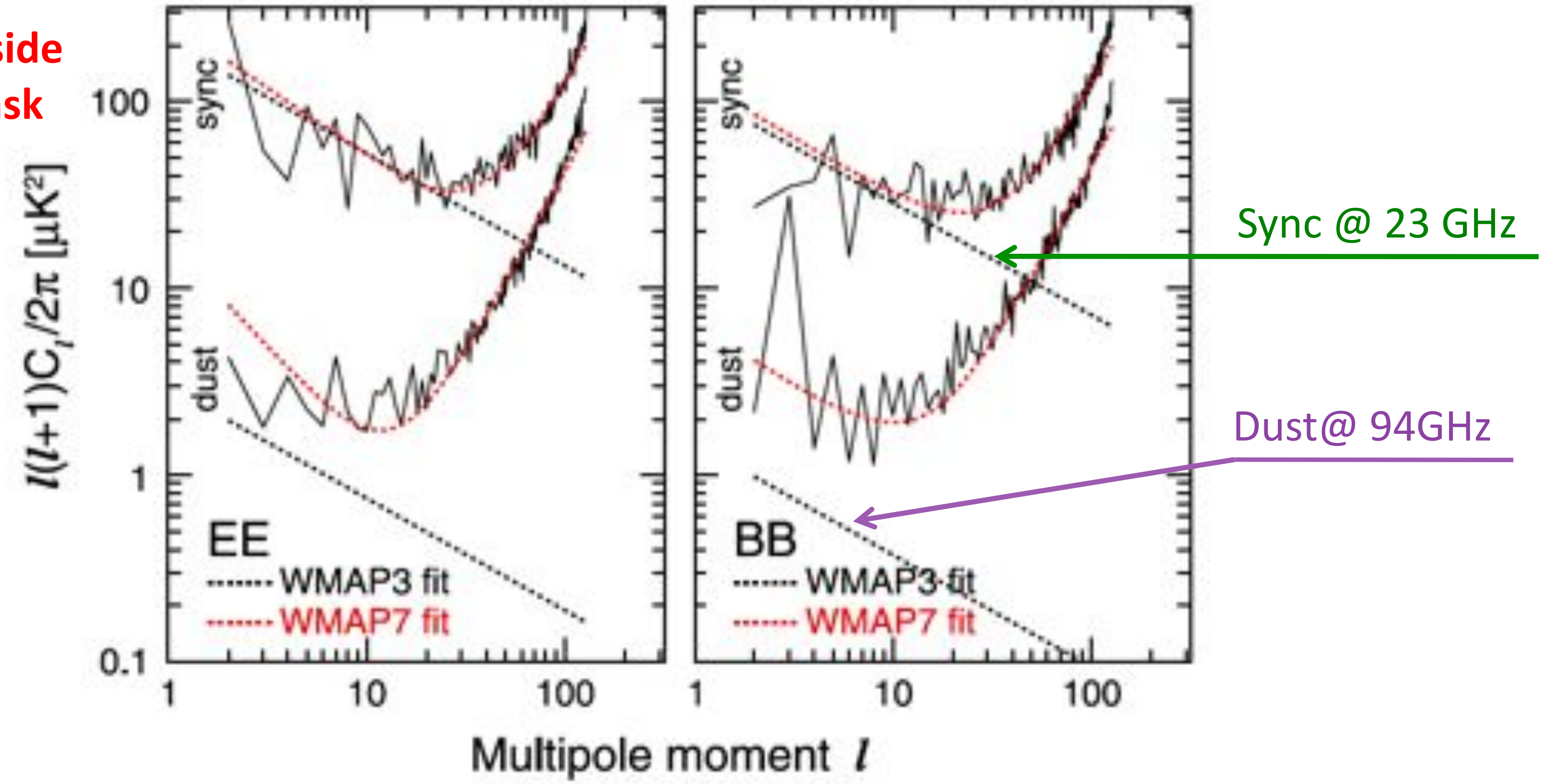
- Foreground level with WMAP kp2 mask as compared to CMB temperature anisotropies

Bennett et al., ApJS Volume 148, Issue 1, pp.97-117 (2003)



The ultimate frontier

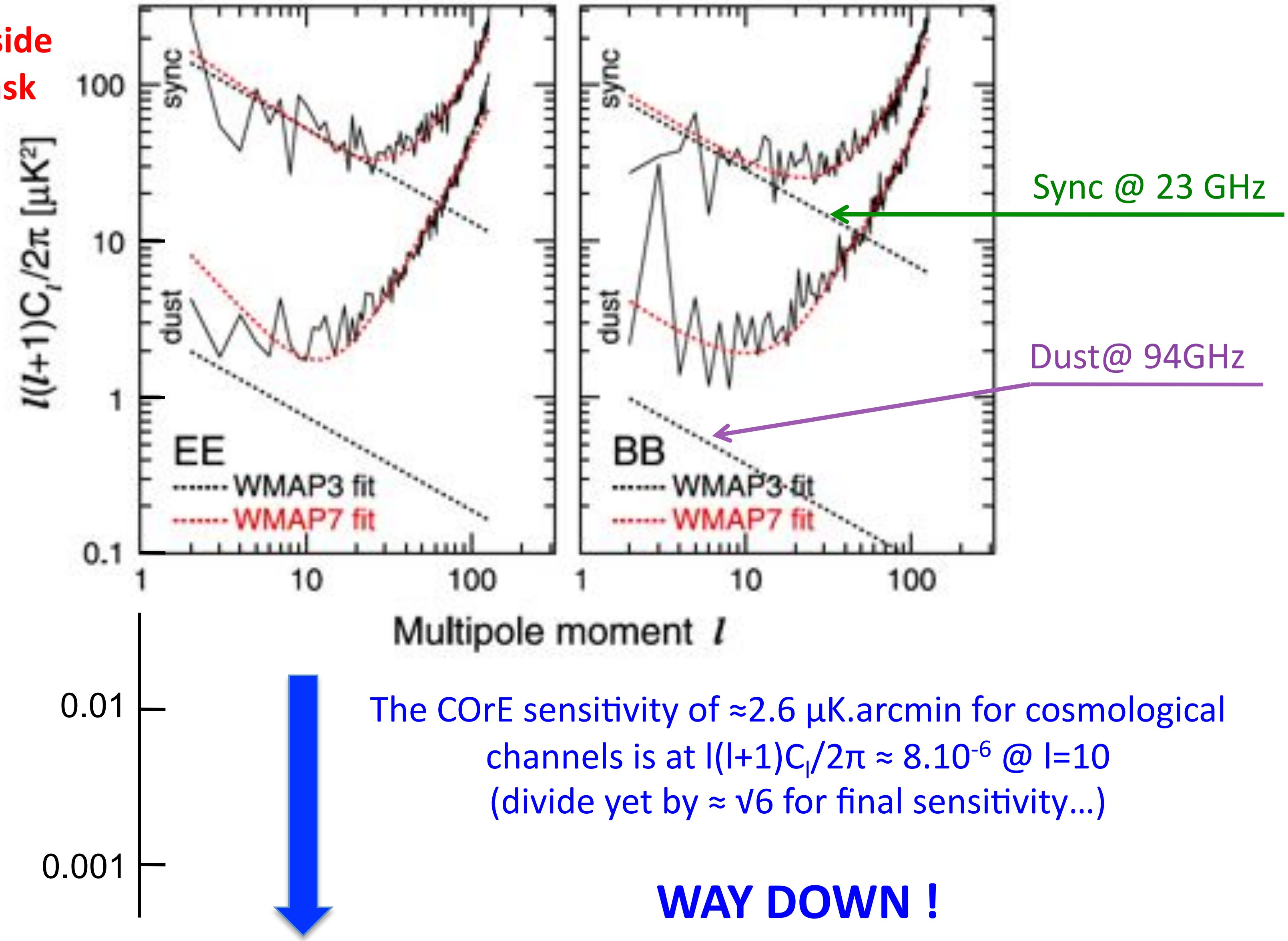
Spectra outside the P06 mask



Gold et al., ApJS Volume 192, Issue 2, article 15 (2012)

The ultimate frontier

Spectra outside the P06 mask



The COrE sensitivity of ≈2.6 μK.arcmin for cosmological channels is at $l(l+1)C_l/2\pi \approx 8.10^{-6}$ @ l=10 (divide yet by ≈ √6 for final sensitivity…)

WAY DOWN !

Outline

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Basic idea for foreground monitoring

Consider a single pixel p. The signal observed at frequency ν, in pixel p, is

$$x(\nu, p) = \sum_i x_i(\nu, p) + n(\nu, p)$$
$$= \sum_i a_i(\nu, p)\, s_i(p) + n(\nu, p),$$

or, in vector-matrix format

$$\boldsymbol{x}(p) = \mathsf{A}(p)\, \boldsymbol{s}(p) + \boldsymbol{n}(p).$$

If we know the frequency dependence of each foreground in that particular pixel, then we can invert the system perfectly.

A ?

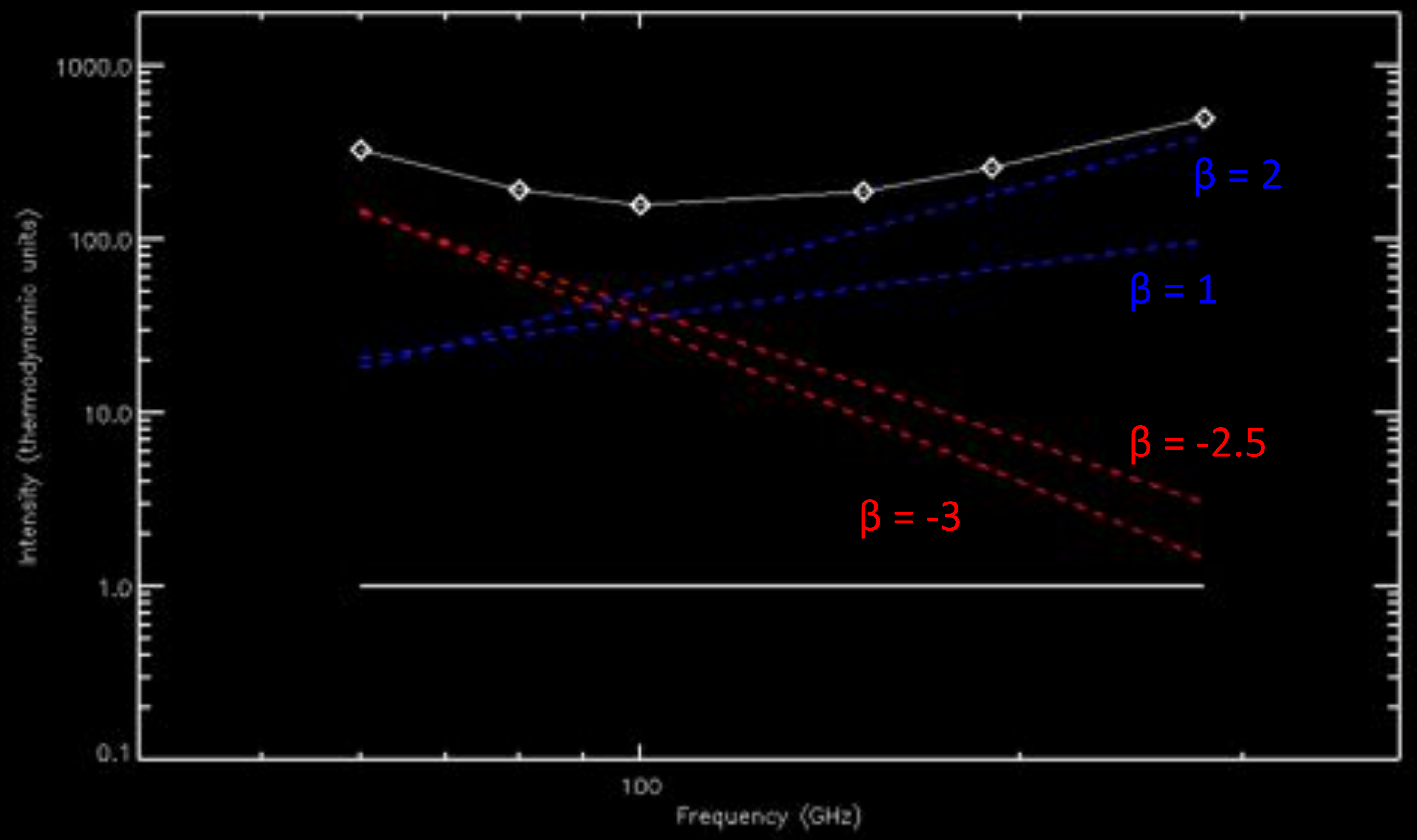
Methods that do use information about A

- Known a priori
  - Not a reasonable option for sensitive polarisation measurements
- Measured from nearby bands ?
- Fitted pixel by pixel
  - Need to trust your model !
- Estimated blindly from statistical independence
  - Need to compute statistics on regions with sufficient number of independent modes

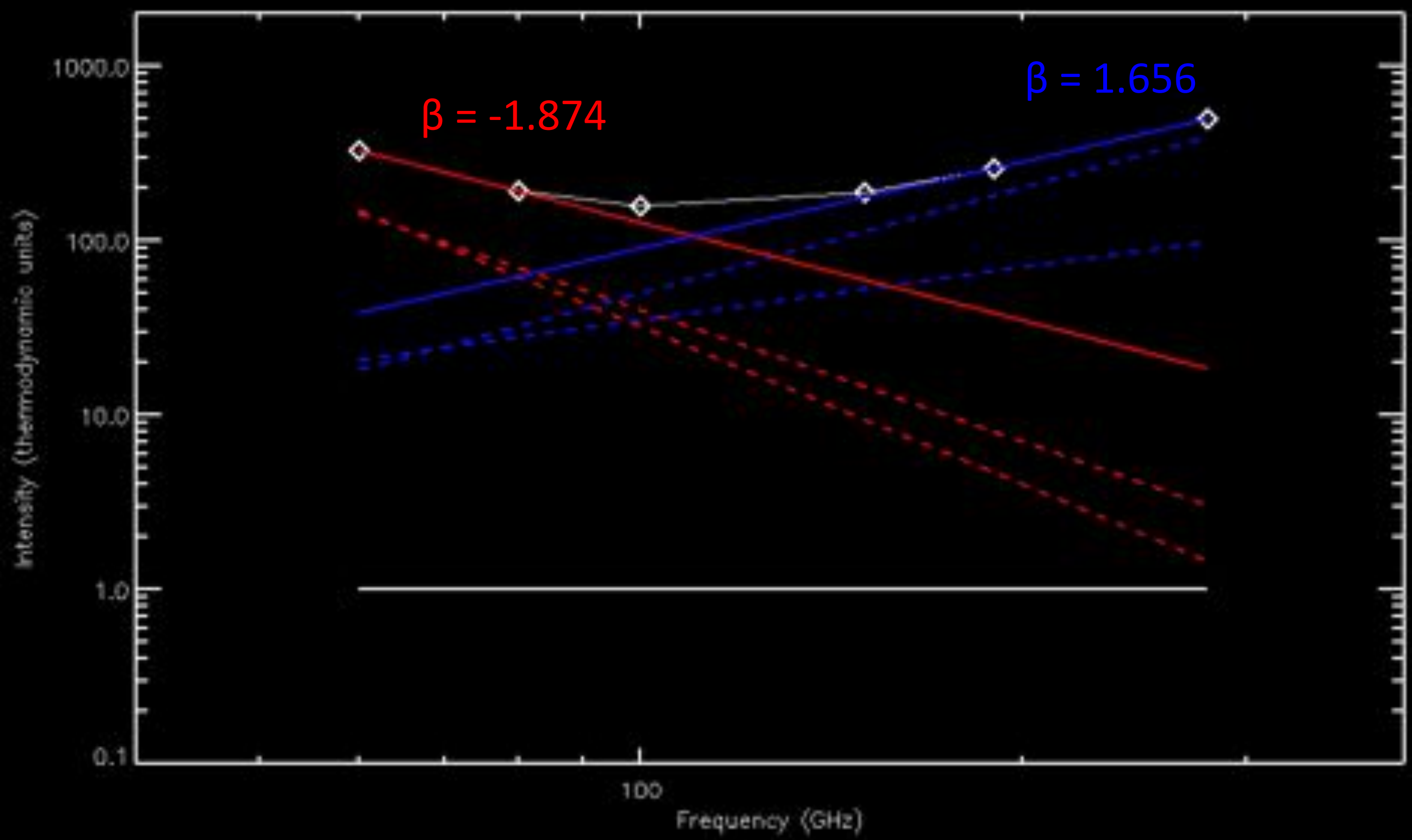
Methods that don’t (at least not directly)

- Template decorrelation (or fitting)
- Internal linear combination

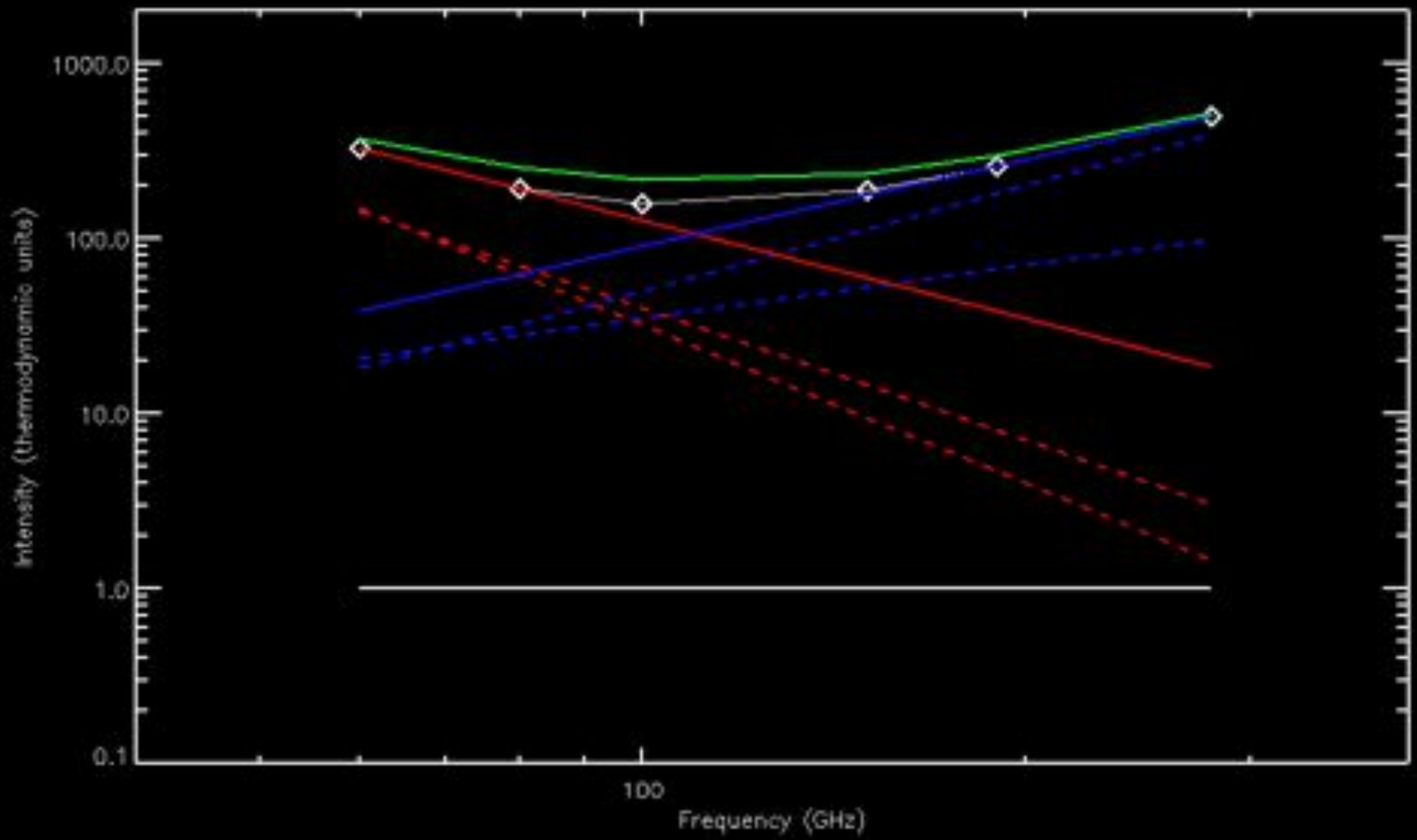
Using nearby bands?

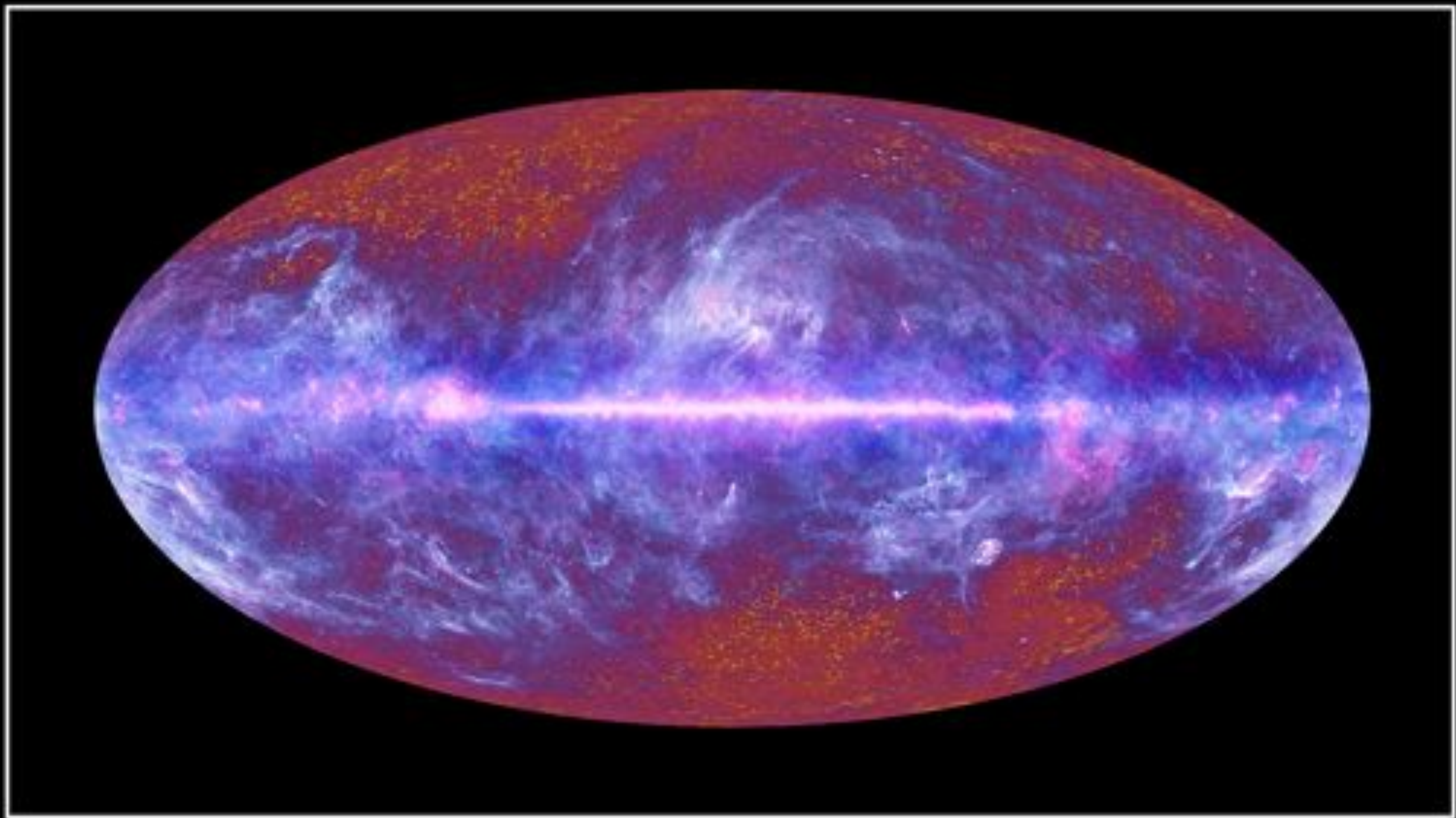


Using nearby bands?



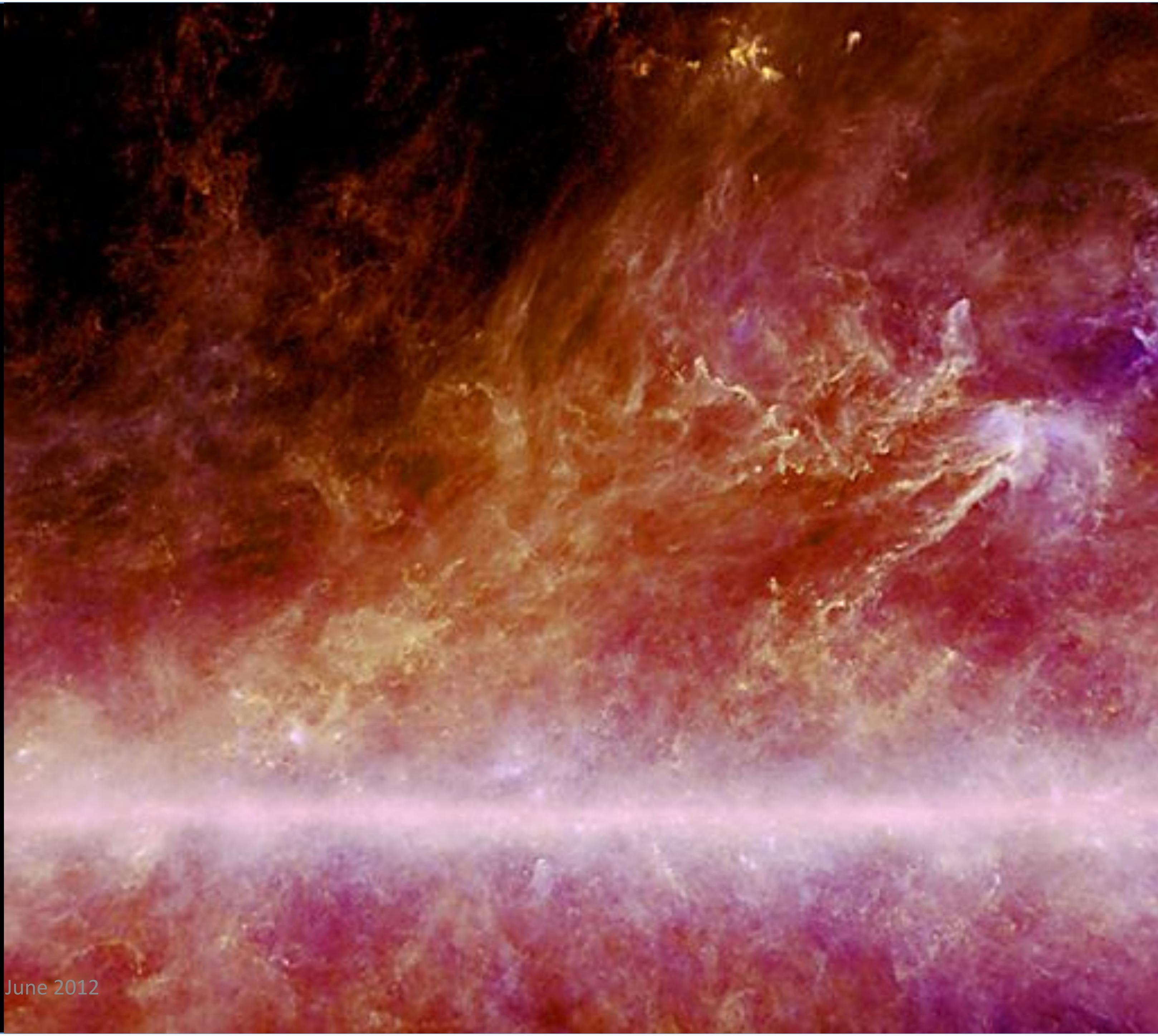
Using nearby bands?





The Planck one-year all-sky survey

esa



Least Square linear combination

Suppose that we know only the frequency dependence of one component of interest,

$$x(\nu, p) = a(\nu)\, s(p) + r(\nu, p)$$

Then the 'optimal' reconstructed signal by linear combination of the input maps is

$$\widehat{s}(p) = \frac{\boldsymbol{a}^t \mathsf{R}_r^{-1}}{\boldsymbol{a}^t \mathsf{R}_r^{-1} \boldsymbol{a}} \boldsymbol{x}(p)$$

Least Square linear combination

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$$\widehat{s}(p) = \frac{a^t R_r^{-1}}{a^t R_r^{-1} a} x(p) = \frac{a^t R_x^{-1}}{a^t R_x^{-1} a} x(p)$$

Woodbury matrix identity and consequences

$$\mathsf{R}_x^{-1} = \left[\boldsymbol{a}\boldsymbol{a}^t\sigma^2_{\rm cmb} + \mathsf{R}_n\right]^{-1}$$

$$= \mathsf{R}_n^{-1} - \sigma^2_{\rm cmb}\frac{\mathsf{R}_n^{-1}\boldsymbol{a}\boldsymbol{a}^t\mathsf{R}_n^{-1}}{1+\sigma^2_{\rm cmb}\boldsymbol{a}^t\mathsf{R}_n^{-1}\boldsymbol{a}}$$

$$\boldsymbol{a}^t\mathsf{R}_x^{-1} = \boldsymbol{a}^t\mathsf{R}_n^{-1} - \sigma^2_{\rm cmb}\frac{\boldsymbol{a}^t\mathsf{R}_n^{-1}\boldsymbol{a}\;\boldsymbol{a}^t\mathsf{R}_n^{-1}}{1+\sigma^2_{\rm cmb}\boldsymbol{a}^t\mathsf{R}_n^{-1}\boldsymbol{a}}$$

$$\boldsymbol{a}^t\mathsf{R}_x^{-1} \propto \boldsymbol{a}^t\mathsf{R}_n^{-1}$$

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Implemented by an ILC, using statistics computed on the data themselves

Least Square linear combination

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May depend on p

Obvious natural solution

Needlet ILC

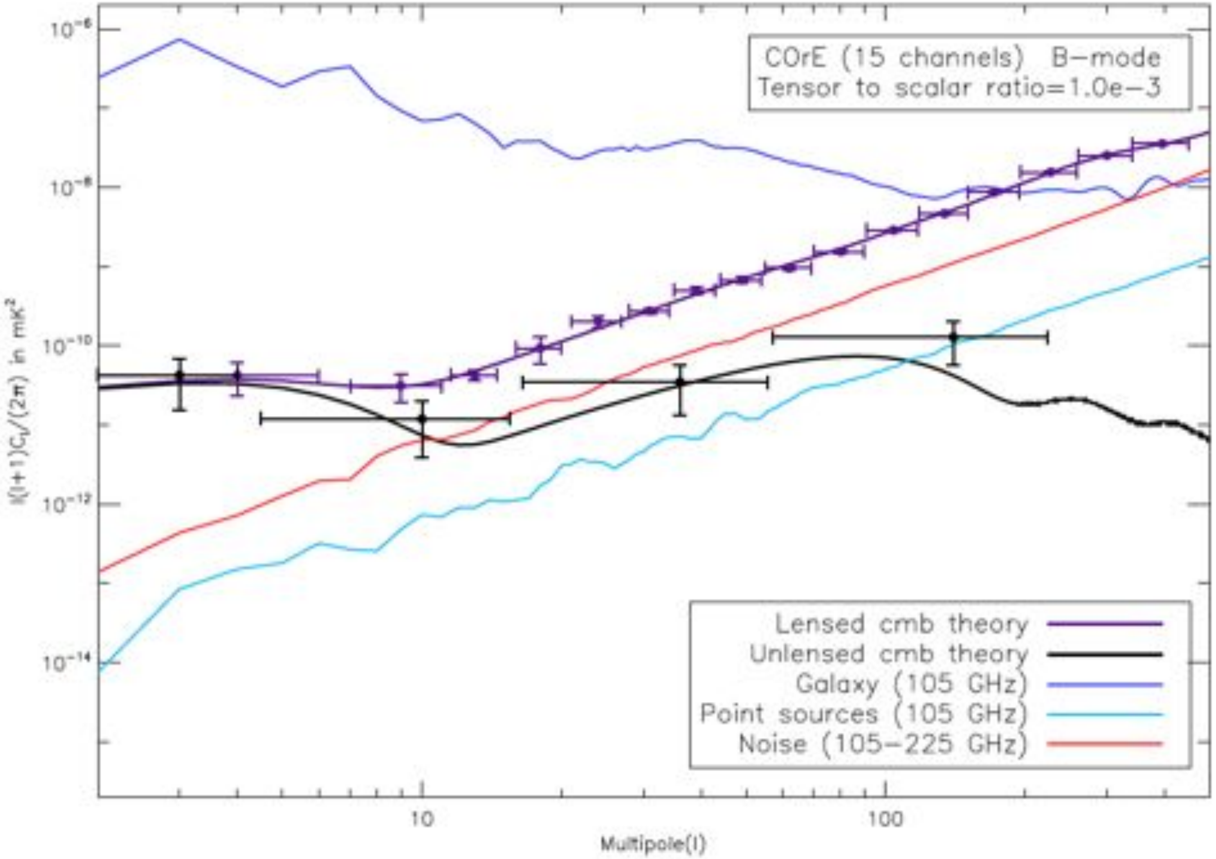
Used on WMAP data →

Delabrouille et al., A&A 493, 835 (2009)
Basak & Delabrouille, MNRAS, 419, 1163 (2012)
Basak & Delabrouille, arXiv 1204:0292

Used for CMB subtraction in Planck
Used for COrE proposal and white paper

How do we know that it works ?

Solution 1:
Simulations



Obvious caveat: are the simulations representative ?

How do we know that it works ?

Solution 2: The dimension of the signal subspace

CMB Foregrounds Noise

$$x(\nu, p) = s(p) + u(\nu, p) + n(\nu, p)$$

We suppose that the three terms are pairwise decorrelated. Then we have

$$\mathrm{R}_x = \mathrm{R}_s + \mathrm{R}_u + \mathrm{R}_n$$

We suppose that R_n is known. Then we can whiten the data by multiplication by $\mathrm{R}_n^{-1/2}$. For the new data set,

$$\mathrm{R}_x = \mathrm{R}_s + \mathrm{R}_u + \mathrm{Id}$$

We can diagonalise R_x. In the basis of diagonalisation, the covariance becomes

$$\mathrm{R}_x = \Delta + \mathrm{Id}$$

Remazeilles, Delabrouille & Cardoso, MNRAS 418, 467 (2011)

How do we know that it works ?

Solution 2: The dimension of the signal subspace

CMB — Foregrounds — Noise

$$x(\nu, p) = s(p) + u(\nu, p) + n(\nu, p)$$

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$$\mathrm{R}_x = \Delta + \mathrm{Id}$$

$$\begin{pmatrix} 1+\delta_1 & & & \\ & 1+\delta_2 & & \\ & & \ddots & \\ & & & 1+\delta_n \end{pmatrix} \longleftarrow \mathrm{R}_x = \Delta + \mathrm{Id}$$

Remazeilles, Delabrouille & Cardoso, MNRAS 418, 467 (2011)

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- → Lessons learnt from temperature
- WMAP polarisation
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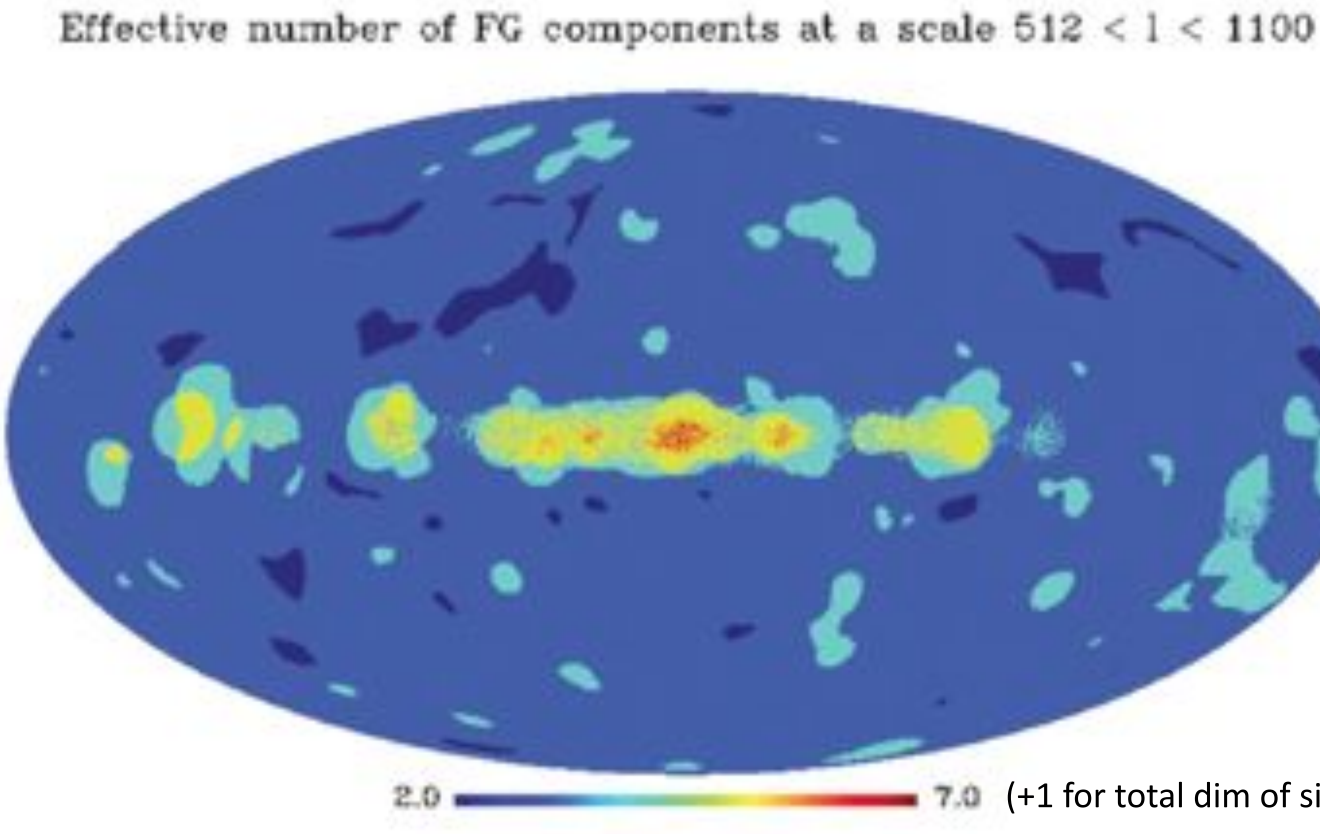
Rank of the signal subspace: example



(+1 for total dim of signal subspace)

We know a posteriori where to mask !

Remazeilles, Delabrouille & Cardoso, MNRAS 418, 467 (2011)

The Planck WG2 experience

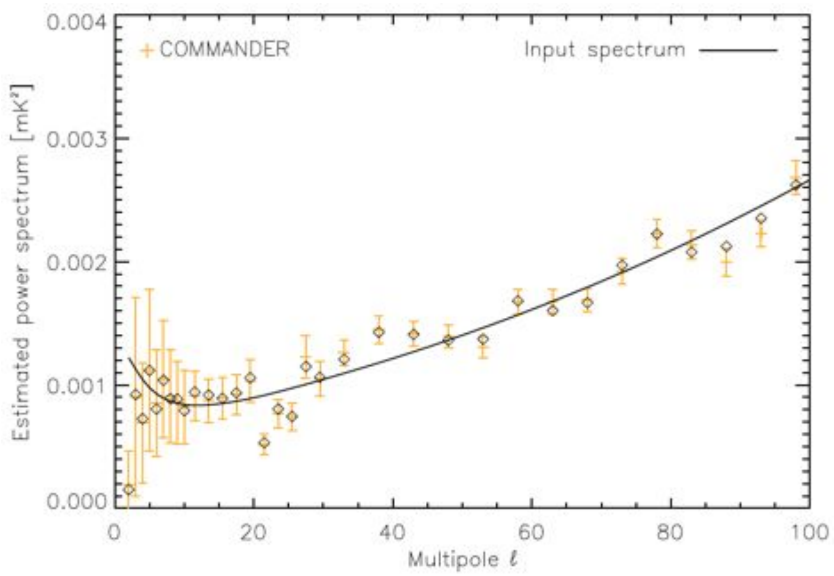
- A specific working group for
  - Development, comparison and validation of component separation methods
  - Collection of external data sets for component separation
  - Sky simulation and modelling (development of the Planck Sky Model)

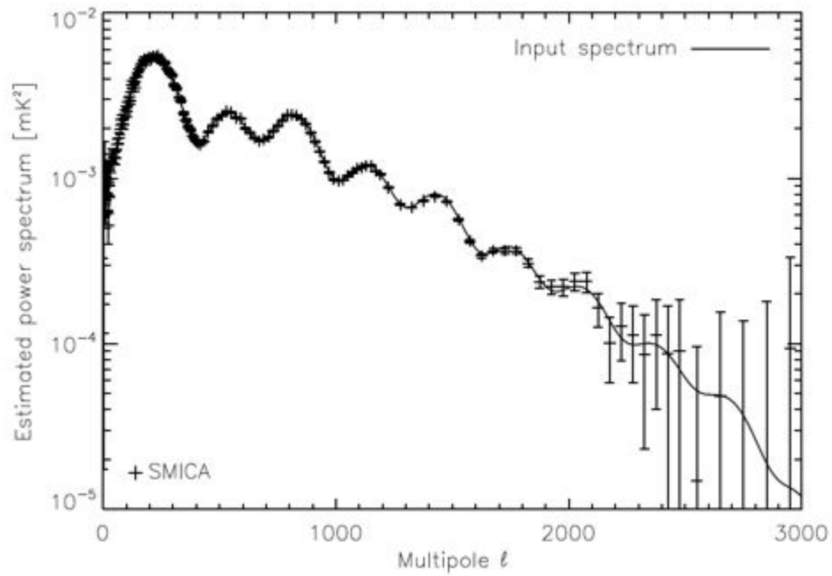
- Temperature challenge
  - Objectives: CMB power spectrum and maps
  - Catalogues of point sources and galaxy clusters
  - Maps of diffuse galactic emission
  - Paper by Leach et al. A&A Volume 491, Issue 2, pp 597-615 (2008)

- Polarisation challenge
  - Technically, most methods which work for T can also work for E or B
  - Main uncertainty: the polarised sky model (not representative enough)

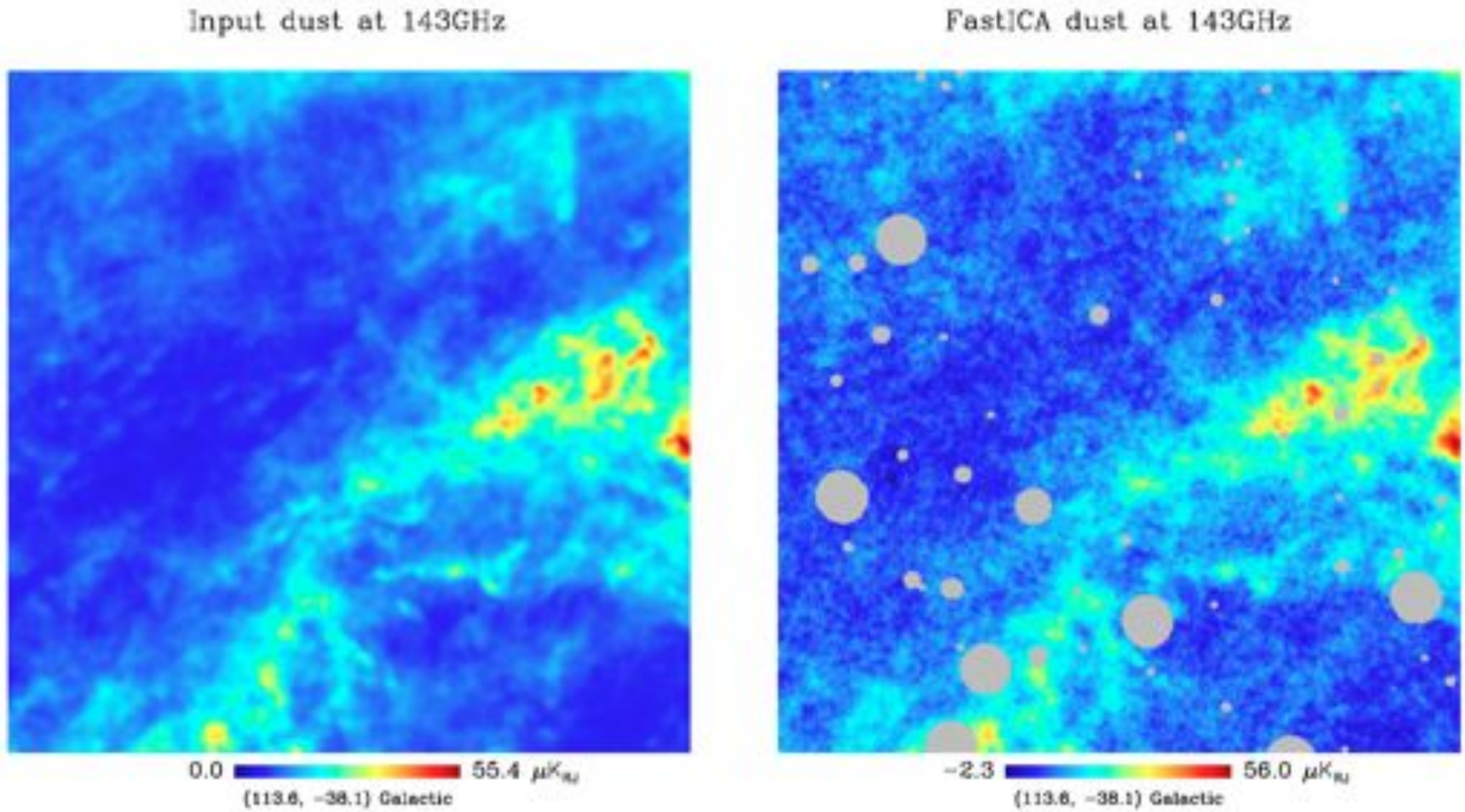
Temperature challenge

- 9 different methods tested and compared
- No single method performs best for all purposes!
- Differences can be fundamental, or due to implementation details
- All methods have been improved during the challenge

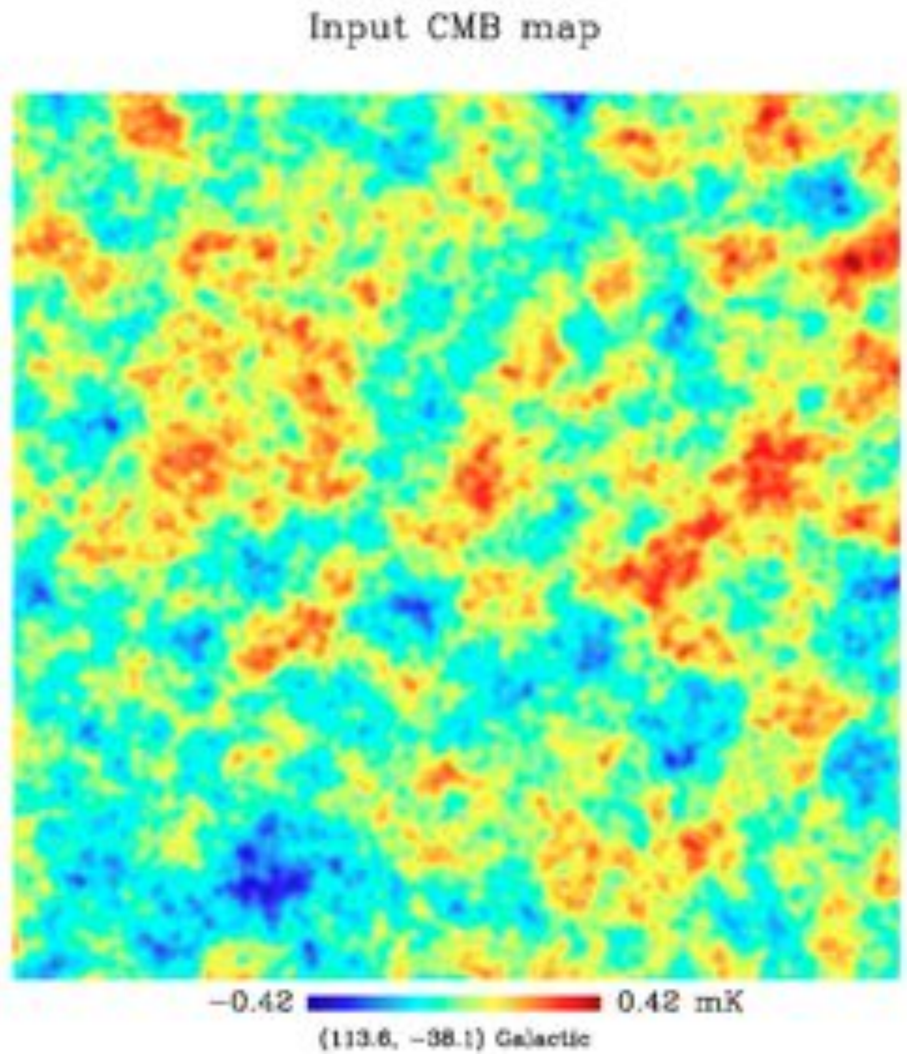


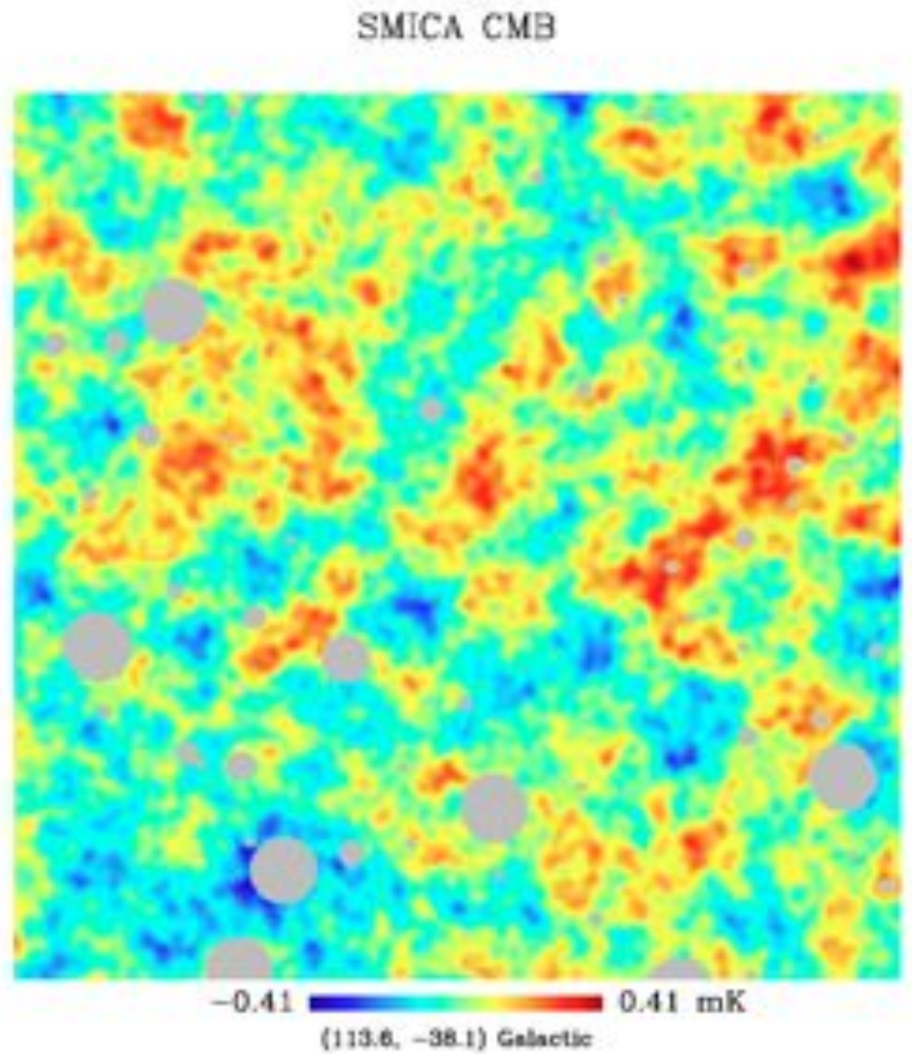


Reconstructed dust map

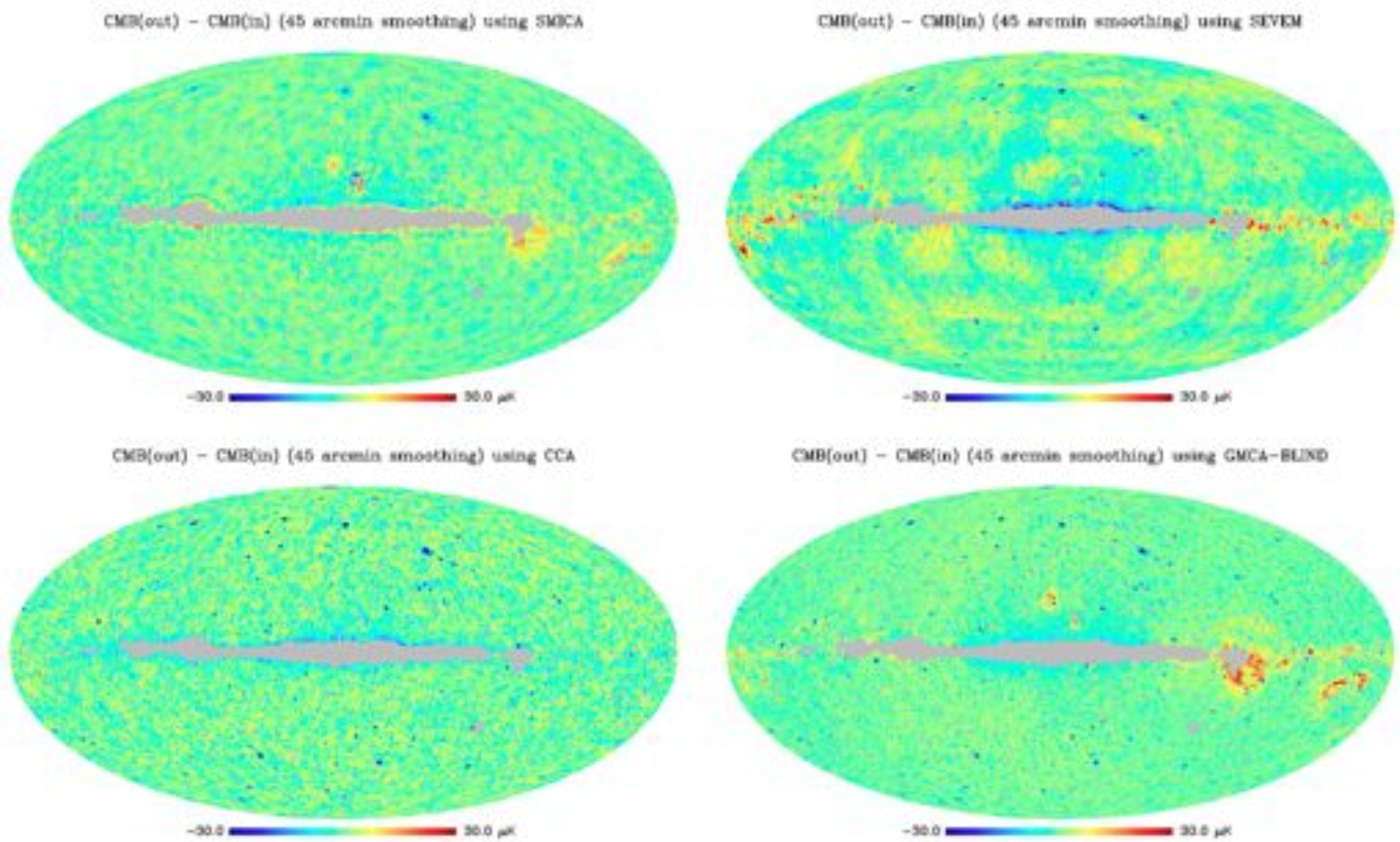


Reconstructed CMB map

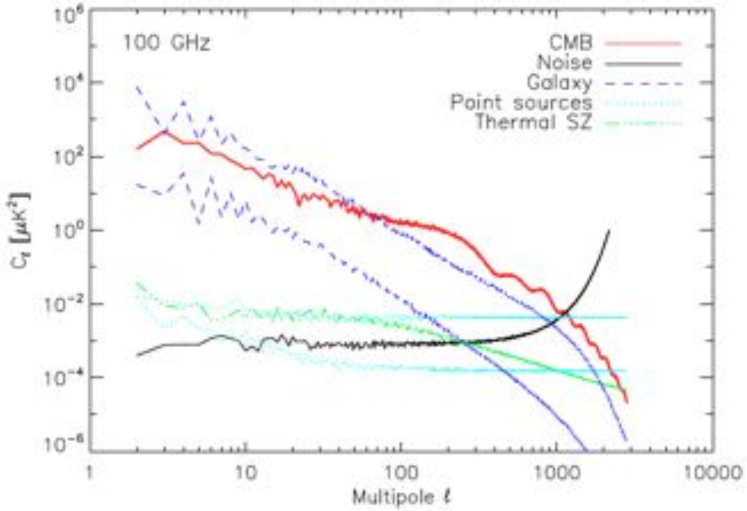




CMB reconstruction error maps

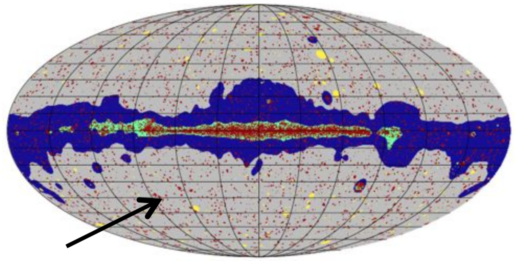


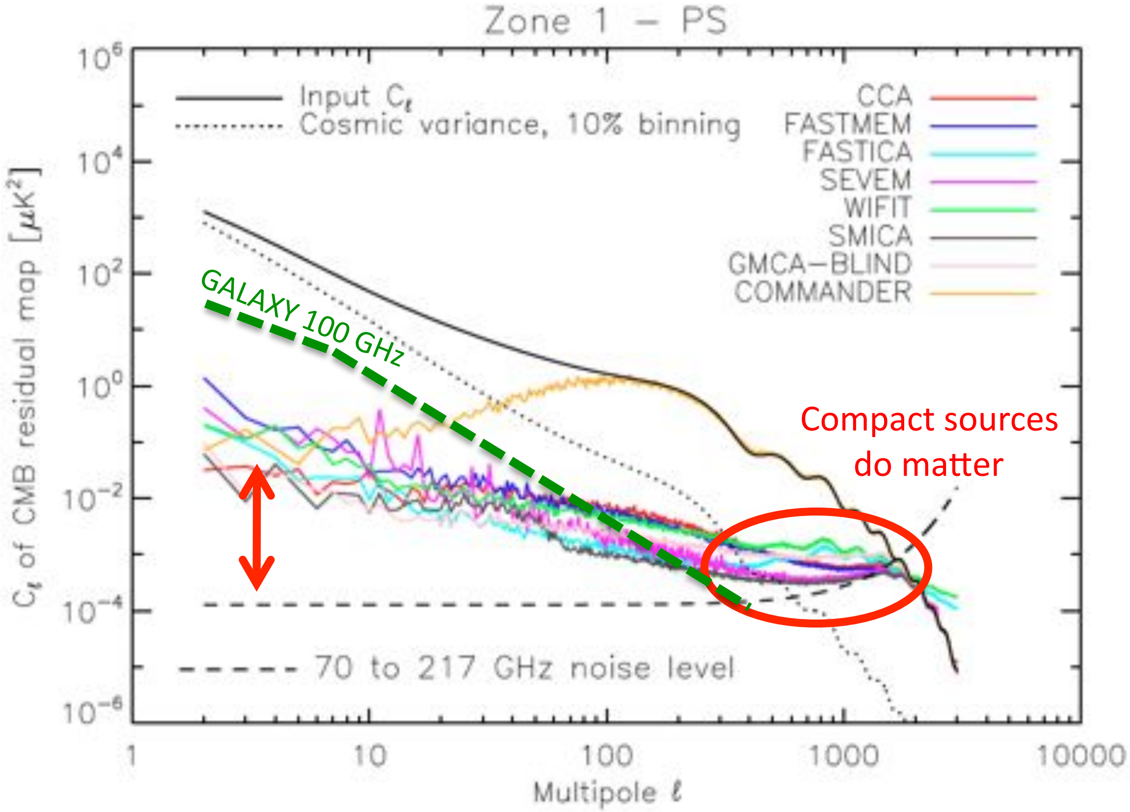
Foreground residuals



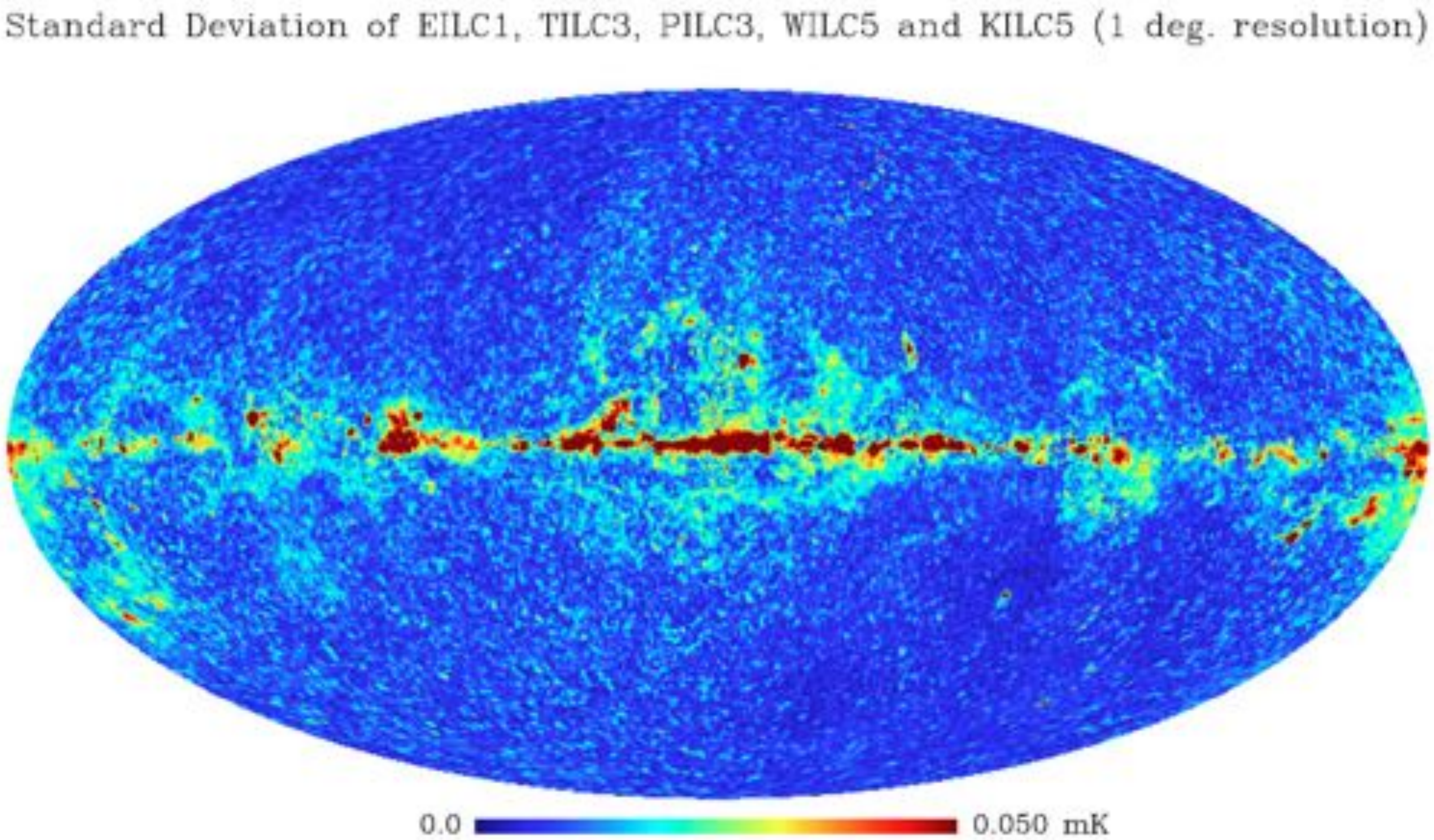
Large scale residuals
two orders of magnitude
above noise

Small scale residuals
above noise and
cosmic variance

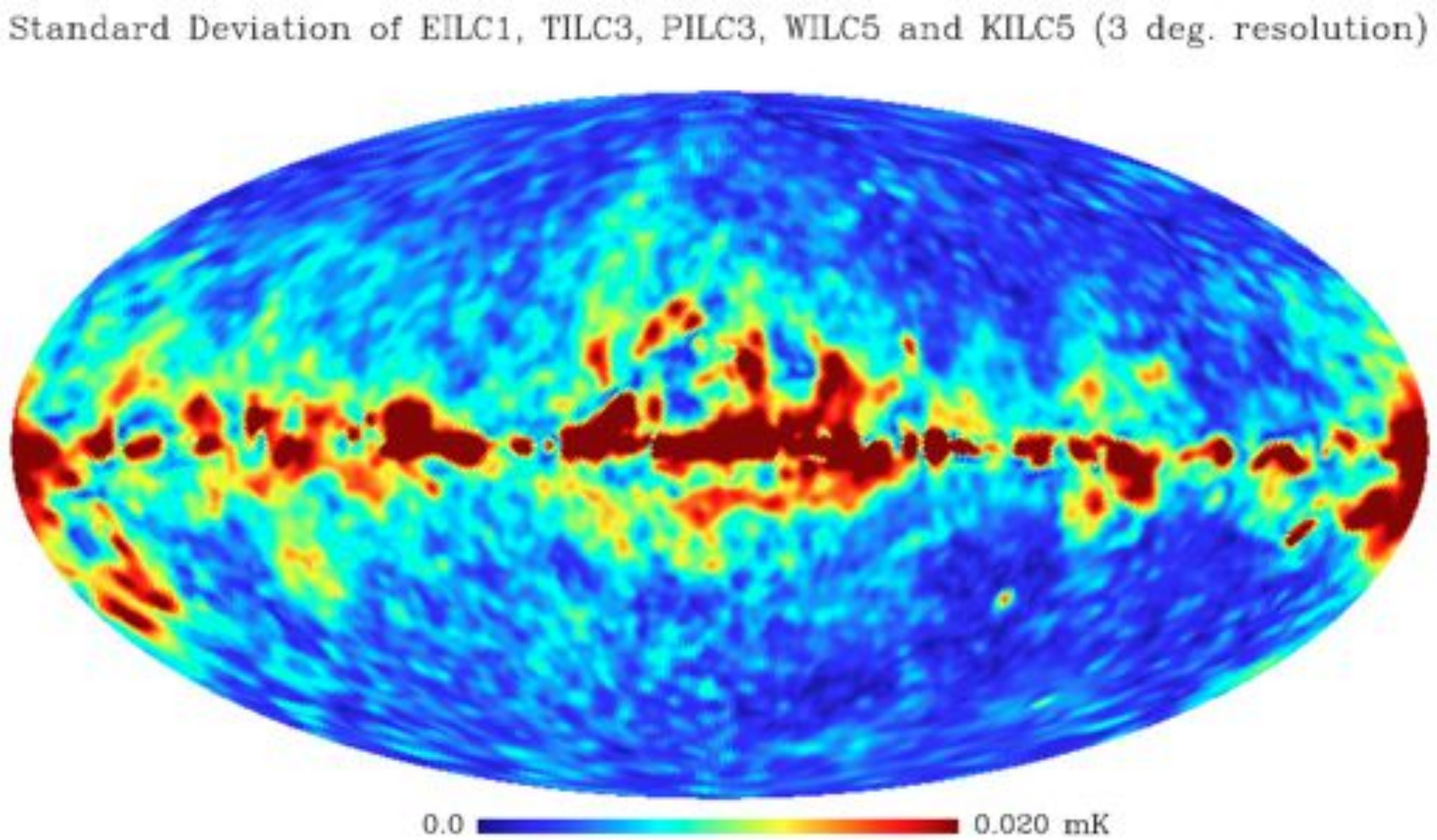




WMAP ILCs


Standard Deviation of EILC1, TILC3, PILC3, WILC5 and KILC5 (1 deg. resolution)
0.0
0.050 mK

WMAP ILCs


Standard Deviation of EILC1, TILC3, PILC3, WILC5 and KILC5 (3 deg. resolution)
0.0
0.020 mK

Outline

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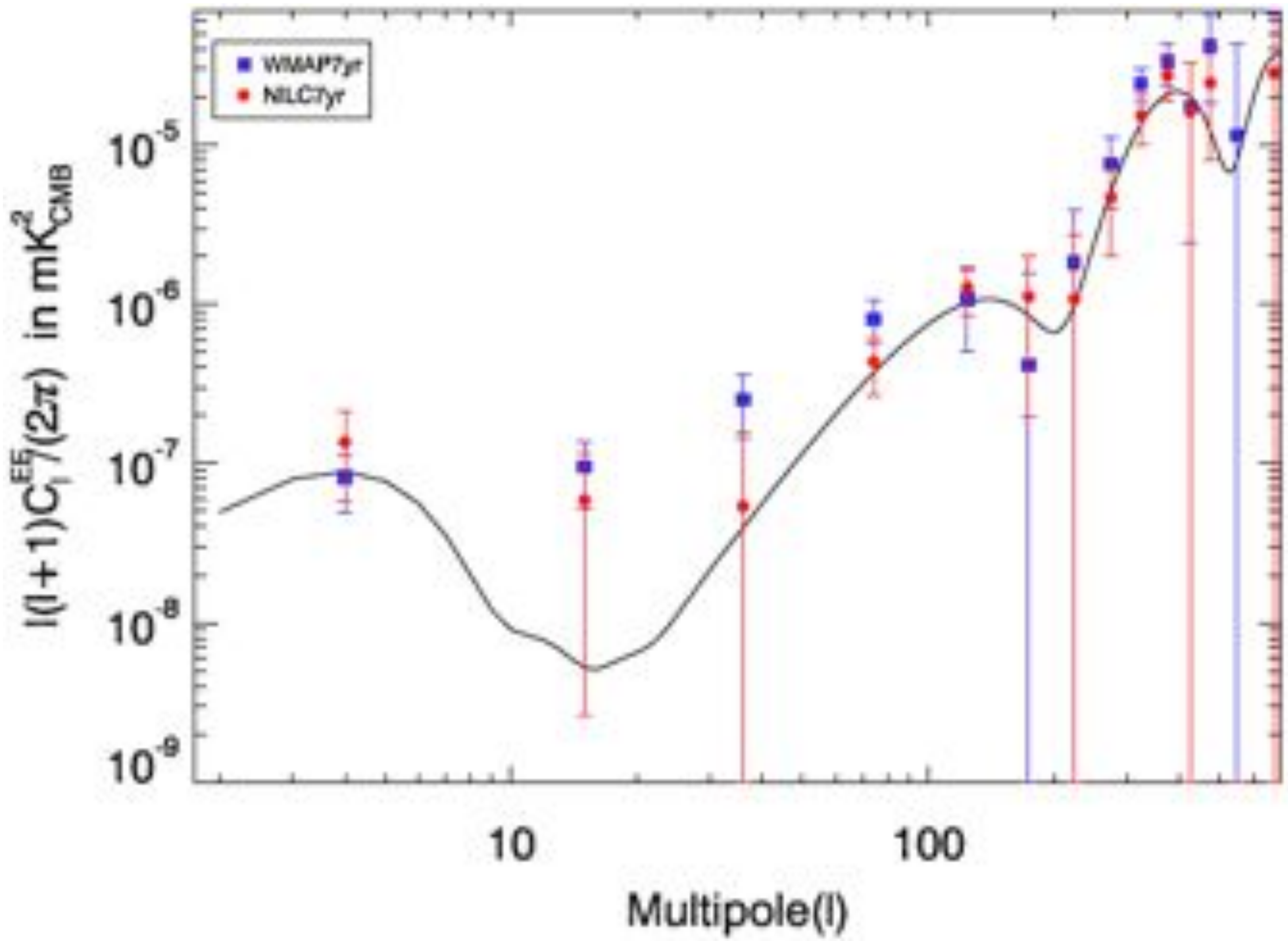
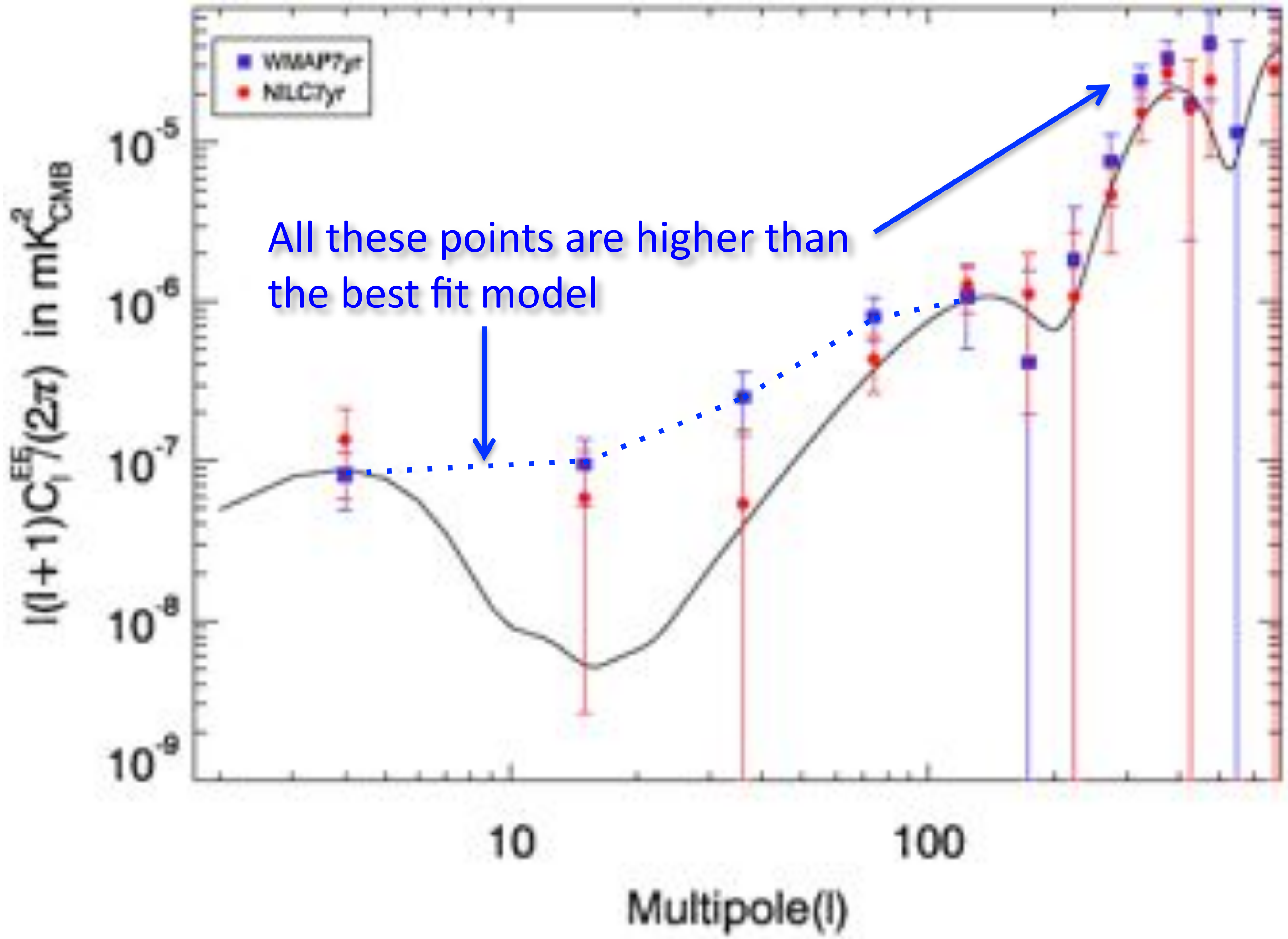
WMAP - C^{EE}



WMAP7yr
NILC7yr
10^-5
10^-6
10^-7
10^-8
10^-9
l(l+1)C_l^EE/(2π) in mK²_CMB
10
100
Multipole(l)

WMAP - C^{EE}


WMAP7yr
NILC7yr
All these points are higher than
the best fit model
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$l(l+1)C_l^{EE}/(2\pi)$ in mK^2_{CMB}
10
100
Multipole(l)

Conclusions

- A forecast can be trusted about as much as the model of sky emission it assumes …
- unless the method does not use take advantage of the simplicity of the model.

- Models assumed for most forecasts are not very reliable…
- however, some of the mechanisms that make component separation possible are well understood.

- Evaluation of performance is possible on simulations, or on real data a posteriori.

- ~~For a mission with enough channels and sensitivity, component separation to measure r cannot really fail~~. Well OK, let's moderate this: There are good reasons to be optimistic.

- Fine tuning the number and location of channels is possible and necessary. It should be done for next answer, in particular in the light of Planck results,
- but it is already clear that less bands than ≈15 is not really reasonable for COrE.

- Think seriously of all the science we will dream of doing with COrE data when it is available, better with as much imagination as possible !

- Simulations are important to understand how and why things work – or don't !